Adv. Alg. With Trig, Glawe **Name:** ______________________________ **Date:** __________ **Pd:**______

Polynomial Long Division Notes (5.5)

So far we have learned how to add, subtract, and multiply polynomials. We can also divide polynomials.

When you divide a polynomial $f(x)$ by a **divisor** $\boldsymbol{d(x)}$, you get a **quotient polynomial** $\boldsymbol{q(x)}$ and a **remainder polynomial** $\boldsymbol{r(x)}$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the remainder must be less than the degree of the divisor. One way to divide polynomials is called **polynomial long division.**

Before we look at an example of how to use polynomial long division, we will first look at an example how to divide two numbers using long division.

Refresher Example: Divide 965 by 5 using long division.

$$\begin{array}{r}
193 \\
5\,\overline{)\,965} \\
\underline{-5} \\
46 \\
\underline{-45} \\
15 \\
\underline{-15} \\
0
\end{array}$$

Divisor: 5
Quotient: 193
Dividend: 965
Remainder: 0

How many times does 5 go into 9?

Multiply 1 by 5

Subtract 5 from 9

Bring down 6

How many times does 5 go into 46?

Multiply 9 by 5

Subtract 45 from 46

Bring down 5

How many times does 5 go into 15?

Multiply 3 by 5

Subtract 15 from 15

Example: **Divide $\boldsymbol{f(x) = 3x^4 - 5x^3 + 4x - 6}$ by $\boldsymbol{x^2 - 3x + 5}$.**

Write polynomial division in the same format you use when dividing numbers. Include a "0" as the coefficient of x^2in the dividend (any of the missing terms). At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor. This gives the next term of the quotient.

$$\begin{array}{rrrrrl}
 & & 3x^2 + & 4x - & 3 & \leftarrow \text{quotient} \\
x^2 - 3x + 5\,\big) & 3x^4 - & 5x^3 + & 0x^2 + & 4x - & 6 \\
 & \underline{3x^4 -} & \underline{9x^3 +} & \underline{15x^2} & & \quad \text{Multiply divisor by } 3x^4/x^2 = 3x^2. \\
 & & 4x^3 - & 15x^2 + & 4x & \quad \text{Subtract. Bring down next term.} \\
 & & \underline{4x^3 -} & \underline{12x^2 +} & \underline{20x} & \quad \text{Multiply divisor by } 4x^3/x^2 = 4x. \\
 & & & -3x^2 - & 16x - & 6 \quad \text{Subtract. Bring down next term.} \\
 & & & \underline{-3x^2 +} & \underline{9x -} & \underline{15} \quad \text{Multiply divisor by } -3x^2/x^2 = -3. \\
 & & & & -25x + & 9 \leftarrow \text{remainder}
\end{array}$$

Answer: $3x^2 + 4x - 3 + \dfrac{-25x+9}{x^2-3x+5}$ $\left(q(x) + \dfrac{r(x)}{d(x)}\right)$

You can always check your answer by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$
\begin{aligned}
&(3x^2 + 4x - 3)(x^2 - 3x + 5) + (-25x + 9) \\
&\quad = 3x^2(x^2 - 3x + 5) + 4x(x^2 - 3x + 5) - 3(x^2 - 3x + 5) - 25x + 9 \\
&\quad = 3x^4 - 9x^3 + 15x^2 + 4x^3 - 12x^2 + 20x - 3x^2 + 9x - 15 - 25x + 9 \\
&\quad = 3x^4 - 5x^3 + 4x - 6 \checkmark
\end{aligned}
$$

Example 2: **Divide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.**

$$
\begin{array}{r l}
x^2 + 7x + 7 & \leftarrow \text{quotient} \\
x - 2 \,\overline{)\, x^3 + 5x^2 - 7x + 2} & \\
\underline{x^3 - 2x^2} \qquad\qquad & \text{Multiply divisor by } x^3/x = x^2. \\
7x^2 - 7x \qquad & \text{Subtract.} \\
\underline{7x^2 - 14x} \qquad & \text{Multiply divisor by } 7x^2/x = 7x. \\
7x + 2 & \text{Subtract.} \\
\underline{7x - 14} & \text{Multiply divisor by } 7x/x = 7. \\
16 & \leftarrow \text{remainder}
\end{array}
$$

Answer: $x^2 + 7x + 7 + \frac{16}{x-2}$

Homework: p. 366: 3-9 (odds)
Divide using polynomial long division.

3) $(x^2 + x - 17) \div (x - 4)$

5) $(x^3 + 3x^2 + 3x + 2) \div (x - 1)$

7) $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$

9) $(5x^4 - 2x^3 - 7x^2 - 39) \div (x^2 + 2x - 4)$