

8.4 Simplify and Multiply Rational Expressions Notes

A rational expression is in **simplified form** if its numerator and denominator have no common factors (other than ± 1). To simplify a rational expression, apply the following property.

Simplifying Rational Expressions:

Let a , b , and c be expressions with $b \neq 0$ and $c \neq 0$. Then the following property applies.

Property $\frac{ac}{bc} = \frac{a}{b}$ Divide out the common factor c .

Examples 1) $\frac{15}{65} = \frac{3 \cdot 5}{13 \cdot 5} = \frac{3}{13}$ Divide out the common factor of 5.

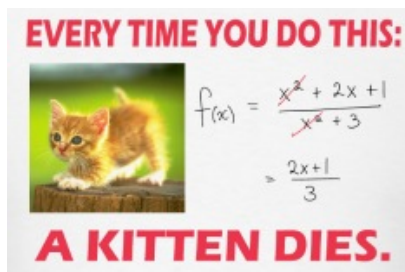
2) $\frac{4(x+3)}{(x-5)(x+3)} = \frac{4}{x-5}$ Divide out the common factor of $x+3$.

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any factors that are common to both the numerator and denominator.

Here is an example:

$$\frac{x^2 + 7x}{x^2} = \frac{x(x+7)}{x \cdot x} = \frac{x+7}{x}$$

Notice that you can divide out common factors in the second expression above. However, you cannot divide out like terms in the third expression.

Remember:

Simplify: $\frac{x^2 - 2x - 15}{x^2 - 9}$

$$\frac{x^2 - 2x - 15}{x^2 - 9} =$$

Factor numerator and denominator.

=

Divide out common factor.

=

Simplified form

Simplify the following expressions, if possible.

1) $\frac{2(x+1)}{(x+1)(x+3)}$

2) $\frac{40x+20}{10x+30}$

3) $\frac{4}{x(x+2)}$

$$4) \frac{x+4}{x^2-16}$$

$$5) \frac{x^2-2x-3}{x^2-x-6}$$

$$6) \frac{2x^2+10x}{3x^2+16x+5}$$

Multiplying Rational Expressions:

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write new fraction in simplified form.

Let a , b , c , and d be expressions with $b \neq 0$ and $d \neq 0$.

Property $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ Simplify $\frac{ac}{bd}$ if possible.

Example
$$\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{10 \cdot 3 \cdot x \cdot x^2 \cdot y^3}{10 \cdot 2 \cdot x \cdot y^3} = \frac{3x^2}{2}$$

1) Multiply: $\frac{x+2}{x^3-27} \cdot (x^2+3x+9)$

$$\frac{x+2}{x^3-27} \cdot (x^2+3x+9) =$$

Write polynomial as a rational expression.

$$=$$

Factor denominator.

$$=$$

Divide out common factor.

$$=$$

Simplified form

2) Multiply: $\frac{3x-3x^2}{x^2+4x-5} \cdot \frac{x^2+x-20}{3x}$

$$\frac{3x-3x^2}{x^2+4x-5} \cdot \frac{x^2+x-20}{3x} = \frac{3x(1-x)}{(x-1)(x+5)} \cdot \frac{(x+5)(x-4)}{3x}$$

Factor numerators and denominators.

$$= \frac{3x(1-x)(x+5)(x-4)}{(x-1)(x+5)(3x)}$$

Multiply numerators and denominators.

$$= \frac{3x(-1)(x-1)(x+5)(x-4)}{(x-1)(x+5)(3x)}$$

Rewrite $1-x$ as $(-1)(x-1)$.

$$= \frac{3x(-1)(x-1)(x+5)(x-4)}{(x-1)(x+5)(3x)}$$

Divide out common factors.

$$= -1(x-4)$$

Simplify.

$$= -x+4$$

Multiply.