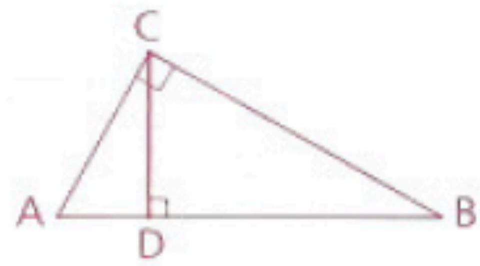
Homework

Read section about altitude-on-hypotenuse theorems in textbook

Part One: Introduction

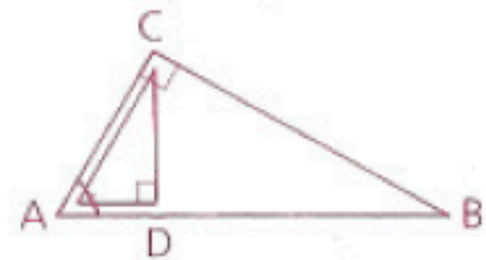
When altitude $\overline{CD}$ is drawn to the hypotenuse of $\triangle ABC$, three similar triangles are formed.

$$\triangle ABC \sim \triangle ACD \sim \triangle CBD$$



$\triangle ABC \sim \triangle ACD$ by AA, and we notice that

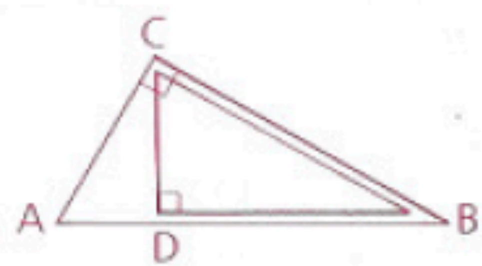
$$\frac{AB}{AC} = \frac{AC}{AD}, \text{ or } (AC)^2 = (AB)(AD)$$



Therefore, AC is the mean proportional between AB and AD.

$\triangle ABC \sim \triangle CBD$ by AA, and we notice that

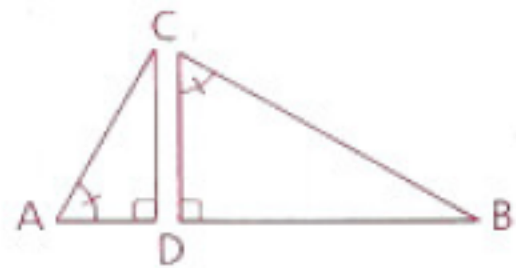
$$\frac{AB}{CB} = \frac{CB}{DB}, \text{ or } (CB)^2 = (AB)(DB)$$



Therefore, CB is the mean proportional between AB and DB.

$\triangle ACD \sim \triangle CBD$ by transitivity of similar triangles, and we notice that

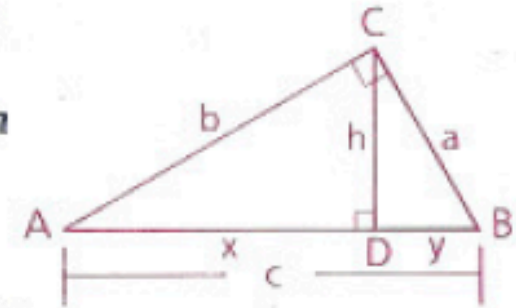
$$\frac{AD}{CD} = \frac{CD}{DB}, \text{ or } (CD)^2 = (AD)\,(DB)$$



Therefore, CD is the mean proportional between AD and DB.

These illustrations prove three closely related theorems, which we will present as one theorem.

Theorem 68 *If an altitude is drawn to the hypotenuse of a right triangle, then*



a *The two triangles formed are similar to the given right triangle and to each other*

$$\triangle ADC \sim \triangle ACB \sim \triangle CDB$$

b *The altitude to the hypotenuse is the mean proportional between the segments of the hypotenuse*

$$\frac{x}{h} = \frac{h}{y}, \text{ or } h^2 = xy$$

c *Either leg of the given right triangle is the mean proportional between the hypotenuse of the given right triangle and the segment of the hypotenuse adjacent to that leg (i.e., the projection of that leg on the hypotenuse)*

$$\frac{y}{a} = \frac{a}{c}, \text{ or } a^2 = yc; \text{ and } \frac{x}{b} = \frac{b}{c}, \text{ or } b^2 = xc$$

Parts **b** and **c** of Theorem 68 can be summarized as follows.

$$h^2 = x \cdot y$$
$$b^2 = x \cdot c$$
$$a^2 = y \cdot c$$