Objective

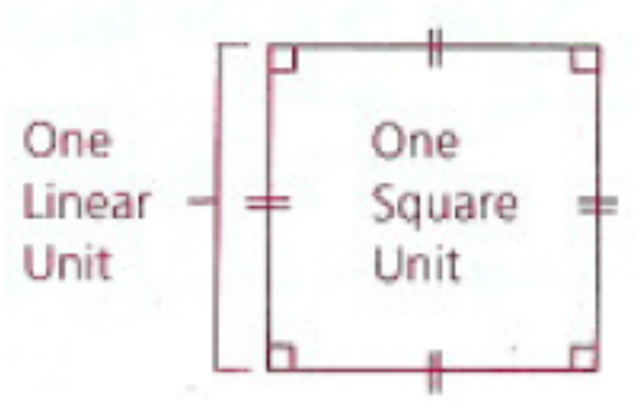
Students will be able to understand the concept of area and find the areas of rectangles, squares, parallelograms, and triangles.

11.1-11.4 Area Quiz on Thursday!

The Concept of Area

When we measure lengths of line segments, we use such standard units as meters, yards, miles, centimeters, and kilometers. These are often called linear units because they are measures of length. The standard unit of area are square units, such as square meters, square yards, and square miles.

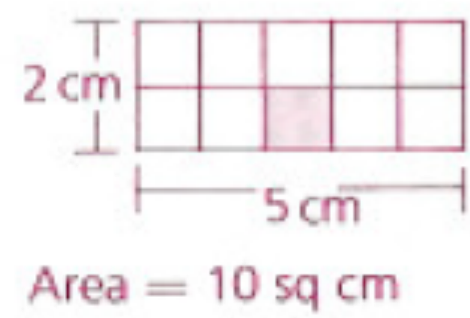
The area of a closed region is the number of square units of space within the boundary of the region.

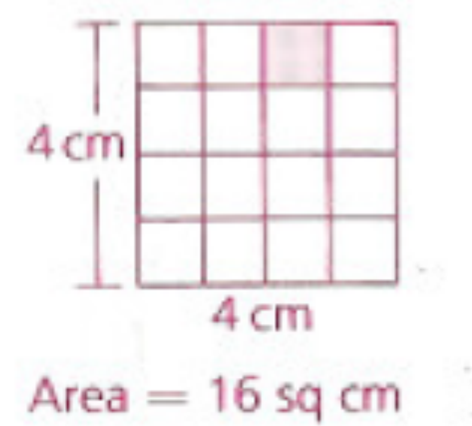


The Areas of Rectangle and Squares

In the figures to the right, there are two ways to find the areas:

1) The number of square units can be counted individually.
2) The areas can be computed by multiplying the number of columns (the measure of the base) by the number of rows (the height).





The Areas of Rectangle and Squares

The area of a rectangle:

$$A = bh$$

where b is the length of the base and h is the height

The area of a square:

$$A = s^2$$

where s is the length of a side

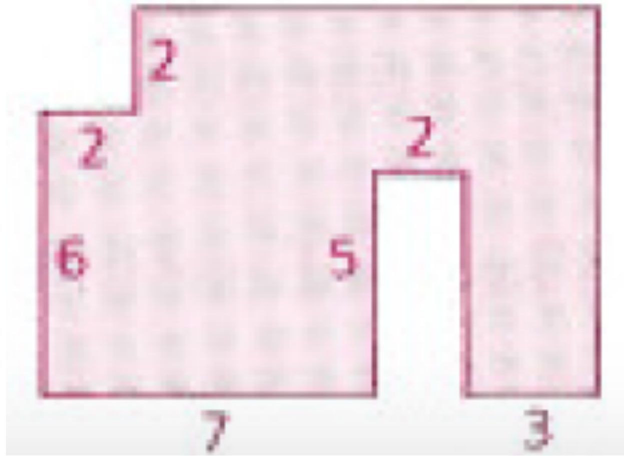
Basic Properties of Area

1) Every closed region has an area.

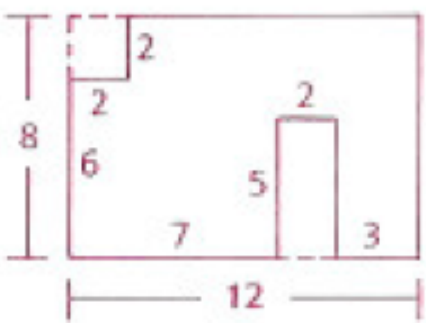
2) If two closed figures are congruent, then their areas are equal.

3) If two closed regions intersect only along a common boundary, then the area of their union is equal to the sum of their individual areas.

Find the area of the shaded region.

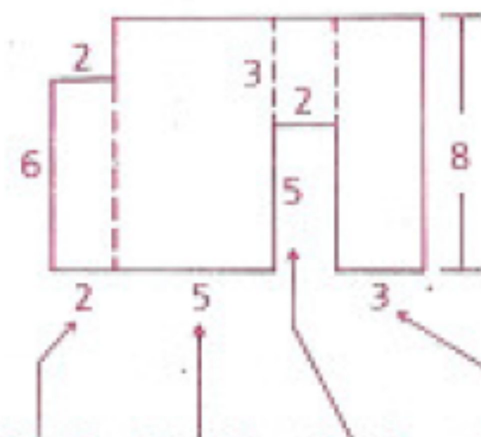


Method One:



Area of large rectangle $= 12 \cdot 8 = 96$
Area of square $= 2^2 = 4$
Area of small rectangle $= 2 \cdot 5 = 10$
Shaded area $= 96 - 4 - 10 = 82$

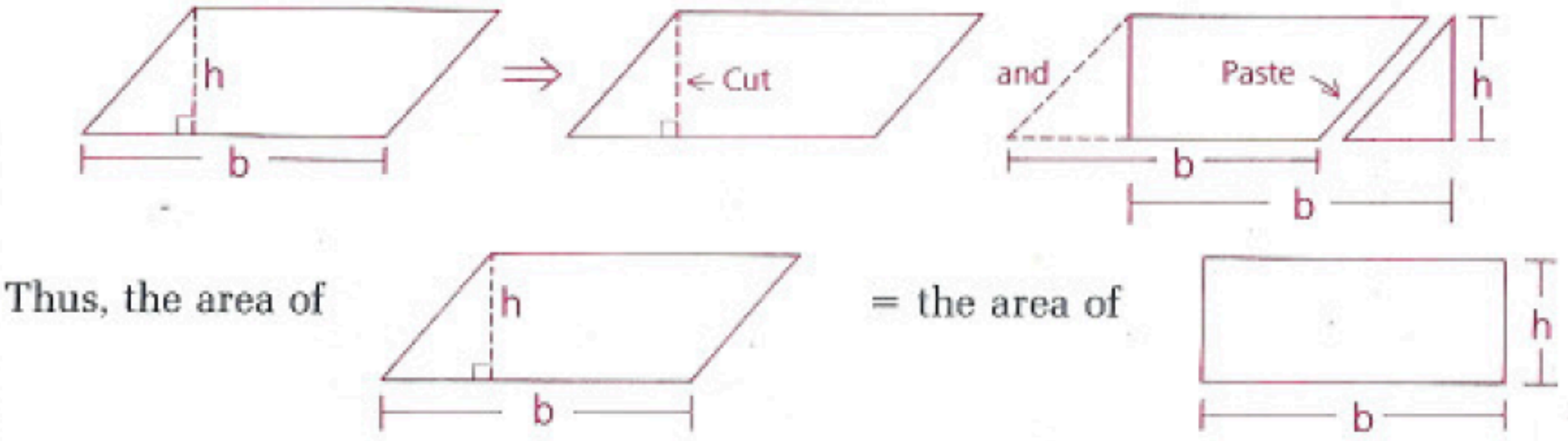
Method Two ("Divide and Conquer"):



$A = 2 \cdot 6 + 5 \cdot 8 + 2 \cdot 3 + 3 \cdot 8$

Shaded area $= 82$

The Area of a Parallelogram

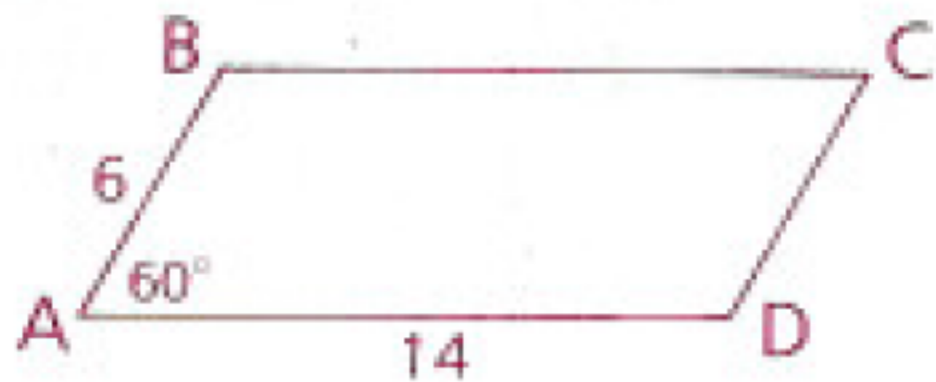


The area of a parallelogram:

$$A = bh$$

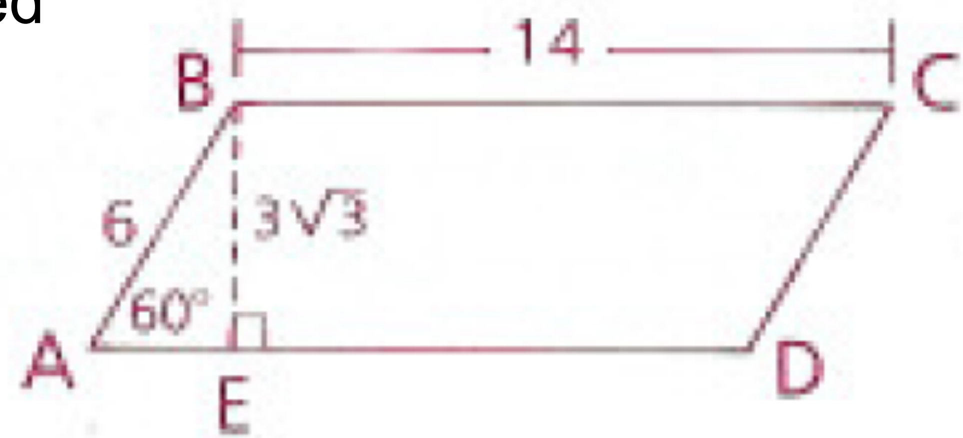
where b is the length of the base and h is the height

Find the area of a parallelogram whose sides are 14 and 6 and whose acute angle is 60º.



Need to find the height of the triangle, so we can draw the altitude.

30º-60º-90º triangle is formed



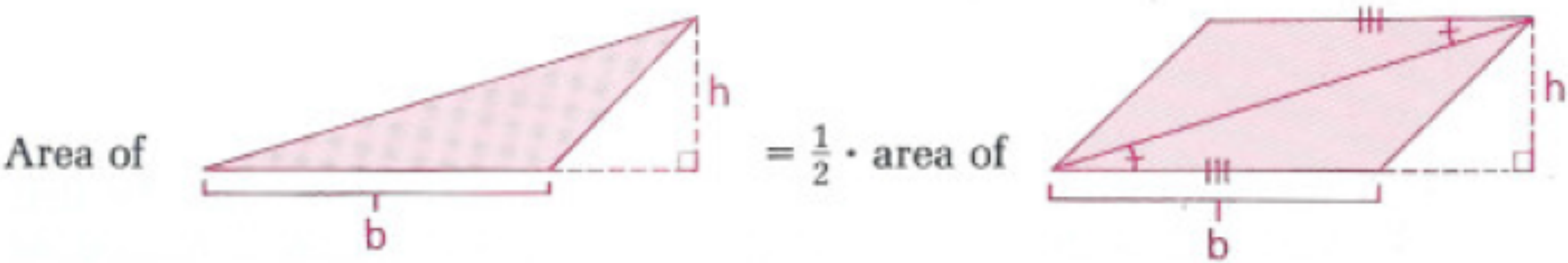
$h = 3\sqrt{3}$

$A = bh = 14(3\sqrt{3})$

$= 42\sqrt{3}$

The Area of a Triangle

The area of any triangle can be shown to be one half of the area of a parallelogram with the same base and height.



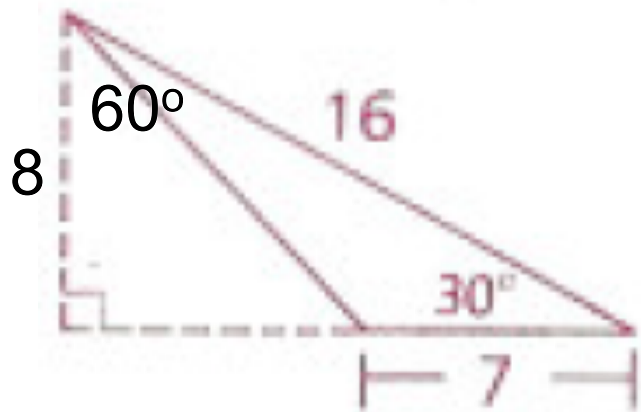
The area of a triangle:

$$A = \tfrac{1}{2}bh$$

where b is the length of the base and h is the height (altitude)

Find the area of the following figures:

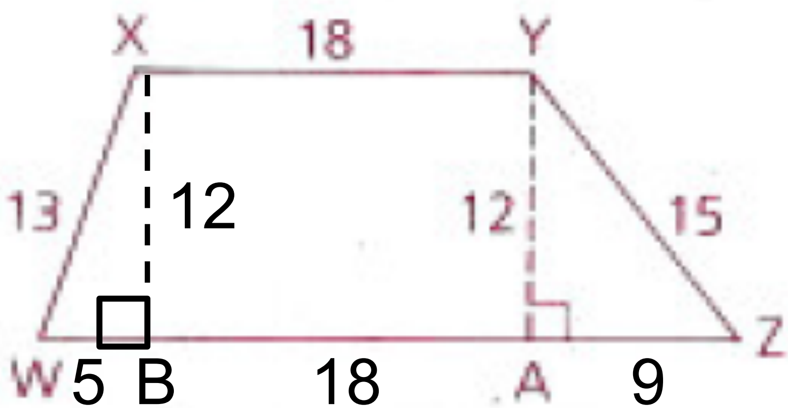
1)



$A = \frac{1}{2}(7)(8)$

$= 28$

2)



$12^2 + (AZ)^2 = 15^2$

$AZ = 9$

$12^2 + (WB)^2 = 13^2$

$WB = 5$

$\triangle XWB = \frac{1}{2}(5)(12) = 30$

$\triangle YAZ = \frac{1}{2}(9)(12) = 54$

$XYAB = 18(12) = 216$

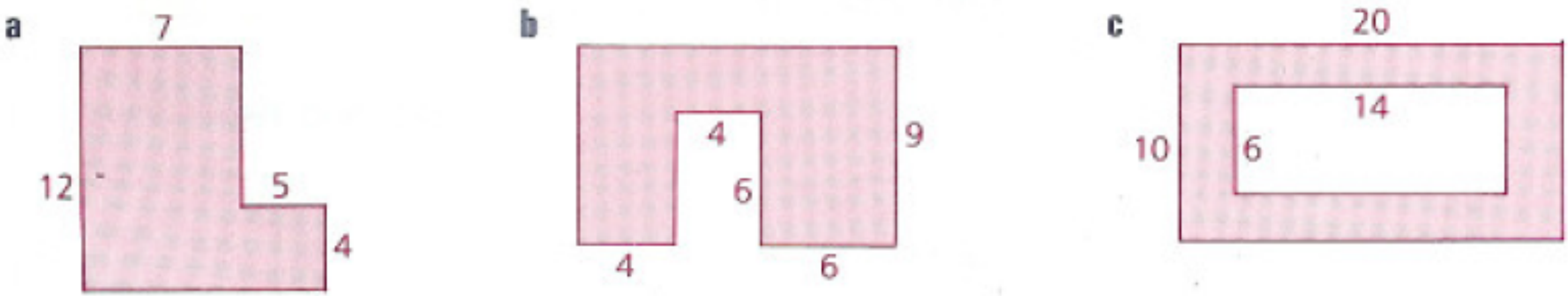
$WZYX = 30 + 54 + 216 = 300$

Homework

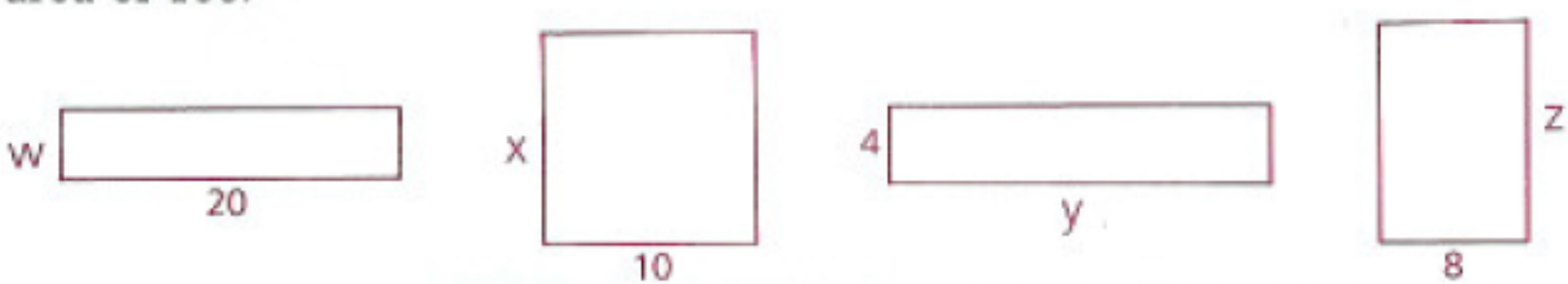
p. 514: 6, 8, 12
p. 520: 17, 21, 22, 23

p. 514: 6, 8, 12

6 Find the area of each shaded region. (Assume right angles.)



8 Each rectangular garden below has an area of 100.



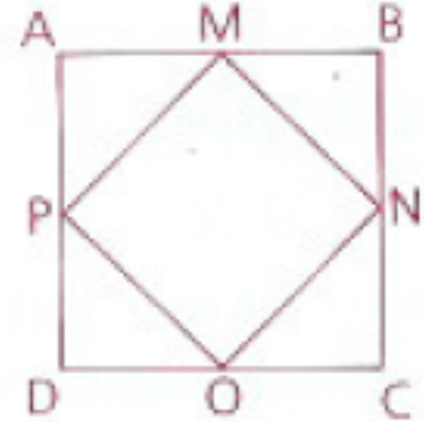
a Find the missing dimension of each.

b What length of fencing is needed to surround each?

c Which figure has the shortest perimeter?

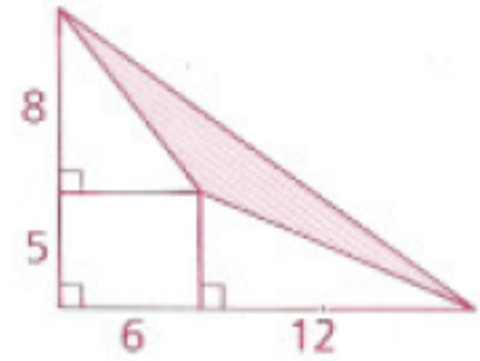
d What do you think must be true about a rectangle that encloses the maximum possible area with the shortest possible perimeter?

12 The area of square ABCD is 64 square units. MNOP is formed by joining the midpoints of the sides of ABCD. Find the area and the perimeter of MNOP.



p. 520: 17, 21, 22, 23

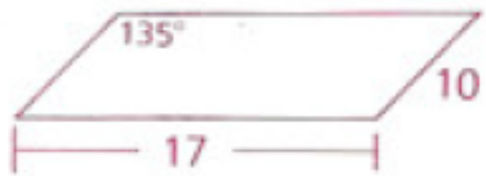
17 Find the area of the shaded triangular region.



21 Find the area of each parallelogram to the nearest tenth.

a

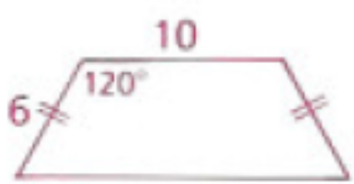
b



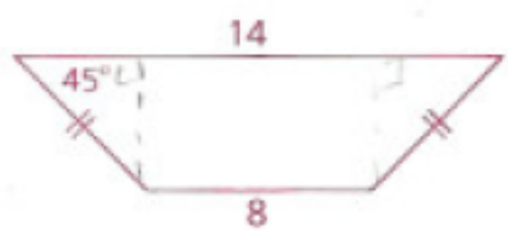
c

22 Find the area of each trapezoid by dividing it into other figures (rectangles and triangles or parallelograms and triangles).

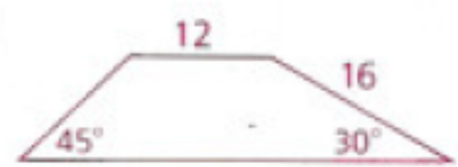
a



b



c



23 Find the area of $\triangle ABC$ with vertices $A = (1, 3)$, $B = (7, 3)$, and $C = (4, -1)$.

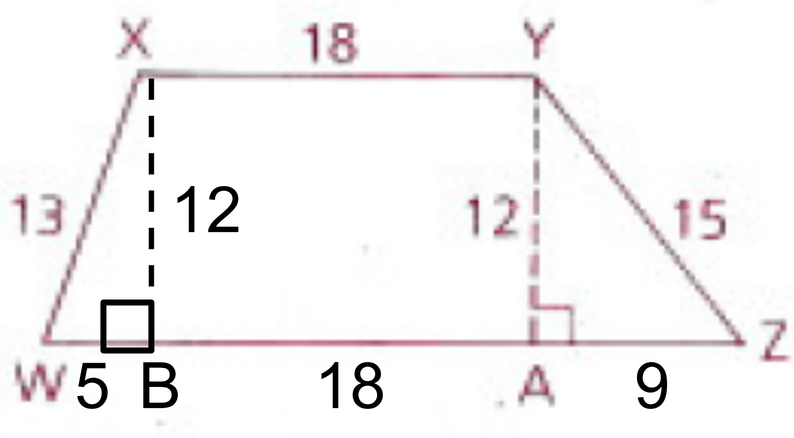
Objective

Students will be able to find the areas of trapezoids and use the measure of a trapezoid's median to find its area.

11.1-11.4 Area Quiz on Thursday!

Science Fair in one week! ☺

Find the area of the trapezoid:



$$12^2 + (AZ)^2 = 15^2$$
$$AZ = 9$$

$$12^2 + (WB)^2 = 13^2$$
$$WB = 5$$

$\triangle XWB = \frac{1}{2}(5)(12) = 30$

$\triangle YAZ = \frac{1}{2}(9)(12) = 54$

$XYAB = 18(12) = 216$

$$WZYX = 30 + 54 + 216 = 300 \text{ units}^2$$

The Area of a Trapezoid

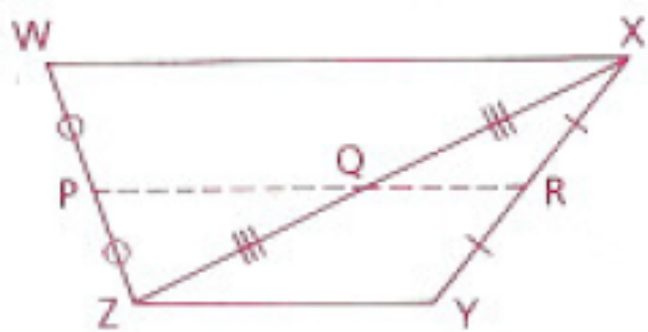
The area of a trapezoid:

$$A = \frac{1}{2}h(b_1 + b_2)$$

where b_1 is the length of the one base, b_2 is the length of the other base, and h is the height

The Median of a Trapezoid

The line segment joining the midpoints of the nonparallel sides (legs) of a trapezoid is called the <u>median</u> of the trapezoid.



In trapezoid WXYZ, P, Q, and R are midpoints of sides of $\triangle WXZ$ and $\triangle XYZ$. P, Q, and R are collinear, because $\overline{PQ}$ and $\overline{QR}$ share Q, and each segment is parallel to $\overline{WX}$ and $\overline{ZY}$. $\overline{PR}$ is the median of trapezoid WXYZ.

By the Midline Theorem, $PQ = \frac{1}{2}(WX)$ and $QR = \frac{1}{2}(YZ)$. Thus, $PR = PQ + QR = \frac{1}{2}(WX) + \frac{1}{2}(YZ) = \frac{1}{2}(WX + YZ)$.

The Median of a Trapezoid

The measure of the median of a trapezoid equals the average of the measures of the bases.

$$M = \frac{1}{2}(b_1 + b_2)$$

where b_1 is the length of the one base and b_2 is the length of the other base

The Median of a Trapezoid

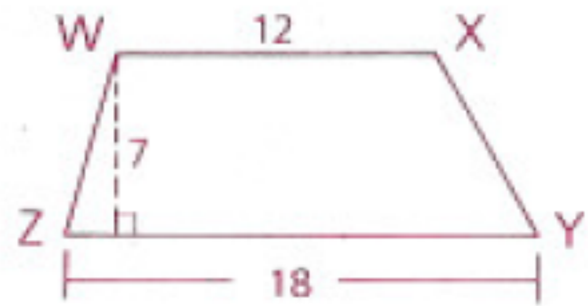
The area of a trapezoid is the product of the median and the height.

$$A_{\text{trap}} = Mh$$

where M is the length of the median and h is the height

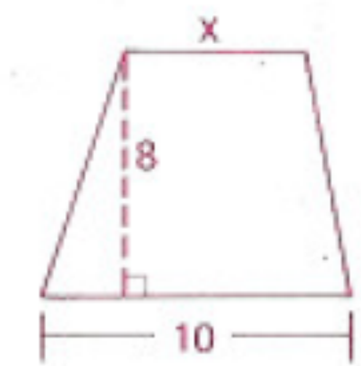
1) Given: Trapezoid WXYZ, with height 7, lower base 18, and upper base 12 cm

Find: The area of WXYZ



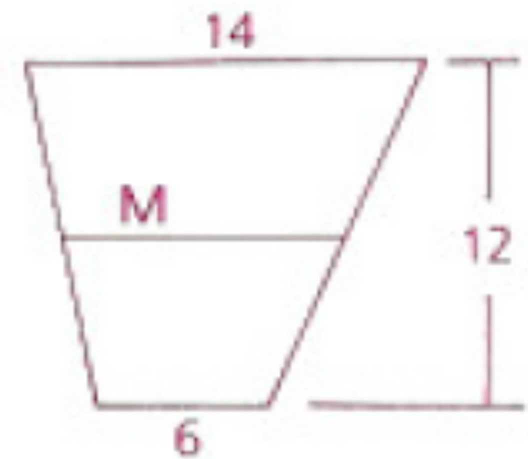
$$A = \tfrac{1}{2}h(b_1 + b_2)$$
$$= \tfrac{1}{2}(7)(18 + 12)$$
$$= 105 \text{ cm}^2$$

2) Find the shorter base of a trapezoid if the trapezoid's area is 52, its altitude is 8, and its longer base is 10.



$$A = \tfrac{1}{2}h(b_1 + b_2)$$
$$52 = \tfrac{1}{2}(8)(10 + x)$$
$$52 = 4(10 + x)$$
$$3 = x$$

3) The height of a trapezoid is 12 m. The bases are 6 and 14 m.



a) Find the median.

$$M = ½(b_1 + b_2)$$
$$= ½(14 + 6)$$
$$= 10 \text{ m}$$

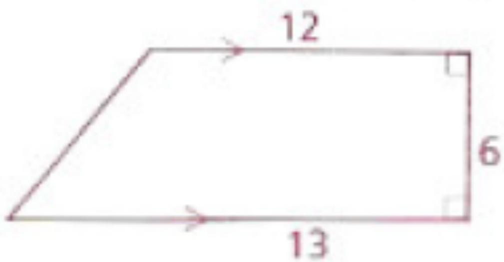
b) Find the area.

$$A_{\text{trap}} = Mh$$
$$= 10(12)$$
$$= 120 \text{ m}^2$$

Homework p.525: 2, 7, 8, 10

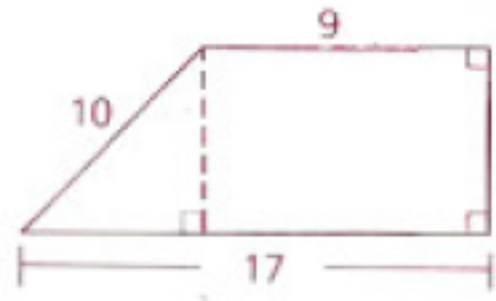
2 Find the area of each trapezoid.

a



b

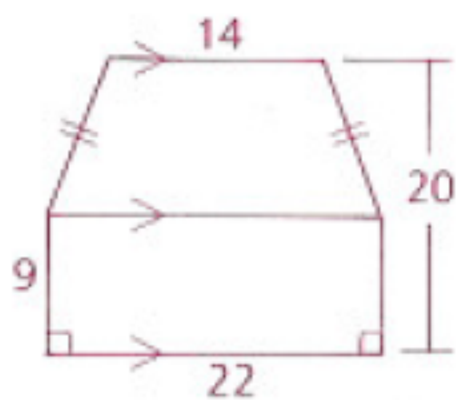
c



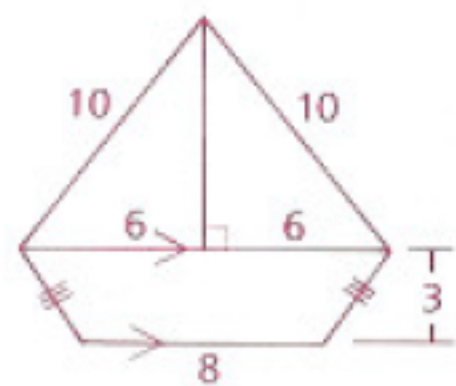
d

7 Find the total area of each figure.

a

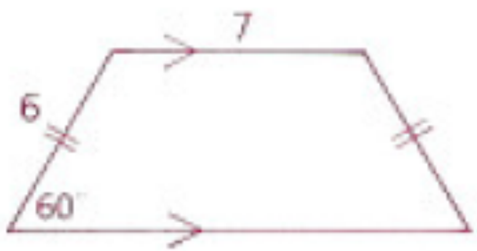


b

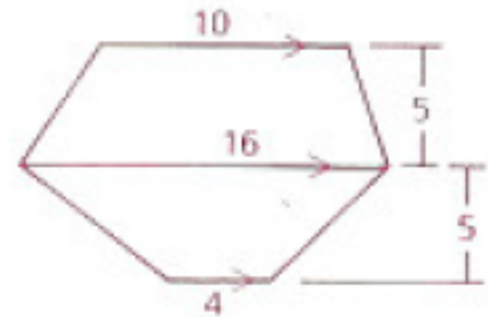


8 Find the total area of each figure.

a



b



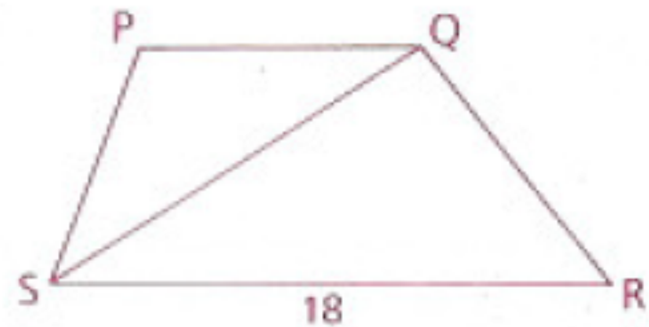
10 The area of triangle PQS is 25.
The median of trapezoid PQRS is 14.
Base $\overline{RS}$ measures 18.

Find: **a** The length of base $\overline{PQ}$

b The height to base $\overline{PQ}$ of $\triangle PQS$

c The height of trapezoid PQRS

d The area of trapezoid PQRS



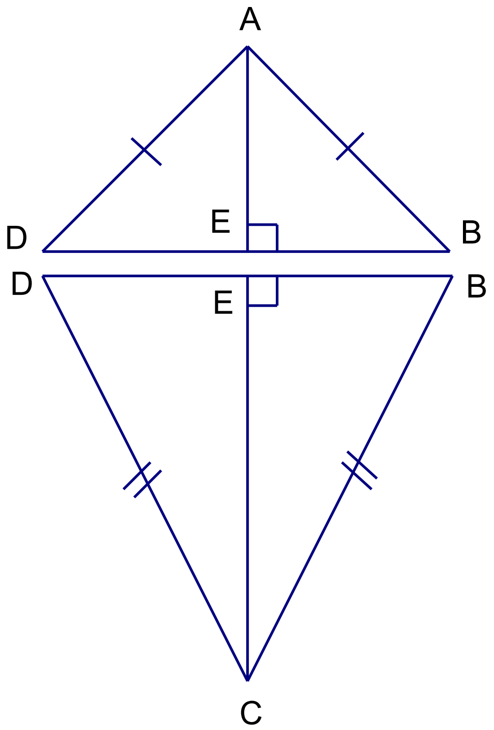
Objective

Students will be able to find the areas of kites.

11.1-11.4 Area Quiz tomorrow!

In-Class Activity

Using centimeter grid paper, draw two perpendicular line segments. Connect the ends of the segments, forming a kite. Cut the kite into two triangles as shown.



1) How does the area of the kite compare to the area of the triangles?

The area of the kite is equal to the sum of the areas of the triangles.

2) Write an expression for the area of each triangle (using the segments).

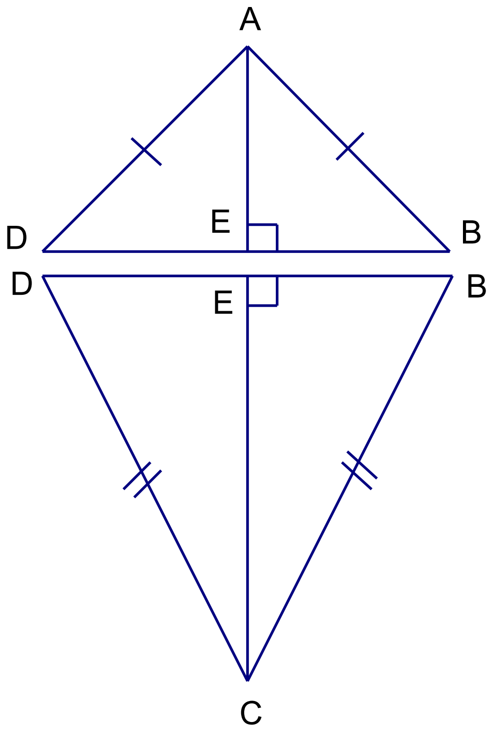
Area of $\triangle ABD = \frac{1}{2}(BD)(AE)$

Area of $\triangle DBC = \frac{1}{2}(BD)(EC)$

In-Class Activity

Using centimeter grid paper, draw two perpendicular line segments. Connect the ends of the segments, forming a kite. Cut the kite into two triangles as shown.

3) Use your answer to Questions 1 and 2 to write a formula for the area of a kite.



Area of ABCD

$= \text{Area of } \triangle ABD + \text{Area of } \triangle DBC$

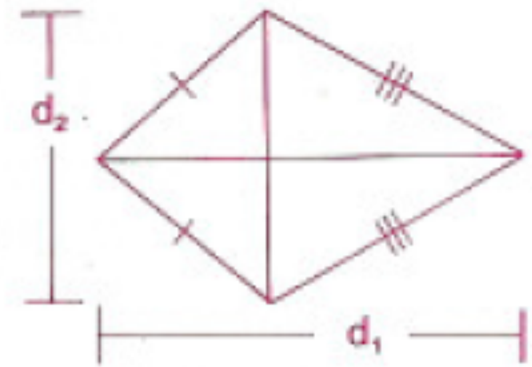
$= \frac{1}{2}(BD)(AE) + \frac{1}{2}(BD)(EC)$

$= \frac{1}{2}(BD)(AE + EC)$

$= \frac{1}{2}(BD)(AC)$

The Area of a Kite

The area of a kite:



$$A = \frac{1}{2}d_1d_2$$

where d_1 is the length of the one diagonal and d_2 is the length of the other diagonal

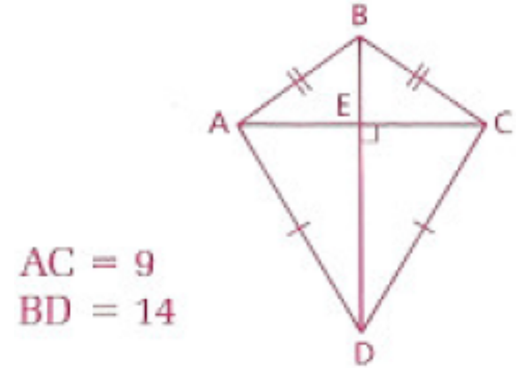
This formula can be applied to any kite, including the special cases of a rhombus and a square.

1) Find the area of a kite with diagonals 9 and 14.

$A = \frac{1}{2}d_1d_2$

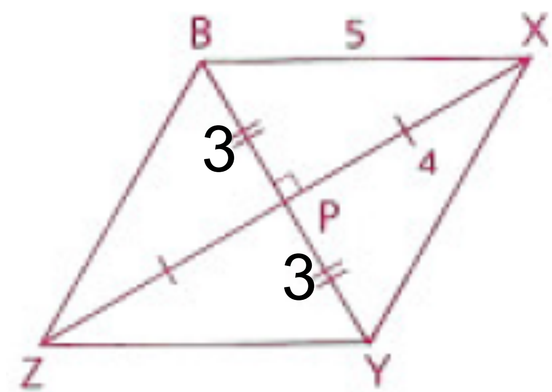
$= \frac{1}{2}(9)(14)$

$= 63 \text{ units}^2$



2) Find the area of a rhombus whose perimeter is 20 and whose longer diagonal is 8.

A rhombus is a ▱, so its diagonals bisect each other. It is also a kite, so its diagonals are ⊥ to each other. Thus, XZ = 8 and XP = 4.



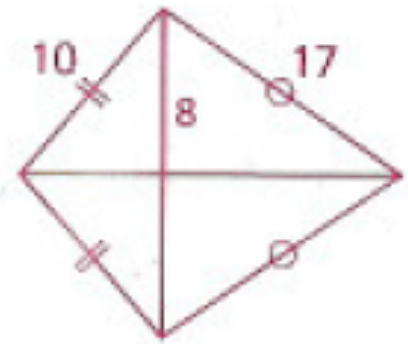
Each side is equal to 5 since the perimeter is 20

$A = \frac{1}{2}d_1d_2 \quad = \frac{1}{2}(8)(6) \quad = 24 \text{ units}^2$

Homework

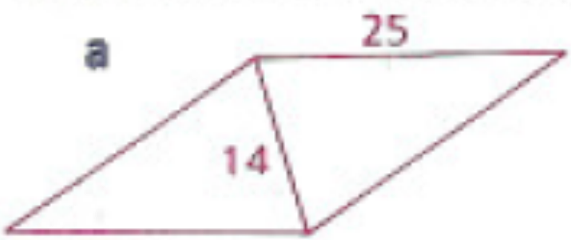
p. 529: 4, 5, 7-9
Study for Area Quiz!!!

4 Find the area of the kite shown.

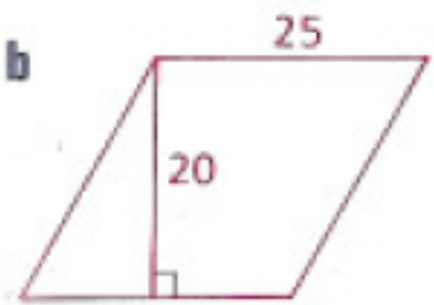


5 Find the area of each rhombus.

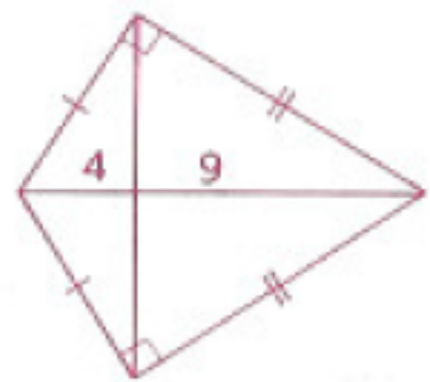
a



b



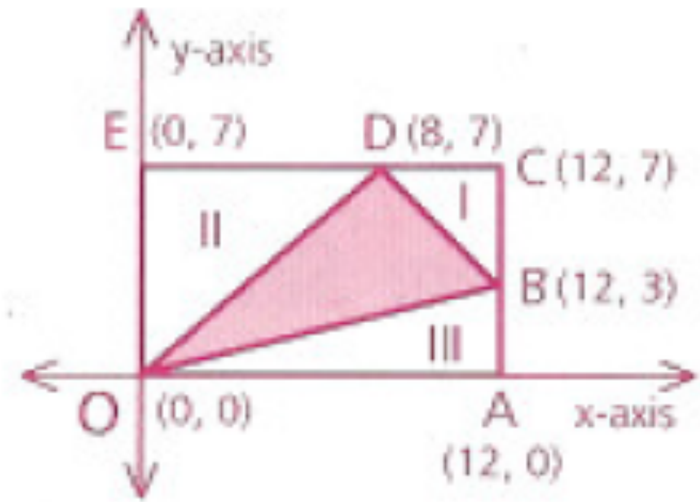
7 Find the area of the kite shown.



8 Find the area of a rhombus with a perimeter of 40 and one angle of 60°.

9 **a** Find the areas of region I, region II, and region III.

b Find the area of $\triangle OBD$.



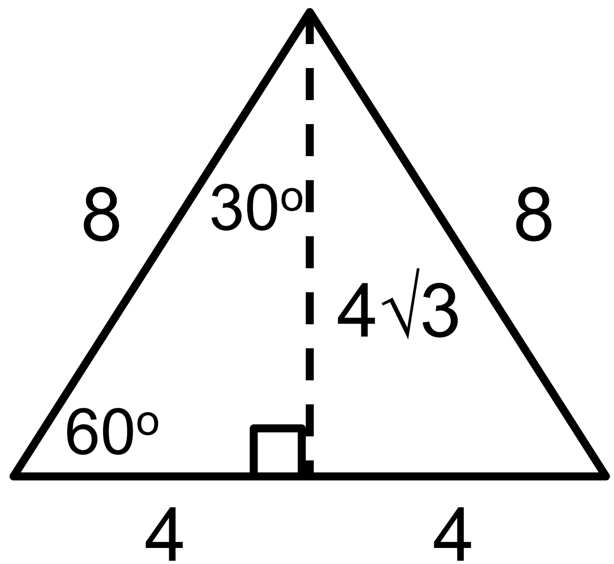
Objective

Students will be able to find the areas of equilateral triangles and other regular polygons.

Area Test on Thursday!

Science Fair is tomorrow!

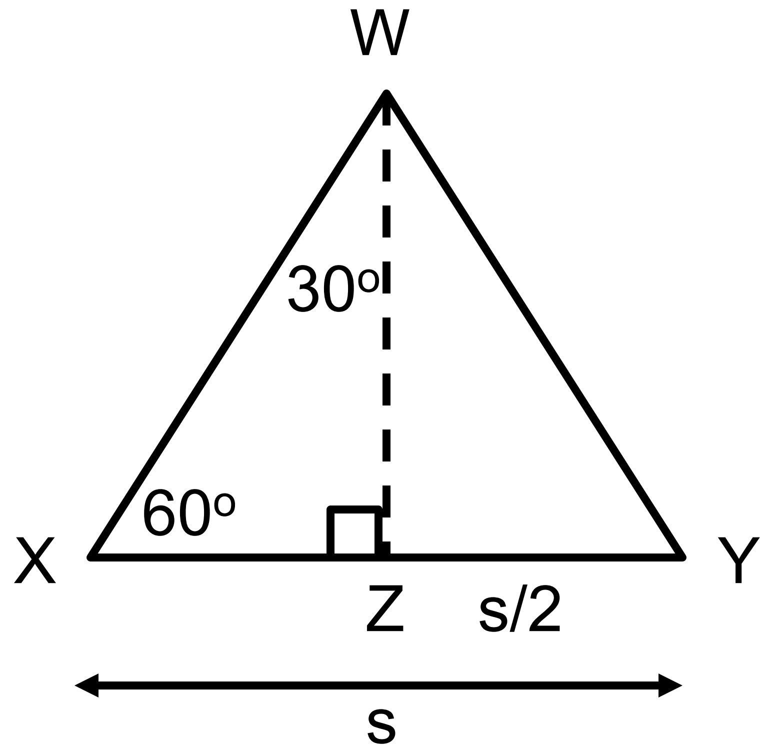
Find the area of an equilateral triangle with sides of length 8.



$$A = \tfrac{1}{2}(8)(4\sqrt{3})$$

$$= 16\sqrt{3} \text{ units}^2$$

Area of an Equilateral Triangle



What is the equation of the area of an equilateral triangle?

$$A = \frac{s^2}{4}\sqrt{3}$$

where *s* is the length of a side

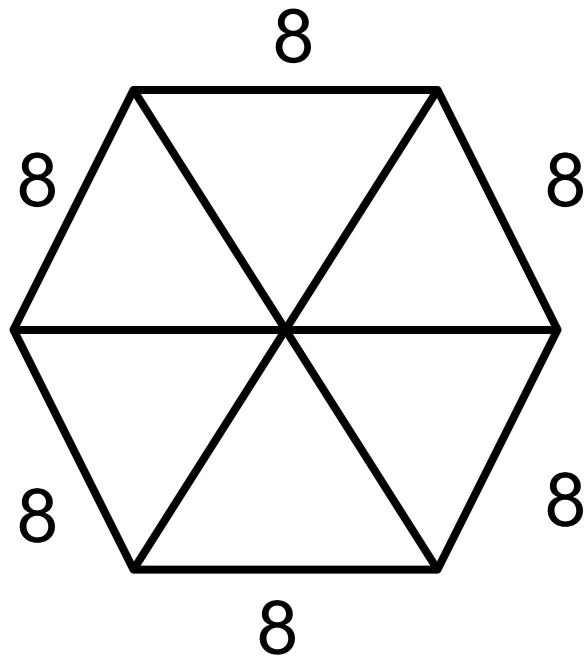
Find the area of an equilateral triangle with sides of length 8.

$$A = \frac{s^2}{4}\sqrt{3}$$

$$= \frac{8^2}{4}\sqrt{3}$$

$$= 16\sqrt{3} \text{ units}^2$$

Find the area of a regular hexagon with sides of length 8.



Make up of 6 equilateral triangles from previous problem:

$$A = 6(16\sqrt{3}) = 96\sqrt{3} \text{ units}^2$$

Area of a Regular Polygon

In a regular polygon, all interior angles are congruent and all sides are congruent.

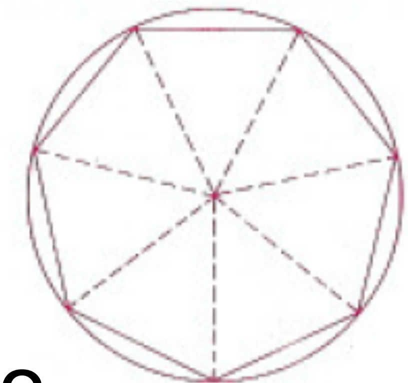
In regular polygon PENTA,

- O is the center
- OA is a radius: a segment joining the center to any vertex in a regular polygon
- OM is an apothem: a segment joining the center to the midpoint of any side in a regular polygon

Area of a Regular Polygon

- All apothems of a regular polygon are congruent
- Only regular polygons have apothems
- An apothem is a radius of a circle inscribed in the polygon
- An apothem is the perpendicular bisector of a side
- A radius of a regular polygon is a radius of a circle circumscribed about the polygon
- A radius of a regular polygon bisects an angle of the polygon

Area of a Regular Polygon



If all of the radii of a regular polygon are drawn, the polygon is divided into congruent isosceles triangles.

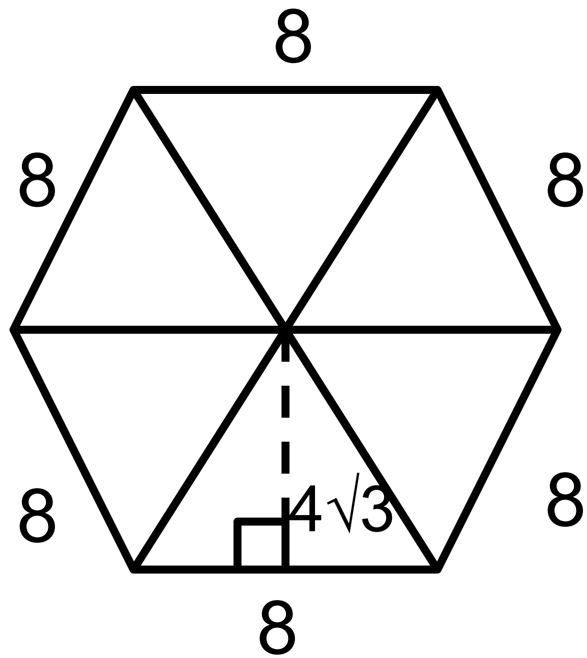
What is an altitude of each triangle?

Think about the pervious problem, what is the equation of the area of a regular polygon?

$$A = \frac{1}{2}ap$$

where *a* is the length of an apothem and *p* is the perimeter

Find the area of a regular hexagon with sides of length 8.



$$A = \frac{1}{2}ap \quad = \frac{1}{2}(4\sqrt{3})(48)$$

$= 96\sqrt{3}$ units^2

Homework

p.533: 3, 12, 13, 15, 17

3 Find the areas of equilateral triangles with the following apothems.

a 6 **b** 4 **c** 3 **d** $2\sqrt{3}$

12 Find the area of

a An equilateral triangle whose side is 9

b A square whose apothem is $7\frac{1}{2}$

c A regular hexagon whose side is 7

13 Find the length of one side and of the apothem of

a A square whose area is 121

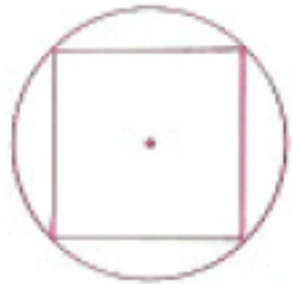
b An equilateral triangle whose area is $36\sqrt{3}$ sq m

c A regular hexagon whose perimeter is 24 cm

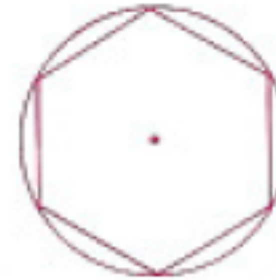
15 A circle of radius 12 is circumscribed about each regular polygon below. Find the area of each polygon.

a

b

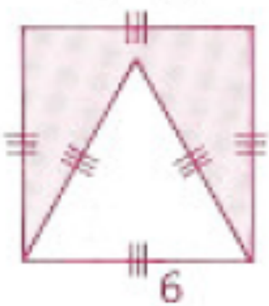


c

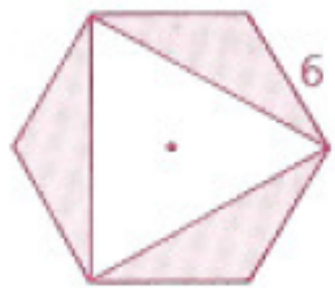


17 Find the area of the shaded region in each polygon. (Assume regular polygons.)

a



b



c