

# Rational Functions (Chapter 8) Review

Advanced Algebra with Trigonometry, Glawe

Perform the indicated operation and simplify.

Name: Key Period: \_\_\_\_\_

$$1) \frac{3x^3y}{4x^5y^2} \cdot \frac{6y^6}{2xy^3} = \frac{18x^3y^7}{8x^6y^5} = \frac{9x^{-3}y^2}{4x^3} = \boxed{\frac{9y^2}{4x^3}}$$

$$2) \frac{x^2+2x-24}{2x+12} \cdot \frac{x^2+6x+8}{x^2-16} = \frac{(x+6)(x-4)}{2(x+6)} \cdot \frac{(x+4)(x+2)}{(x-4)(x+4)} = \frac{(x+6)(x-4)(x+4)(x+2)}{2(x+6)(x-4)(x+4)} = \boxed{\frac{x+2}{2}}$$

$$3) \frac{x^2-3x-10}{3x+9} \div \frac{x^2+5x+6}{2x^2-x-6} = \frac{x^2-3x-10}{3x+9} \cdot \frac{2x^2-x-6}{x^2+5x+6} = \frac{(x-5)(x+2)}{3(x+3)} \cdot \frac{(2x+3)(x-2)}{(x+2)(x+3)} = \frac{(x-5)(x+2)(2x+3)(x-2)}{3(x+3)(x+2)(x+3)} = \boxed{\frac{(x-5)(2x+3)(x-2)}{3(x+3)^2}}$$

$$4) \frac{7}{5x} - \frac{4}{3x} = \frac{3}{3} \cdot \frac{7}{5x} - \frac{4}{3x} \cdot \frac{5}{5} = \frac{21}{15x} - \frac{20}{15x} = \boxed{\frac{1}{15x}}$$

$$5) \frac{x}{x-5} + \frac{2x+1}{x+3} \cdot \frac{x-5}{x-5} = \frac{x(x+3)}{(x+3)(x-5)} + \frac{(2x+1)(x-5)}{(x+3)(x-5)} = \frac{x^2+3x}{(x+3)(x-5)} + \frac{2x^2-10x+x-5}{(x+3)(x-5)} = \frac{x^2+3x+2x^2-9x-5}{(x+3)(x-5)} = \boxed{\frac{3x^2-6x-5}{(x+3)(x-5)}}$$

$$6) \frac{5x}{x^2+x-12} - \frac{4}{x+4} = \frac{5x}{(x+4)(x-3)} - \frac{4}{x+4} \cdot \frac{x-3}{x-3} = \frac{5x-4(x-3)}{(x+4)(x-3)} = \frac{5x-4x+12}{(x+4)(x-3)} = \boxed{\frac{x+12}{(x+4)(x-3)}}$$

$$7) \text{Simplify } \frac{x-4}{x} \cdot \frac{3}{x} + \frac{x+1}{x-4} \cdot \frac{x}{x} = \frac{3(x-4)}{x(x-4)} + \frac{x(x+1)}{x(x-4)} = \frac{3x-12+x^2+x}{x(x-4)} = \frac{x^2+4x-12}{x(x-4)} = \frac{(x+6)(x-2)}{x(x-4)} = \boxed{\frac{x+6}{2x}}$$

$$\text{LCD: } X(X-3)$$

Solve the equation. Check for extraneous solutions.

$$9) \frac{9}{x^2-6x+9} = \frac{3x}{x^2-3x}$$

$$X=0: \frac{9}{0^2-6(0)+9} = \frac{3(0)}{0^2-3(0)} \quad \frac{9}{9} = \frac{0}{0} \text{ NO}$$

$$10) \left( \frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3} \right) X(X-3) \quad \frac{2X(X-3)}{(X-3)} + \frac{X(X-3)}{X} = \frac{X(X-1)(X-3)}{(X-3)}$$

$$9(X^2-3X) = 3X(X^2-6X+9) \quad X=6: \frac{9}{6^2-6(6)+9} = \frac{3(6)}{6^2-3(6)} \quad \frac{9}{36-36+9} = \frac{18}{36-18}$$

$$9X^2-27X = 3X^3-18X^2+27X$$

$$0 = 3X^3-27X^2+54X$$

$$0 = 3X(X^2-9X+18) \quad \frac{18}{-6, 3} \quad \frac{9}{9-18+9} = \frac{9}{9-9} \quad \frac{9}{0} = \frac{9}{0} \text{ NO}$$

$$0 = 3X(X-6)(X-3)$$

$$3X=0, \quad X-6=0, \quad X-3=0 \quad X=0, \quad X=6, \quad X=3$$

$$2X+X-3 = X(X-1)$$

$$3X-3 = X^2-X$$

$$0 = X^2-4X+3$$

$$0 = (X-3)(X-1)$$

$$0 = X-3, \quad 0 = X-1$$

$$X=3, \quad X=1$$

$$X=3: \frac{2}{3-3} + \frac{1}{3} = \frac{3-1}{3-3}$$

$$\frac{2}{0} + \frac{1}{3} = \frac{2}{0} \text{ NO}$$

$$X=1: \frac{2}{1-3} + \frac{1}{1} = \frac{1-1}{1-3}$$

$$\frac{2}{-2} + 1 = \frac{0}{-2} \quad -1+1=0$$

Sketch the asymptotes and graph the rational function. Identify the vertical asymptote and horizontal asymptote,  $0=0$  ✓ and state the domain/range. Plot a minimum of two points on each branch.

$$11) y = \frac{-3}{x+2} - 1$$

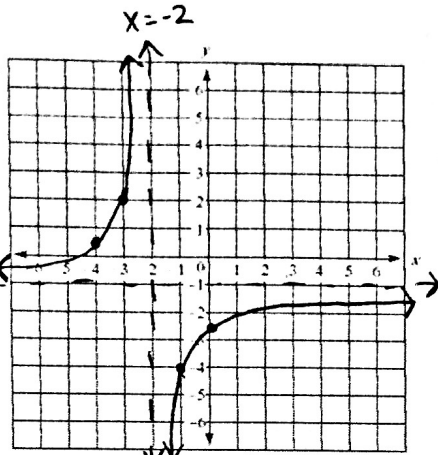
$$x+2=0$$

$$x=-2$$

$$\text{v.a. } x=-2$$

$$\text{h.a. } y=-1$$

x	y
-4	1/2
-3	2
-1	-4
0	-5/2



$$\text{Domain: } (-\infty, -2) \cup (-2, \infty)$$

$$\text{Range: } (-\infty, -1) \cup (-1, \infty)$$

$$x=-4: y = \frac{-3}{-4+2} - 1$$

$$= \frac{-3}{-2} - 1$$

$$= \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$

$$12) y = \frac{2x+1}{4x-8}$$

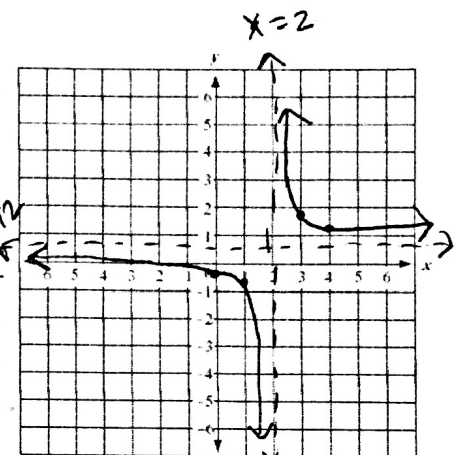
$$4x-8=0 \quad \frac{4x}{4} = \frac{8}{4} \quad x=2$$

$$\text{v.a. } x=2$$

$$\text{h.a. } y = \frac{1}{2}$$

$$y = \frac{2}{4} = \frac{1}{2}$$

x	y
0	-1/8
1	-3/4
3	1/4
4	9/8



$$\text{Domain: } (-\infty, 2) \cup (2, \infty)$$

$$\text{Range: } (-\infty, 1/2) \cup (1/2, \infty)$$

$$x=0: y = \frac{2(0)+1}{4(0)-8} = \frac{1}{-8}$$

13) How does the graph of  $y = \frac{3}{x+5} + 2$  compare to the graph of  $y = \frac{3}{x}$  ?

left five units, up two units

14) How does the graph of  $y = \frac{3x-1}{x-2}$  compare to the graph of  $y = \frac{2x}{x}$  ?

~~right five units~~

Find the least common multiple of the polynomials.

$$15) 3x^2 \text{ and } 3x-12$$

$$3(x-4)$$

$$\text{LCM: } 3x^2(x-4)$$

$$16) x^2-36 \text{ and } 2x+12$$

$$(x-6)(x+6) \quad 2(x+6)$$

$$\text{LCM: } 2(x-6)(x+6)$$