Objective

Students will be able to use trigonometric functions to find lengths.

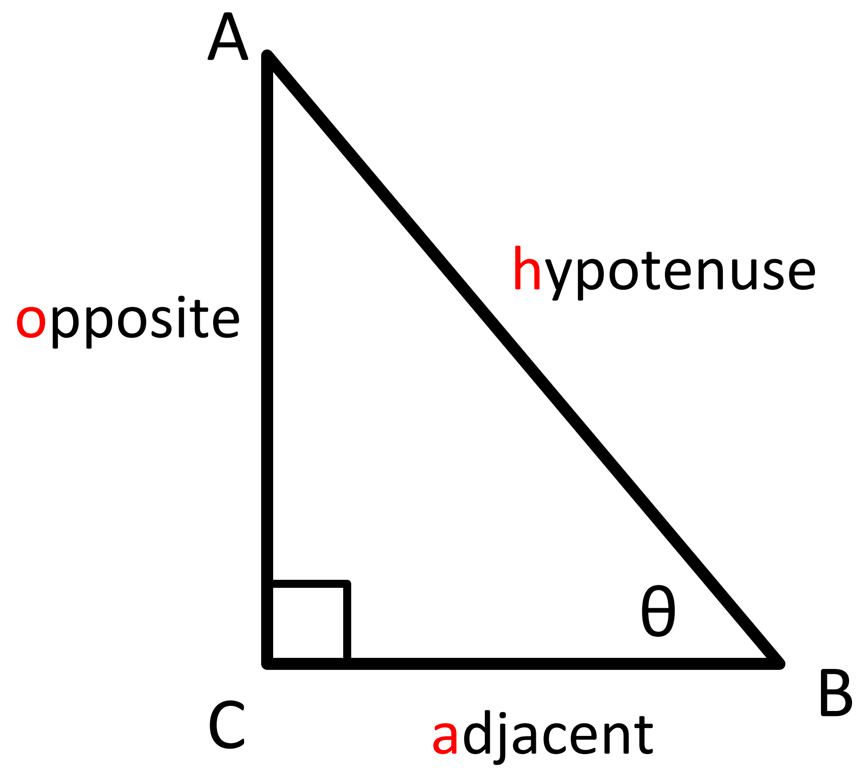
Three Trigonometric Ratios-
sine, cosine, and tangent

Let θ be an acute angle of a right triangle, then:

$$\sin\theta = \frac{opposite}{hypotenuse}$$ SOH

$$\cos\theta = \frac{adjacent}{hypotenuse}$$ CAH

$$\tan\theta = \frac{opposite}{adjacent}$$ TOA



SOH-CAH-TOA

**only for right triangles

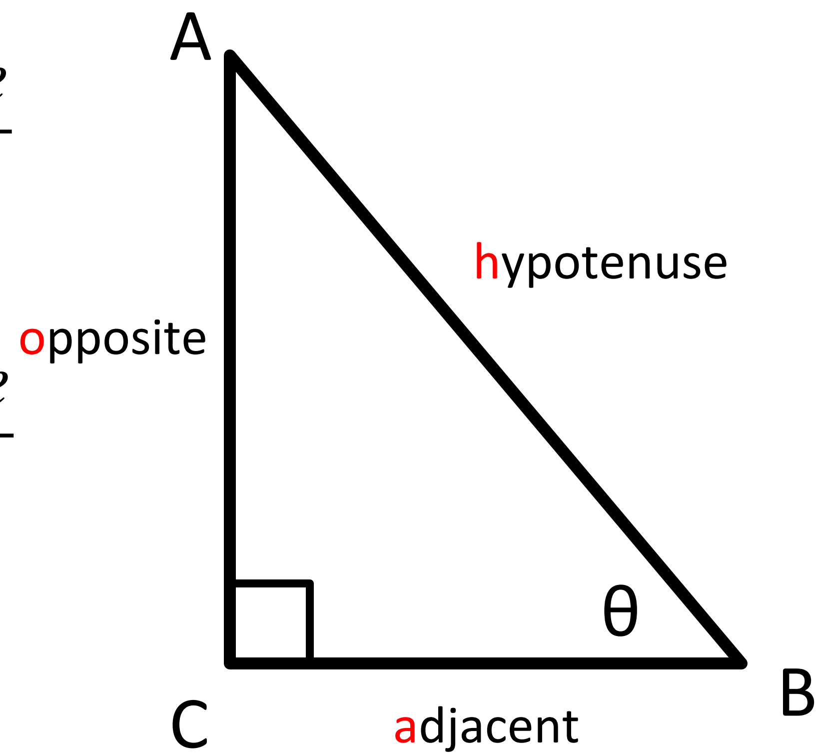
Three More Trigonometric Ratios- cosecant, secant, and cotangent

Let θ be an acute angle of a right triangle, then:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \csc\theta = \frac{hypotenuse}{opposite}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \sec\theta = \frac{hypotenuse}{adjacent}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \cot\theta = \frac{adjacent}{opposite}$$



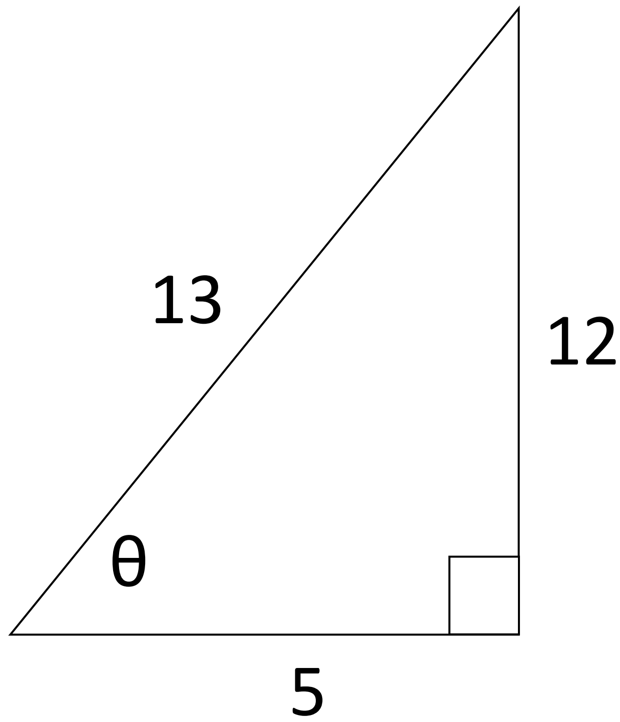
**only for right triangles

Evaluate the six trig values for angle θ

Find the third side of the right triangle by using the Pythagorean theorem:

$$a^2 + b^2 = c^2 \qquad 5^2 + 12^2 = c^2$$

$$\sqrt{5^2 + 12^2} = \sqrt{169} = 13$$



**Pythagorean triple

$$\sin\theta = \frac{opp}{hyp} = \frac{12}{13} \qquad \csc\theta = \frac{hyp}{opp} = \frac{13}{12}$$

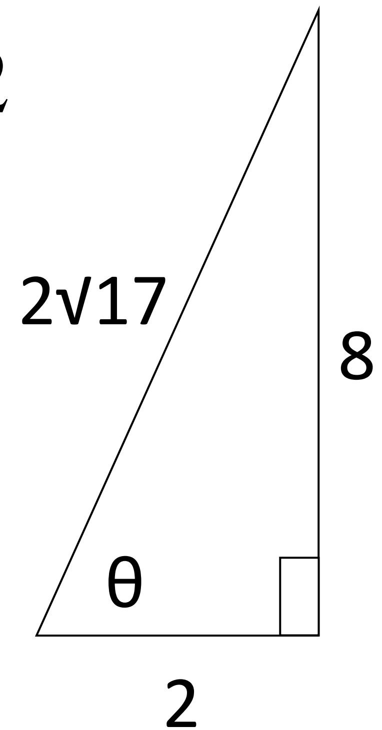
$$\cos\theta = \frac{adj}{hyp} = \frac{5}{13} \qquad \sec\theta = \frac{hyp}{adj} = \frac{13}{5}$$

$$\tan\theta = \frac{opp}{adj} = \frac{12}{5} \qquad \cot\theta = \frac{adj}{opp} = \frac{5}{12}$$

Evaluate the six trig values for angle θ

Find the third side of the right triangle by using the Pythagorean theorem: $a^2 + b^2 = c^2$

$8^2 + b^2 = (2\sqrt{17})^2 \quad 64 + b^2 = 68 \quad b^2 = 4 \quad b = 2$



$$\sin\theta = \frac{opp}{hyp} = \frac{8}{2\sqrt{17}} = \frac{4}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{2}{2\sqrt{17}} = \frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$

$$\tan\theta = \frac{opp}{adj} = \frac{8}{2} = 4$$

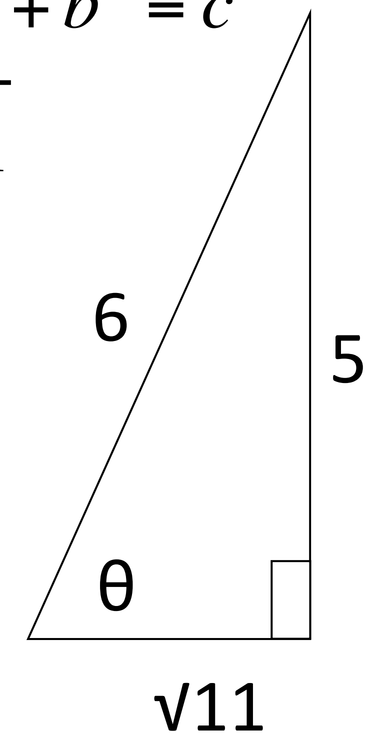
$$\cot\theta = \frac{adj}{opp} = \frac{2}{8} = \frac{1}{4}$$

$$\csc\theta = \frac{hyp}{opp} = \frac{2\sqrt{17}}{8} = \frac{\sqrt{17}}{4}$$

$$\sec\theta = \frac{hyp}{adj} = \frac{2\sqrt{17}}{2} = \sqrt{17}$$

Find the other five trig values if $\sin\theta = \dfrac{5}{6}$

Draw picture and find the third side of the right triangle by using the Pythagorean theorem: $a^2 + b^2 = c^2$



$5^2 + b^2 = 6^2 \quad 25 + b^2 = 36 \quad b^2 = 11 \quad b = \sqrt{11}$

$$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{11}}{6}$$

$$\tan\theta = \frac{opp}{adj} = \frac{5}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\csc\theta = \frac{hyp}{opp} = \frac{6}{5}$$

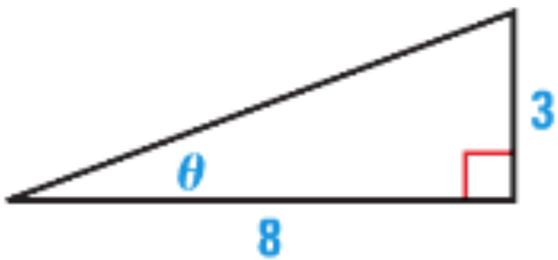
$$\sec\theta = \frac{hyp}{adj} = \frac{6}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$$

$$\cot\theta = \frac{adj}{opp} = \frac{\sqrt{11}}{5}$$

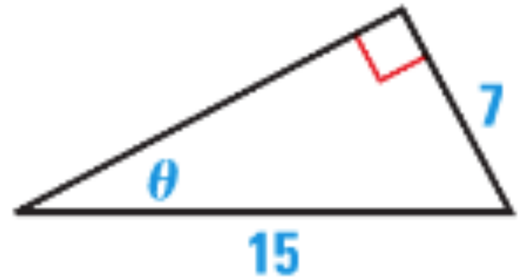
Homework p. 856: 4, 6, 10 – 14 (evens)

EVALUATING FUNCTIONS Evaluate the six trigonometric functions of the angle θ.

4.



6.



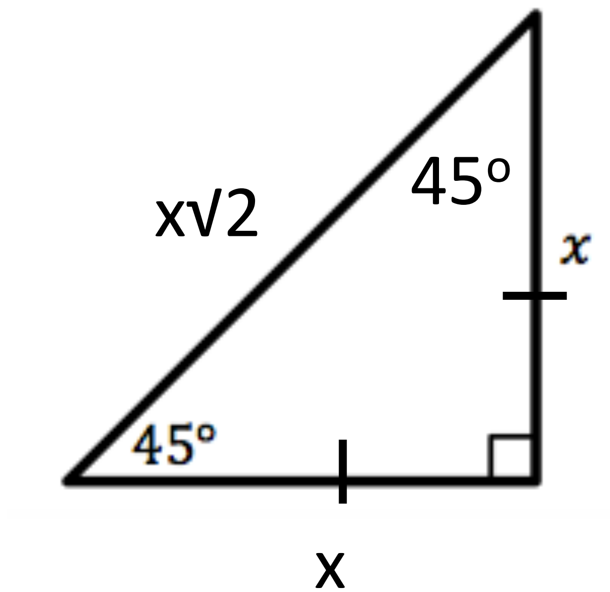
FINDING VALUES Let θ be an acute angle of a right triangle. Find the values of the other five trigonometric functions of θ.

10. $\cos\theta = \frac{5}{8}$ **12.** $\csc\theta = \frac{10}{7}$ **14.** $\cot\theta = \frac{6}{11}$

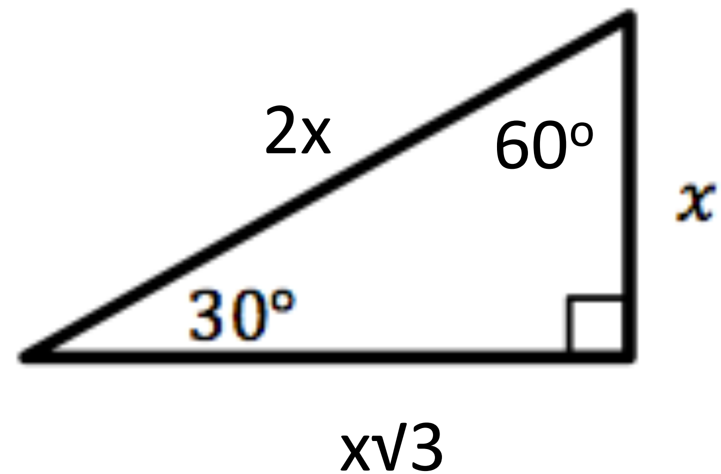
Objective

Students will be able to use trigonometric functions to find lengths.

Special Right Triangles



45^o-45^o-90^o Triangles

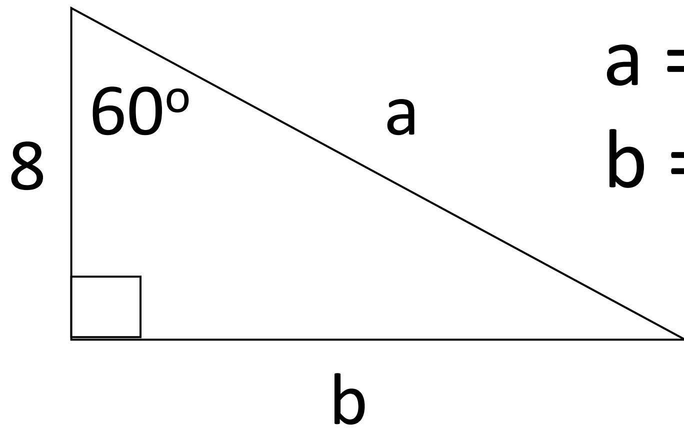


30^o-60^o-90^o Triangles

**you do not need to memorize these because you can find them using the Pythagorean theorem, but they are time savers if you do

Find the unknown side lengths of the following triangles.

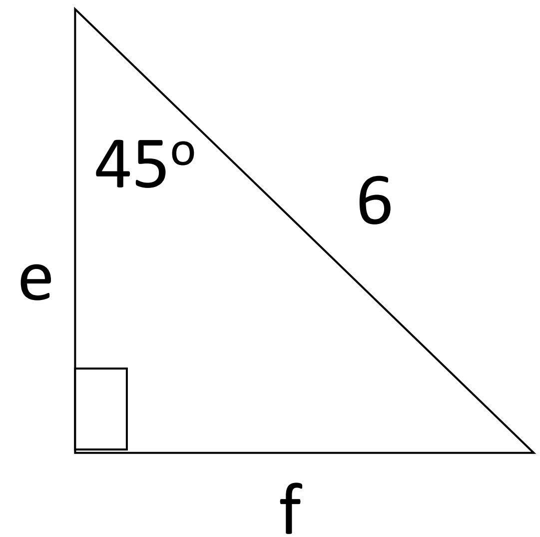
1)



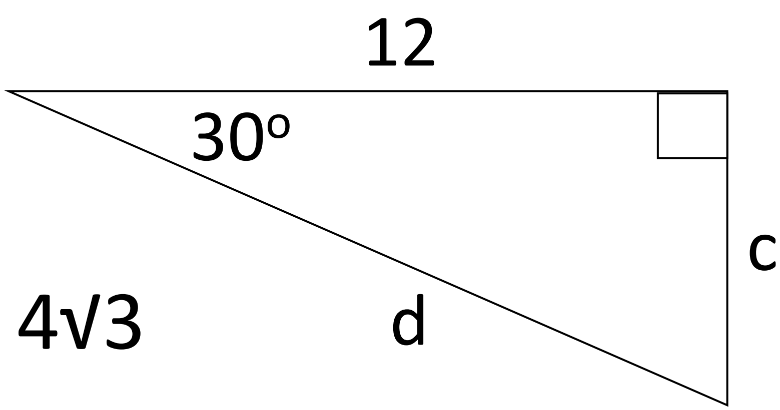
$a = 16$

$b = 8\sqrt{3}$

3)



2)



$c = 4\sqrt{3}$

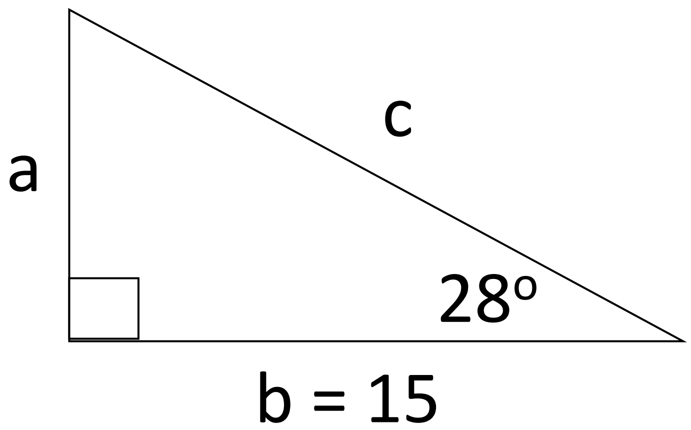
$d = 8\sqrt{3}$

$e = 3\sqrt{2}$

$f = 3\sqrt{2}$

Find all angles and sides of the triangle.

1)



$$\tan 28^\circ = \frac{a}{15}$$

$$a = 15\tan 28^\circ$$

$$a \approx 7.98$$

$$\cos 28^\circ = \frac{15}{c}$$

$$c\cos 28^\circ = 15$$

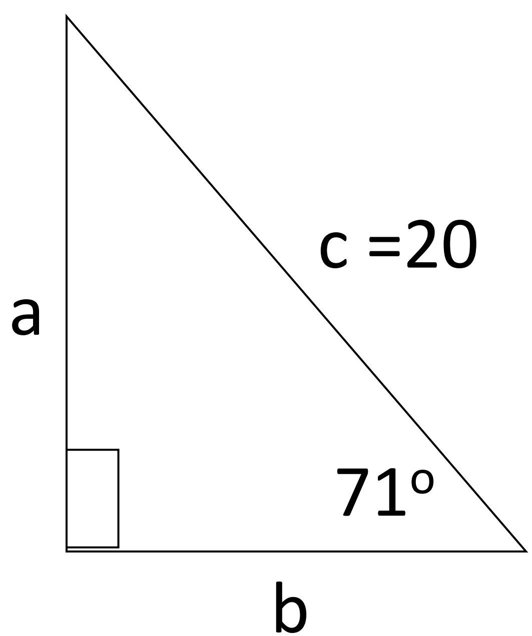
$$c = \frac{15}{\cos 28^\circ}$$

$$c \approx 16.99$$

B = 180° – 90° – 28°

B = 62°

2)



$$\sin 71^\circ = \frac{a}{20}$$

$$a = 20\sin 71^\circ$$

$$a \approx 18.9$$

$$\cos 71^\circ = \frac{b}{20}$$

$$b = 20\cos 71^\circ$$

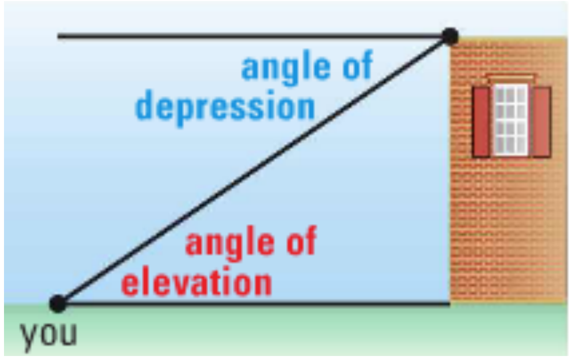
$$b \approx 6.51$$

B = 180° – 90° – 71° B = 19°

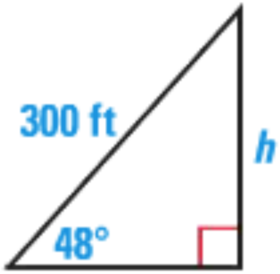
**make sure your calculators are in degree mode!

Angles of Sight

If you look at a point above you, such as the top of a building, the angle that your line of sight makes with a line parallel to the ground is called the angle of elevation. At the top of the building, the angle between a line parallel to the ground and your line of sight is called the angle of depression.



A parasailer is attached to a boat with a rope 300 feet long. The angle of elevation from the boat to the parasailer is 48°. Estimate the parasailer's height above the boat.



$$\sin 48^\circ = \frac{h}{300}$$

$$h = 300 \sin 48^\circ$$

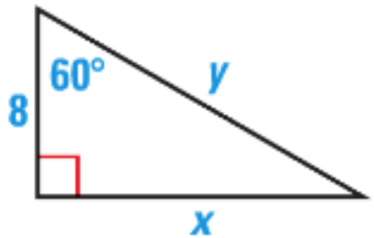
$$h \approx 223$$

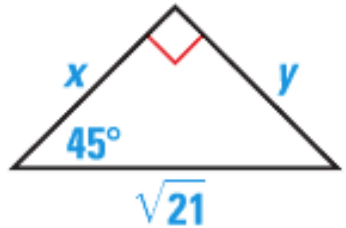
the parasailer is about 223 feet above the boat

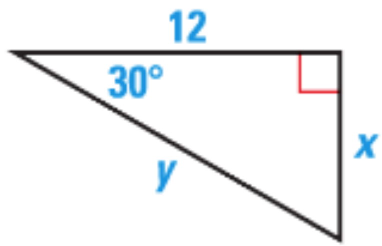
Homework

p. 856: 17-19, 22, 24, 30

FINDING SIDE LENGTHS Find the exact values of x and y.

17. 

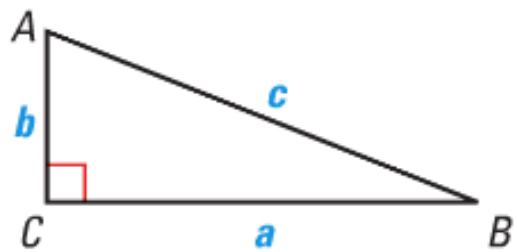
18. 

19. 

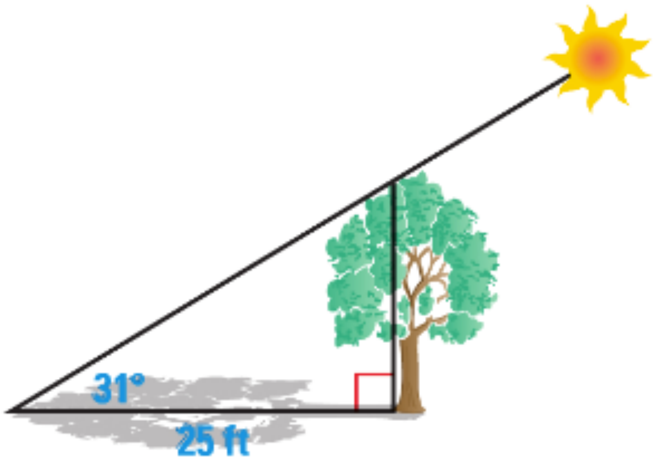
SOLVING TRIANGLES Solve $\triangle ABC$ using the diagram and the given measurements.

22. $B = 53°, a = 12$

24. $A = 67°, b = 7$



30. **TREE HEIGHT** A tree casts the shadow shown. What is the height of the tree?



Objective

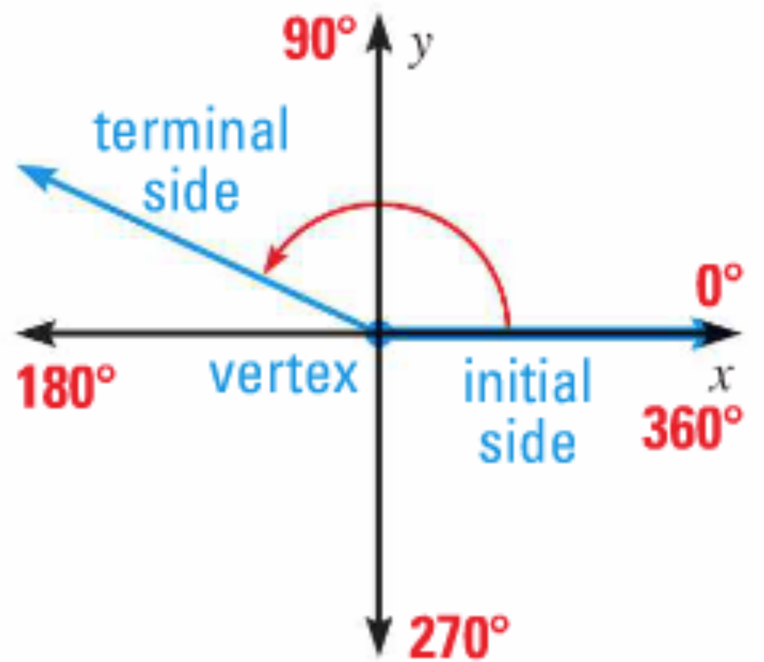
Students will be able to use general angles that may be measured in radians.

Trigonometry and Angle Measurement (13.1/13.2) Quiz on Tuesday!

Angles in Standard Position

In a coordinate plane, an angle can be formed by fixing one ray, called the <u>initial side</u>, and rotating the other ray, called the <u>terminal side</u>, about the vertex.

An angle is in <u>standard position</u> if its vertex is at the origin and its initial side lies on the positive x-axis.

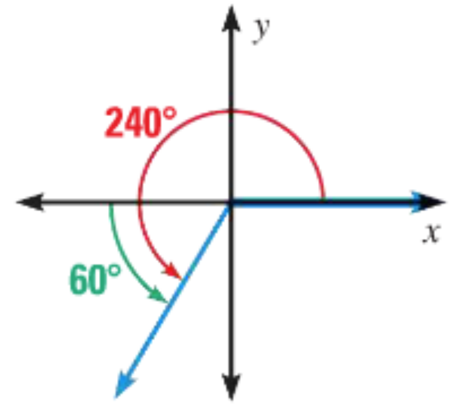


<u>Positive angle</u>: go counter-clockwise (left)
<u>Negative angle</u>: go clockwise (right)

Draw an angle with the given measure in standard position:

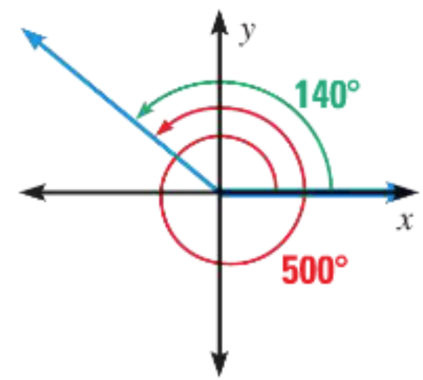
1) 240º

Because 240° is 60° more than 180°, the terminal side is 60° counterclockwise past the negative x-axis.



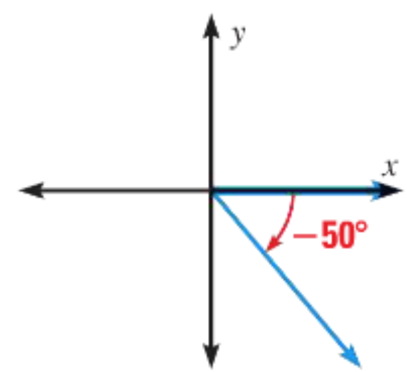
2) 500º

Because 500° is 140° more than 360°, the terminal side makes one whole revolution counterclockwise plus 140° more.



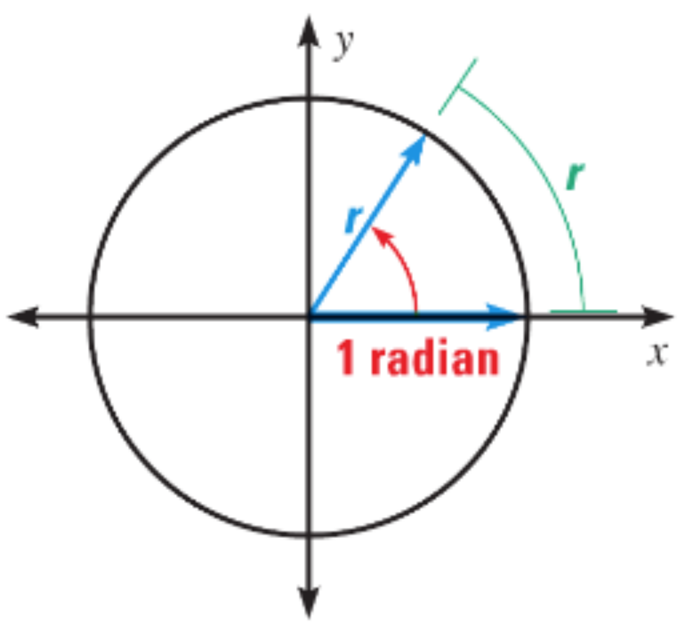
3) -50º

Because −50° is negative, the terminal side is 50° clockwise from the positive x-axis.



Radian Measure

Angles can also be measured in *radians*. To define a radian, consider a circle with radius *r* centered at the origin as shown. One radian is the measure of an angle in standard position whose terminal side intercepts an arc of length *r*.

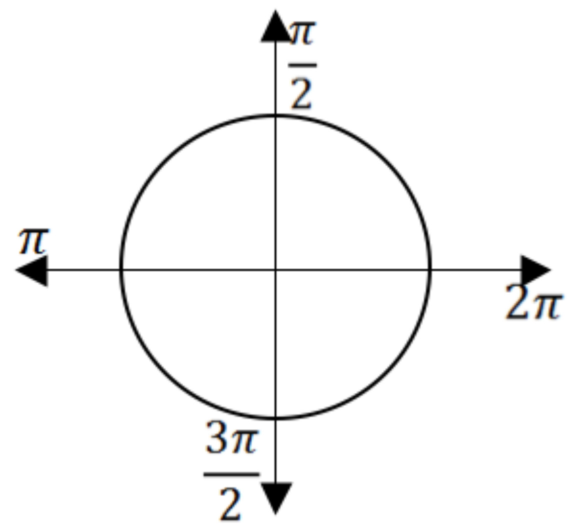


Converting Between Degrees and Radians

Because the circumference of a circle is $2\pi r$, there are 2π radians in a full circle. Degree measure and radian measure are therefore related by the equation $360^o = 2\pi$ radians, or $180^o = \pi$ radians.

Degrees to radians:

multiply degree measure by $\frac{\pi radians}{180^o}$



Radians to degrees:

multiply radian measure by $\frac{180^o}{\pi radians}$

Convert between degrees and radians:

1) 125°

$$=125^o\left(\frac{\pi radians}{180^o}\right) \quad =\frac{25\pi}{36}radians$$

2) $-\dfrac{5\pi}{4}radians$

$$=-\frac{5\pi}{4}radians\left(\frac{180^o}{\pi radians}\right) = -225^o$$

Coterminal Angles

Angles whose terminal sides are the same ($\pm$ 360° or $\pm$ 2π)

Find one positive and one negative angle that is coterminal with:

1) -45°

-45° + 360° = 315°

-45° – 360° = -405°

2) $\dfrac{5\pi}{3}$

$$\frac{5\pi}{3}+2\pi=\frac{5\pi}{3}+\frac{6\pi}{3}=\frac{11\pi}{3}$$

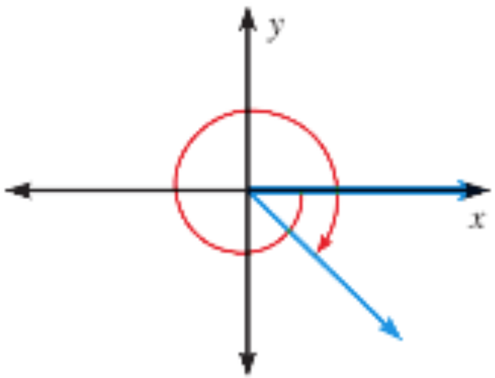
$$\frac{5\pi}{3}-2\pi=\frac{5\pi}{3}-\frac{6\pi}{3}=-\frac{\pi}{3}$$

Homework p. 862: 3-5, 16, 20, 24, 28

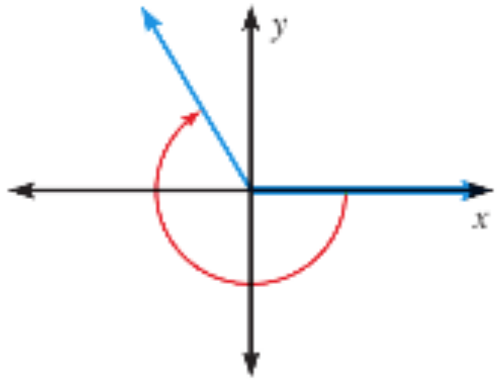
VISUAL THINKING **Match the angle measure with the angle.**

3. -240° **4.** 600° **5.** $-\frac{9\pi}{4}$

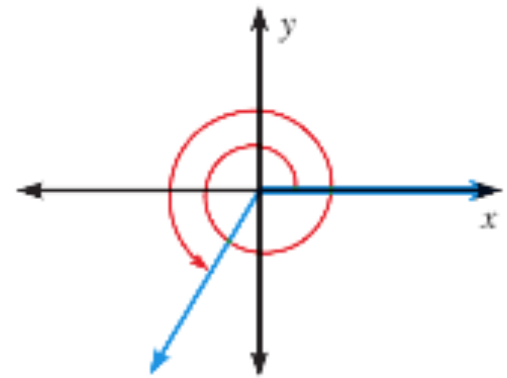
A.



B.



C.



FINDING COTERMINAL ANGLES **Find one positive angle and one negative angle that are coterminal with the given angle.**

16. 255° **20.** $-\frac{7\pi}{6}$

CONVERTING MEASURES **Convert the degree measure to radians or the radian measure to degrees.**

24. 315° **28.** $-\frac{\pi}{4}$

Objective

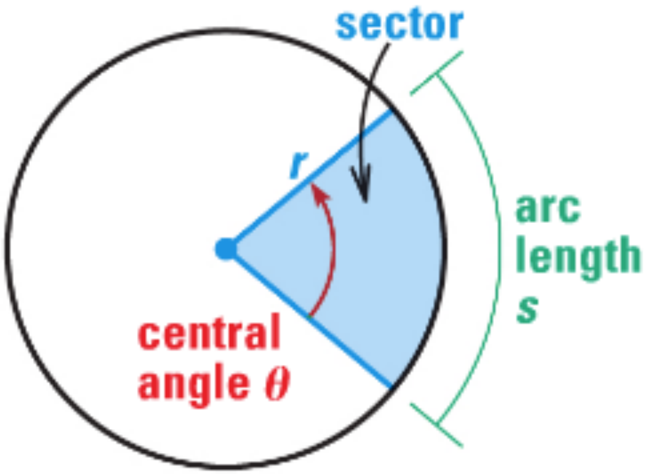
Students will be able to use general angles that may be measured in radians.

Trigonometry and Angle Measurement (13.1/13.2) Quiz on Tuesday!

Arc Length and Area of a Sector

A <u>sector</u> is a region of a circle that is bounded by two radii and an arc of a circle. The <u>central angle</u> θ of a sector is the angle formed by the two radii.

The arc length *s* and area *A* of a sector with radius *r* and central angle θ (measured in radians) are as follows.



Arc length: $s = r\theta$

Area: $A = \frac{1}{2}r^2\theta$

Find the arc length and area of a sector with the given radius, r, and central angle θ.

1) r = 3 m, θ = 5π/12

$$s = r\theta = 3\left(\frac{5\pi}{12}\right) = \frac{5\pi}{4} \approx 3.93 \text{ meters}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2\left(\frac{5\pi}{12}\right) = \frac{9}{2}\left(\frac{5\pi}{12}\right) = \frac{15\pi}{8} \approx 5.89 \text{ m}^2$$

2) r = 15 cm, θ = 45°

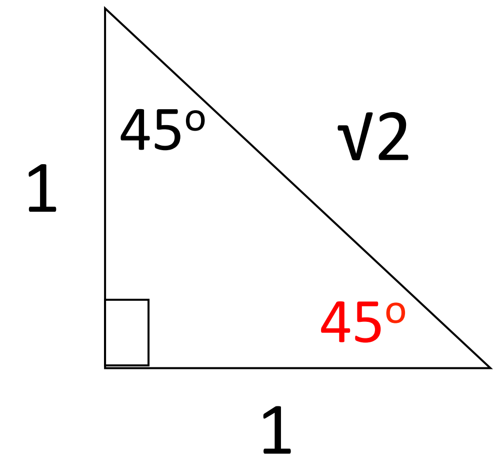
$$45^o = 45^o\left(\frac{\pi radians}{180^o}\right) = \frac{\pi}{4} radians$$

$$s = r\theta = 15\left(\frac{\pi}{4}\right) = \frac{15\pi}{4} \approx 11.78 \text{ cm}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(15)^2\left(\frac{\pi}{4}\right) = \frac{225}{2}\left(\frac{\pi}{4}\right) = \frac{225\pi}{8} \approx 88.36 \text{ cm}^2$$

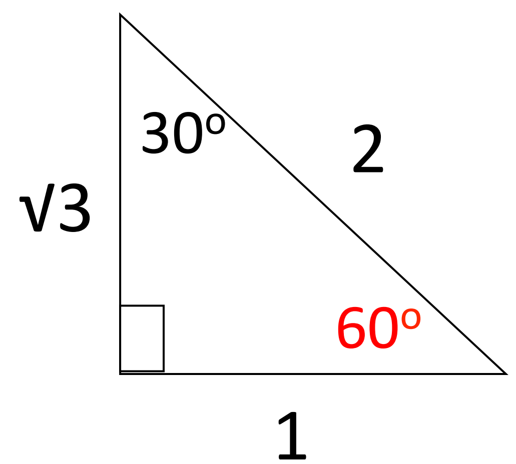
Evaluate the trig functions without using a calculator. Sketch the right triangle used to solve.

1) $\cos\dfrac{\pi}{4} = \cos 45^\circ = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$



$$= \frac{\pi}{4} radians \left(\frac{180^o}{\pi radians} \right) = 45^\circ$$

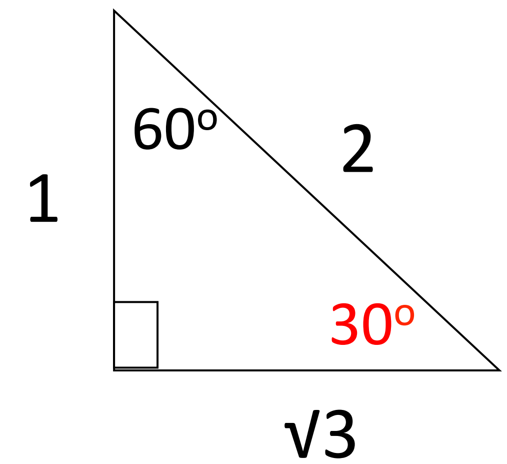
2) $\sin\dfrac{\pi}{3} = \sin 60^\circ = \dfrac{\sqrt{3}}{2}$



$$= \frac{\pi}{3} radians \left(\frac{180^o}{\pi radians} \right) = 60^\circ$$

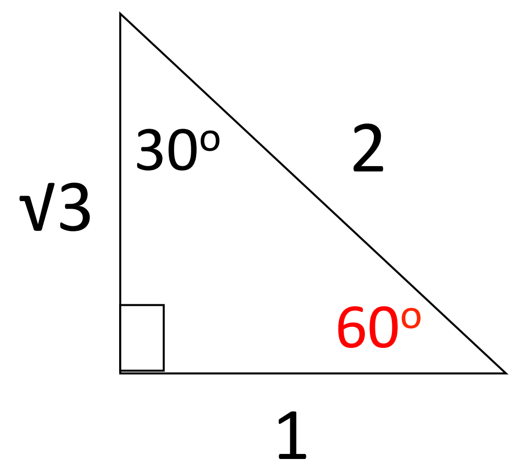
Evaluate the trig functions without using a calculator. Sketch the right triangle used to solve.

$$3)\quad \sec\frac{\pi}{6} = \sec 30^\circ = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$



$$= \frac{\pi}{6} radians\left(\frac{180^o}{\pi radians}\right) = 30^\circ$$

$$4)\quad \tan\frac{\pi}{3} = \tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$



$$= \frac{\pi}{3} radians\left(\frac{180^o}{\pi radians}\right) = 60^\circ$$

Homework

p. 863: 32, 36, 39, 40, 41, extra prb.

FINDING ARC LENGTH AND AREA **Find the arc length and area of a sector with the given radius r and central angle θ.**

32. $r = 4$ in., $\theta = \frac{\pi}{6}$

36. $r = 18$ m, $\theta = 25°$

Evaluate the trig functions without using a calculator. Sketch the right triangle used to solve.

39. $\cos \frac{\pi}{3}$

40. $\sin \frac{\pi}{4}$

41. $\tan \frac{\pi}{6}$

ALSO: $\csc \frac{\pi}{3}$

Objective

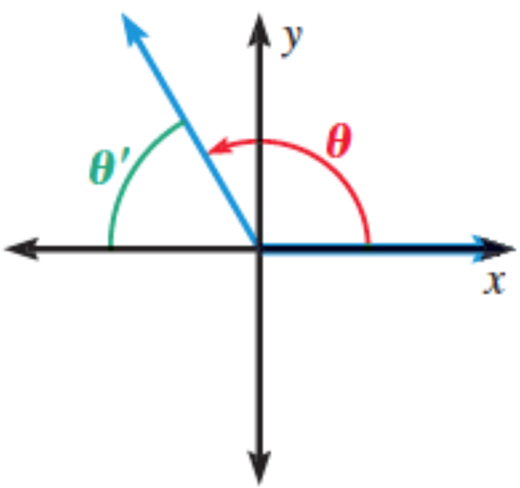
Students will be able to find reference angles and find values of trigonometric functions for general angles in Quadrant I.

Reference Angle Relationships

Let θ be an angle in standard position. The <u>reference angle</u> for θ is the acute angle θ’ formed by the terminal side of θ and the *x*-axis.

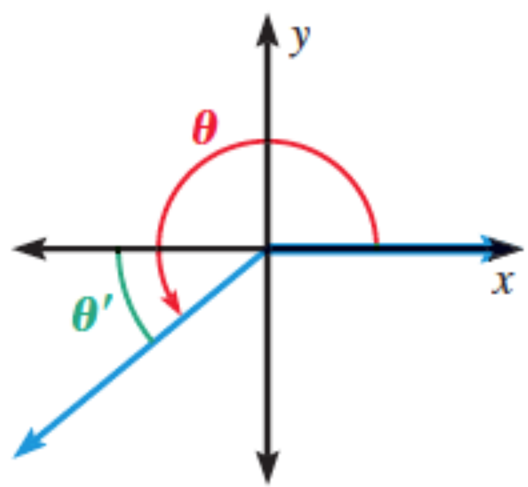
**In Quadrant I, θ = θ’

Quadrant II



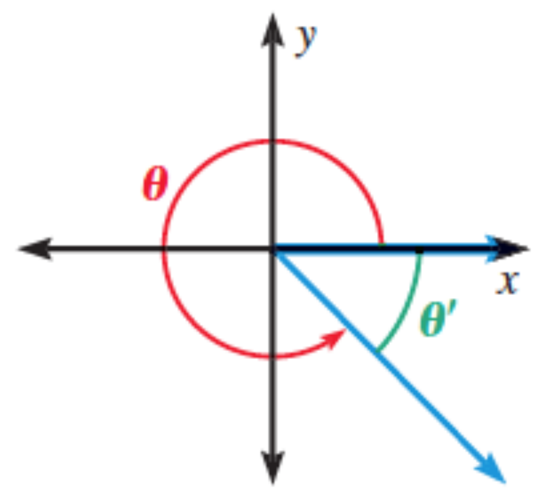
Degrees: $\theta' = 180° - \theta$
Radians: $\theta' = \pi - \theta$

Quadrant III



Degrees: $\theta' = \theta - 180°$
Radians: $\theta' = \theta - \pi$

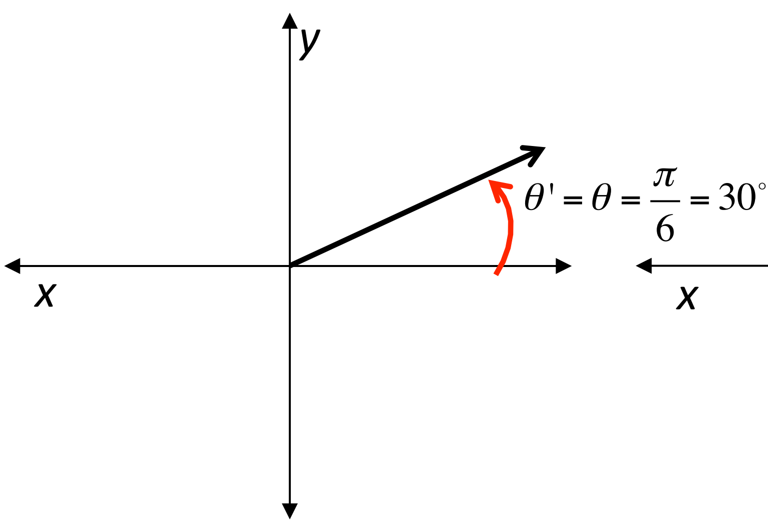
Quadrant IV

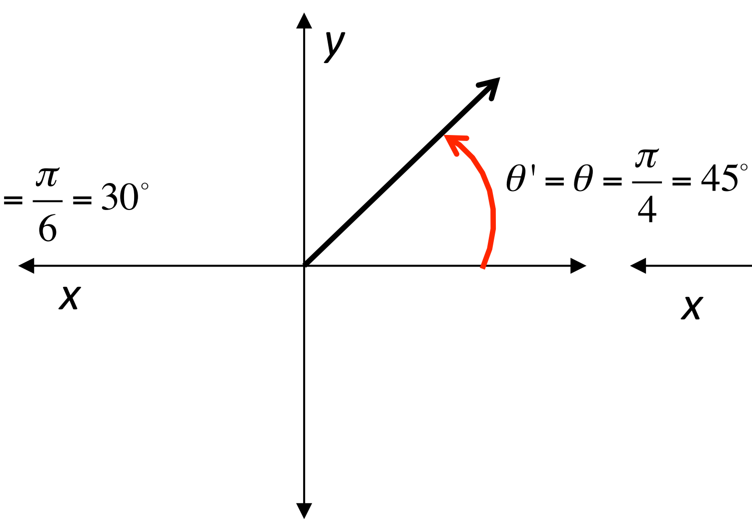


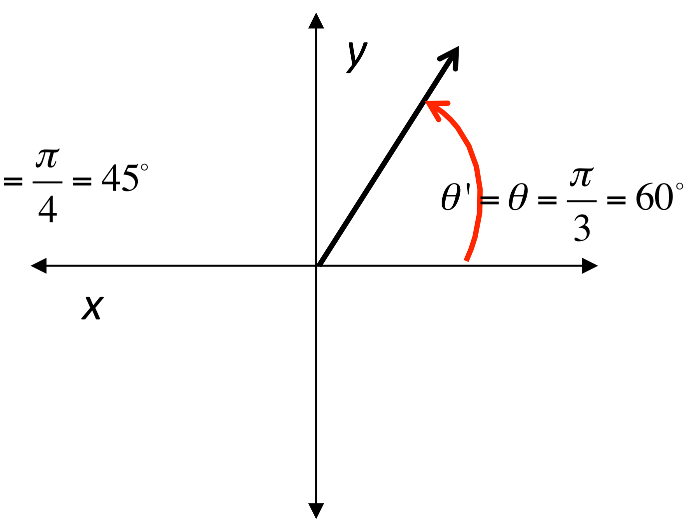
Degrees: $\theta' = 360° - \theta$
Radians: $\theta' = 2\pi - \theta$

Reference Angles in Quadrant I

$$\theta = \frac{\pi}{6} = 30^\circ \qquad \theta = \frac{\pi}{4} = 45^\circ \qquad \theta = \frac{\pi}{3} = 60^\circ$$







Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ. (1, 1)

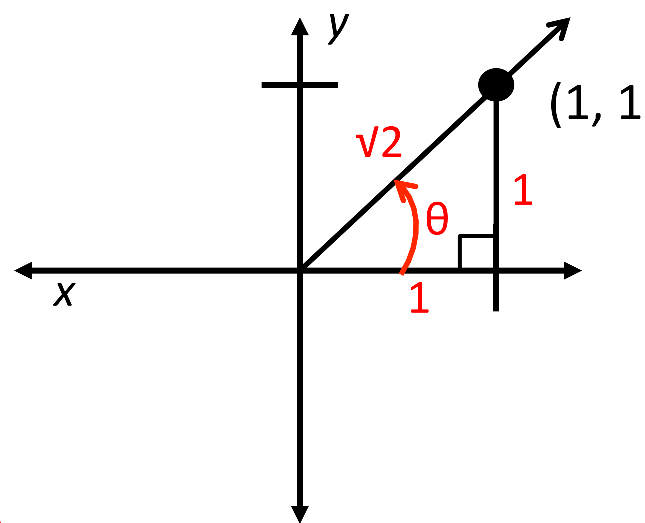
1) Draw point on coordinate plane

2) Draw terminal side and create right triangle

3) Label side lengths off of point

4) Use Pythagorean theorem to find missing side (hypotenuse of triangle)

5) Find six trig functions of θ



$$= \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\sin\theta = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan\theta = \frac{1}{1} = 1$$

$$\csc\theta = \frac{\sqrt{2}}{1} = \sqrt{2} \qquad \sec\theta = \frac{\sqrt{2}}{1} = \sqrt{2} \qquad \cot\theta = \frac{1}{1} = 1$$

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.

(3, 4)

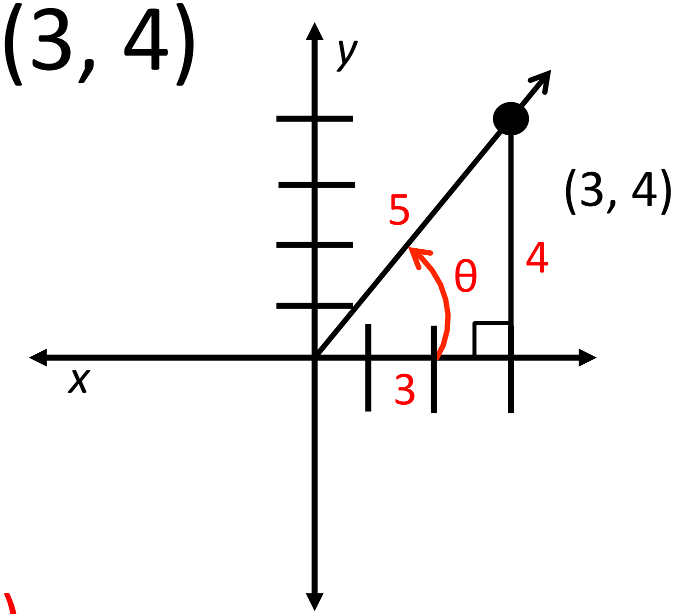
1) Draw point on coordinate plane

2) Draw terminal side and create right triangle

3) Label side lengths off of point

4) Use Pythagorean theorem to find missing side (hypotenuse of triangle)

5) Find six trig functions of θ



$$= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\sin\theta = \frac{4}{5} \qquad \cos\theta = \frac{3}{5} \qquad \tan\theta = \frac{4}{3}$$

$$\csc\theta = \frac{5}{4} \qquad \sec\theta = \frac{5}{3} \qquad \cot\theta = \frac{3}{4}$$

Homework

Evaluating Trig Functions in QI Worksheet

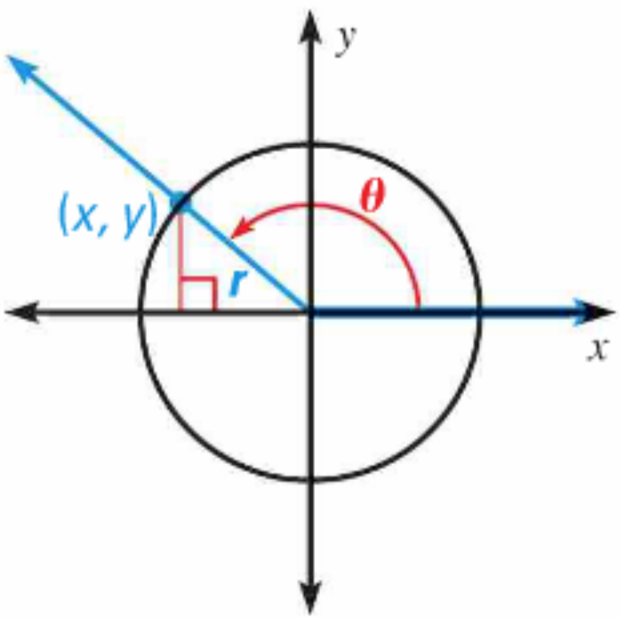
Objective

Students will be able to find values of trigonometric functions for general angles.

Evaluate Trigonometric Functions of Any Angle (13.1-13.3) Test next Friday!

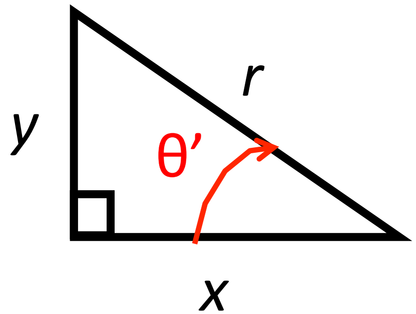
General Definitions of Trigonometric Functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as follows:



$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$



Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ. (-5, 3)

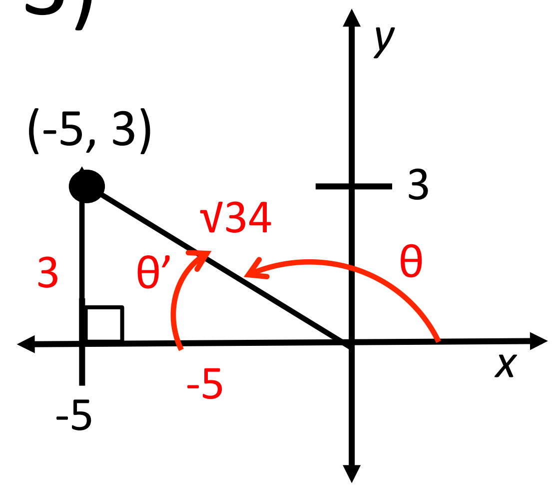
1) Draw point on coordinate plane

2) Draw terminal side and create right triangle

3) Label side lengths off of point

4) Use Pythagorean theorem to find missing side (hypotenuse of triangle)

5) Find six trig functions of θ

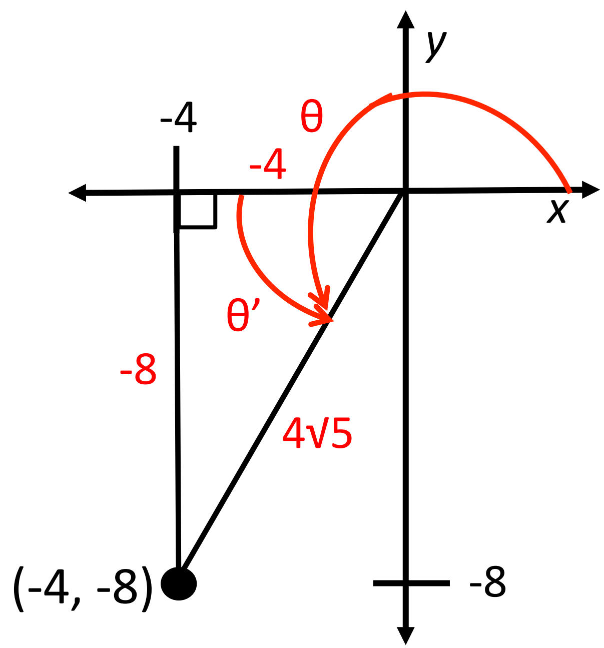


$$= \sqrt{(-5)^2 + 3^2} = \sqrt{34}$$

$$\sin\theta = \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{3\sqrt{34}}{34} \qquad \cos\theta = -\frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = -\frac{5\sqrt{34}}{34} \qquad \tan\theta = -\frac{3}{5}$$

$$\csc\theta = \frac{\sqrt{34}}{3} \qquad \sec\theta = -\frac{\sqrt{34}}{5} \qquad \cot\theta = -\frac{5}{3}$$

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ. (-4, -8)



$$= \sqrt{(-8)^2 + (-4)^2}$$

$$= \sqrt{80}$$

$$= 4\sqrt{5}$$

$$\sin\theta = -\frac{2\sqrt{5}}{5} \qquad \cos\theta = -\frac{\sqrt{5}}{5} \qquad \tan\theta = 2$$

$$\csc\theta = -\frac{\sqrt{5}}{2} \qquad \sec\theta = -\sqrt{5} \qquad \cot\theta = \frac{1}{2}$$

Homework

p. 870: 4, 6-10

USING A POINT **Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.**

4. $(-9, 12)$ **6.** $(5, -12)$ **7.** $(2, -2)$

8. $(-6, 9)$ **9.** $(-3, -5)$ **10.** $(5, -\sqrt{11})$

Objective

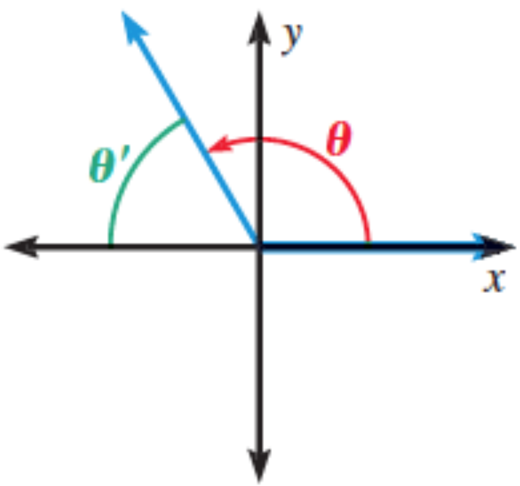
Students will be able to find reference angles.

Evaluate Trigonometric Functions of Any Angle (13.1-13.3) Test next Friday!

Reference Angle Relationships

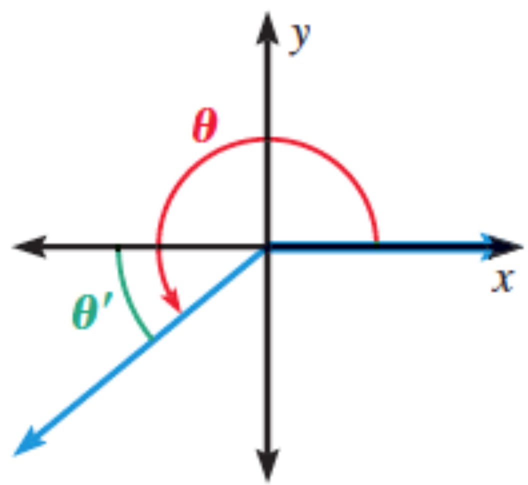
Let θ be an angle in standard position. The <u>reference angle</u> for θ is the acute angle θ' formed by the terminal side of θ and the *x*-axis.

Quadrant II



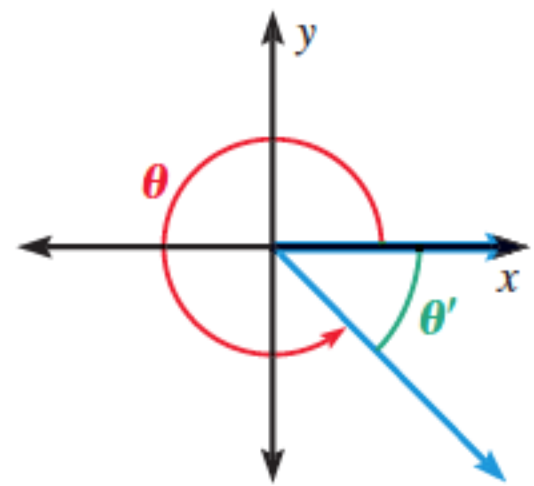
Degrees: $\theta' = 180° - \theta$
Radians: $\theta' = \pi - \theta$

Quadrant III



Degrees: $\theta' = \theta - 180°$
Radians: $\theta' = \theta - \pi$

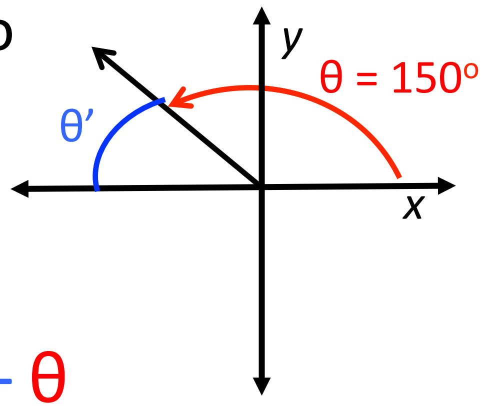
Quadrant IV



Degrees: $\theta' = 360° - \theta$
Radians: $\theta' = 2\pi - \theta$

Sketch the following angles and then find its reference angle.

1) 150°



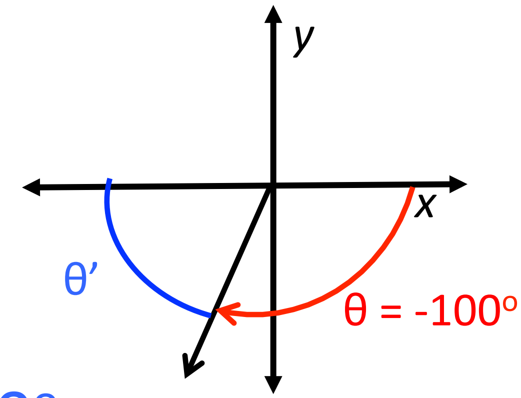
$\theta' = 180^\circ - \theta$

$= 180^\circ - 150^\circ = 30^\circ$

2) -100°

$\theta = -100^\circ + 360^\circ$

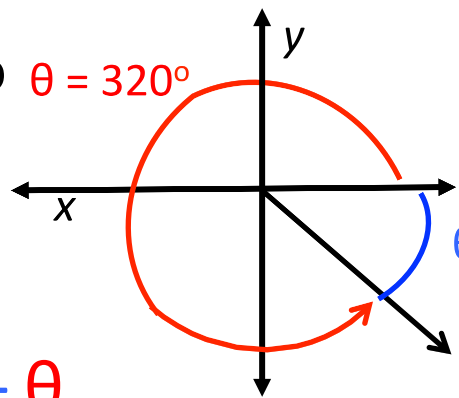
$= 260^\circ$



$\theta' = \theta - 180^\circ$

$= 260^\circ - 180^\circ = 80^\circ$

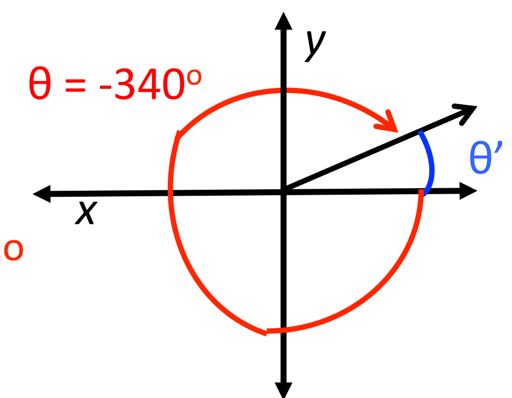
3) 320°



$\theta' = 360^\circ - \theta$

$= 360^\circ - 320^\circ = 40^\circ$

4) -340°



$\theta = -340^\circ + 360^\circ$

$= 20^\circ$

$\theta' = \theta = 20^\circ$

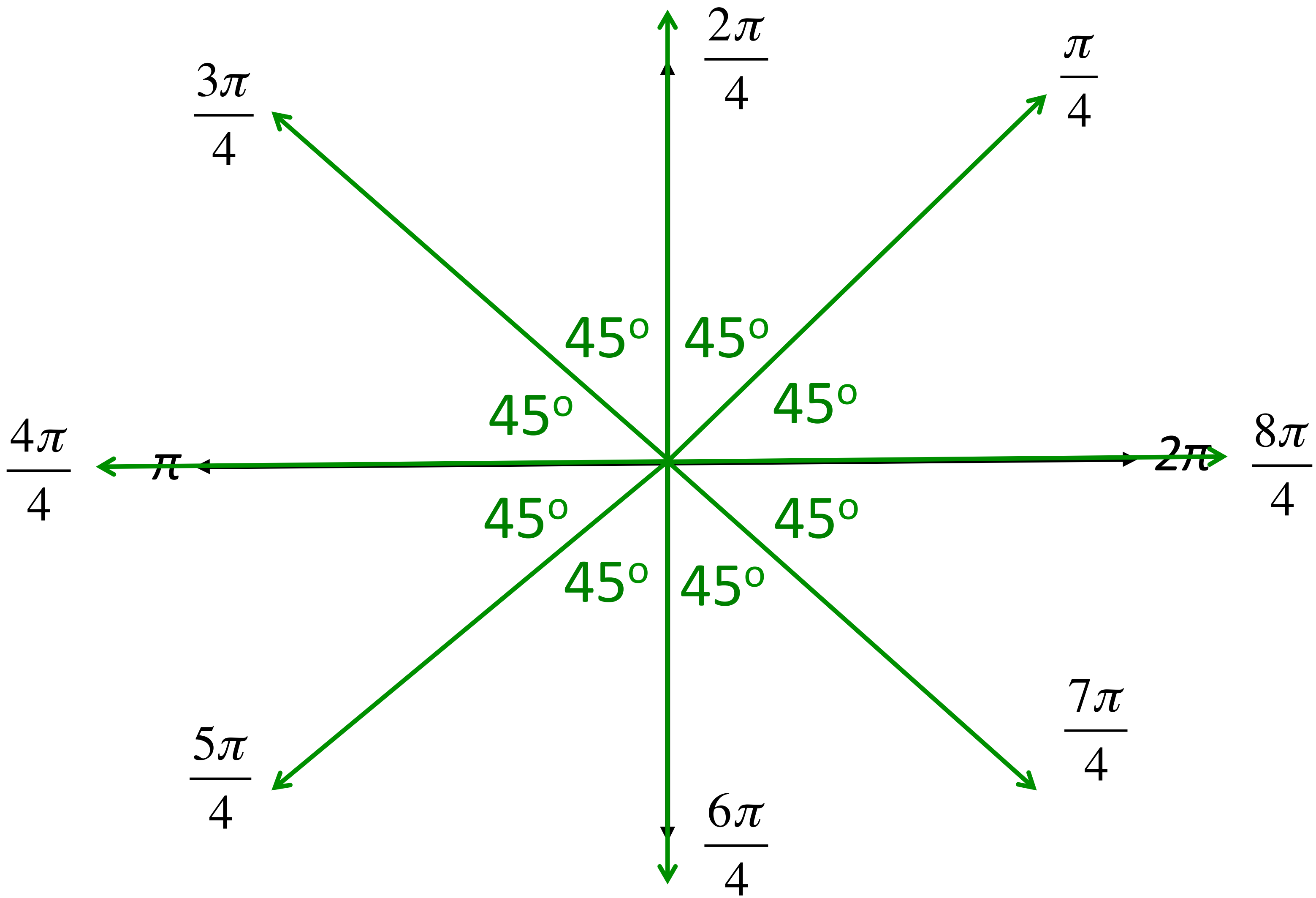
Reference Angles in Radians

On your worksheet, start by splitting up π or 180^o into fourths by counting by $\pi/4$ ($\pi/4$, $2\pi/4$, $3\pi/4$, etc.) and labeling. Then continue doing so between π and 2π. Label the angles that you have created in degrees and notice your reference angles for $\pi/4$.

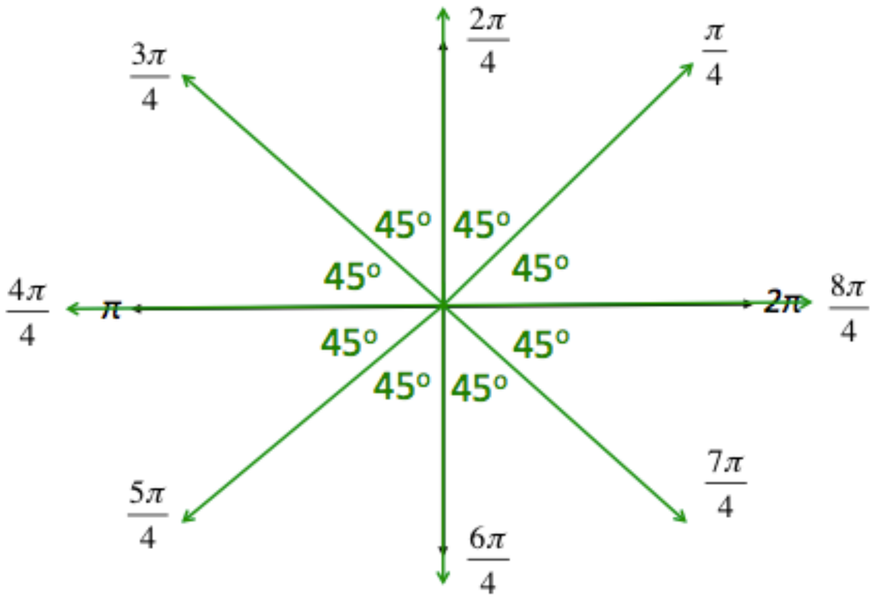
Then use the reference angle to sketch a right triangle to find sin $7\pi/4$ (remember to use what you know about special right triangles!)

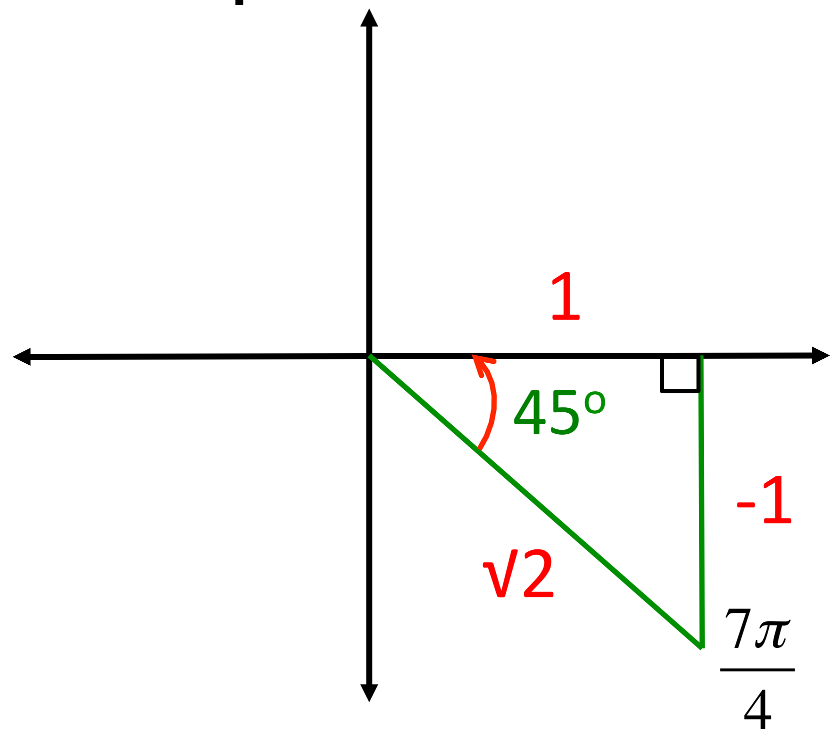
Once you have finished $\pi/4$, then do the same thing with $\pi/6$ but this time you are splitting π up into sixth and counting by $\pi/6$ and finding tan $7\pi/6$, and then with $\pi/3$ by splitting up π into thirds and counting by $\pi/3$ and finding cos $2\pi/3$.

Split up π into fourths (counting by $\frac{\pi}{4}$)



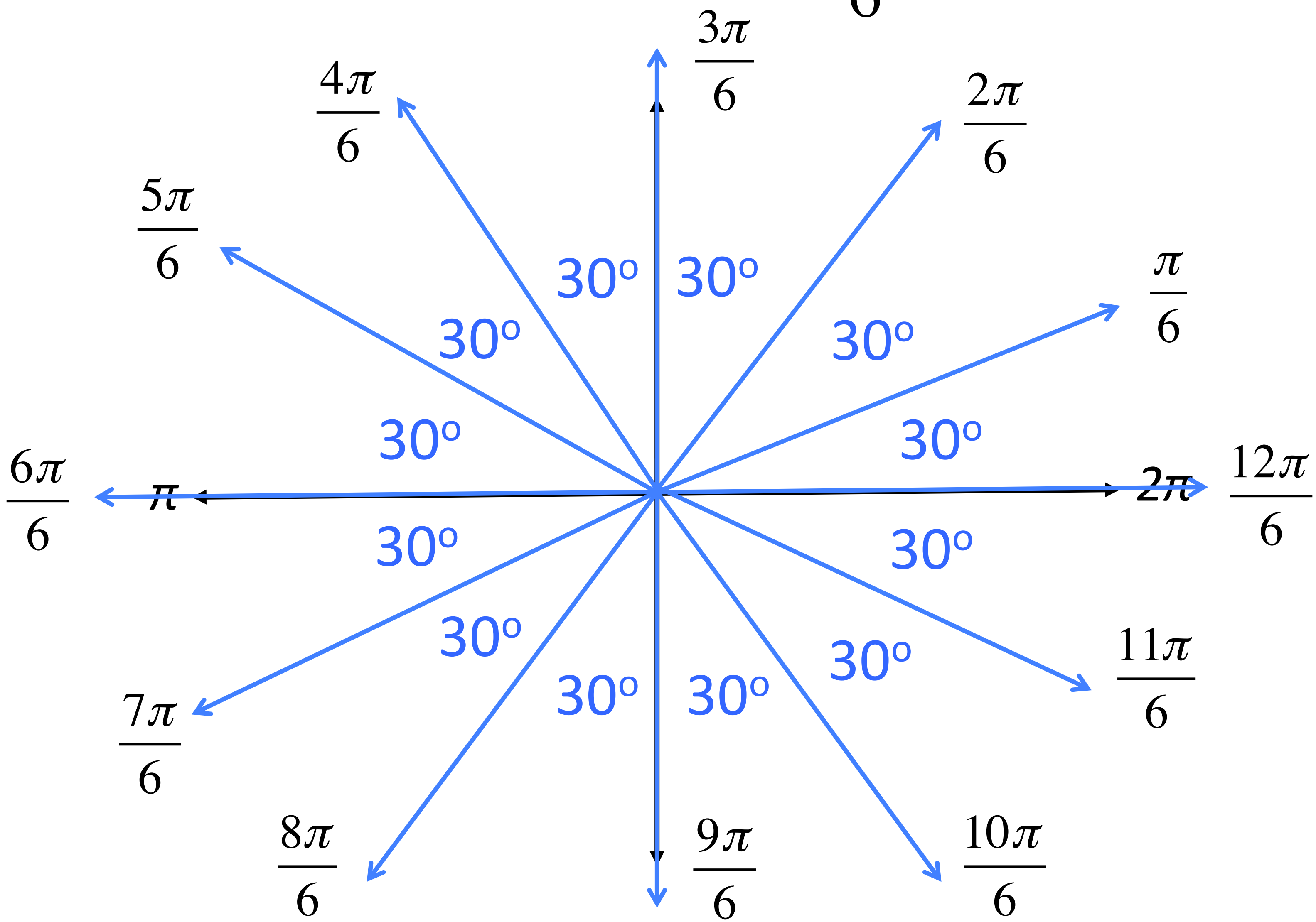
Use the reference angle to sketch a right triangle to find $\sin\frac{7\pi}{4}$.



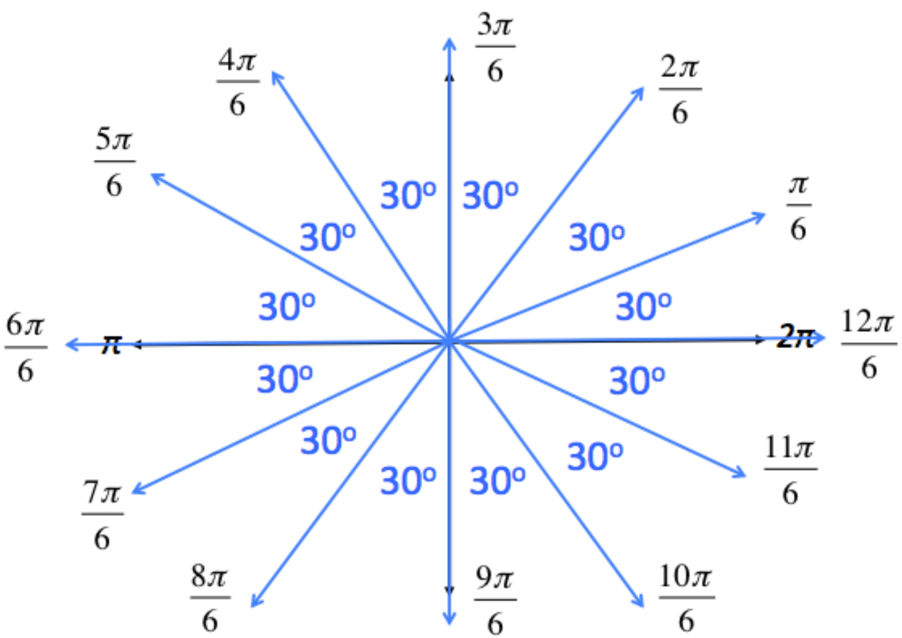


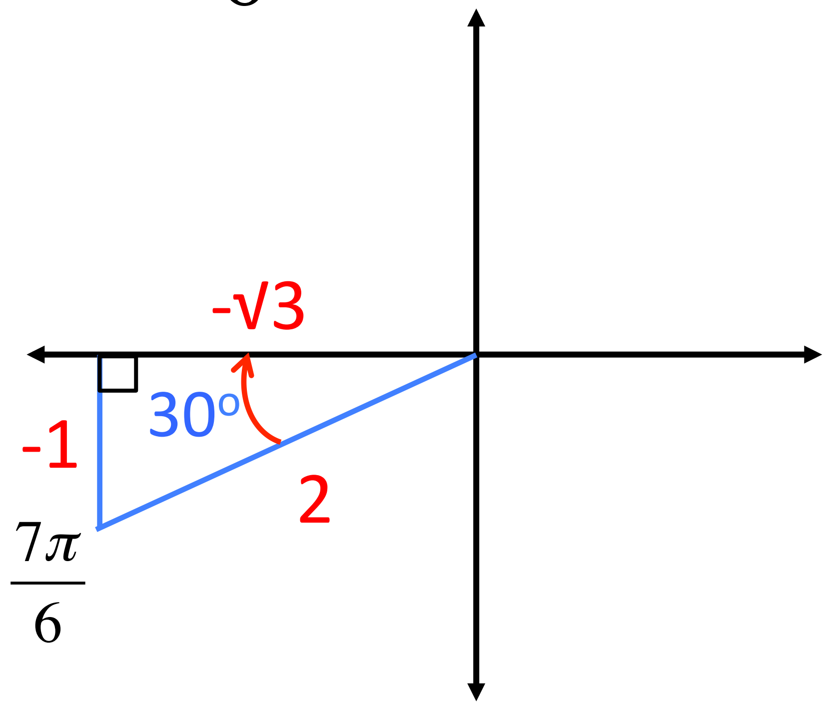
$$\sin\frac{7\pi}{4} = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Split up π into sixths (counting by $\frac{\pi}{6}$)



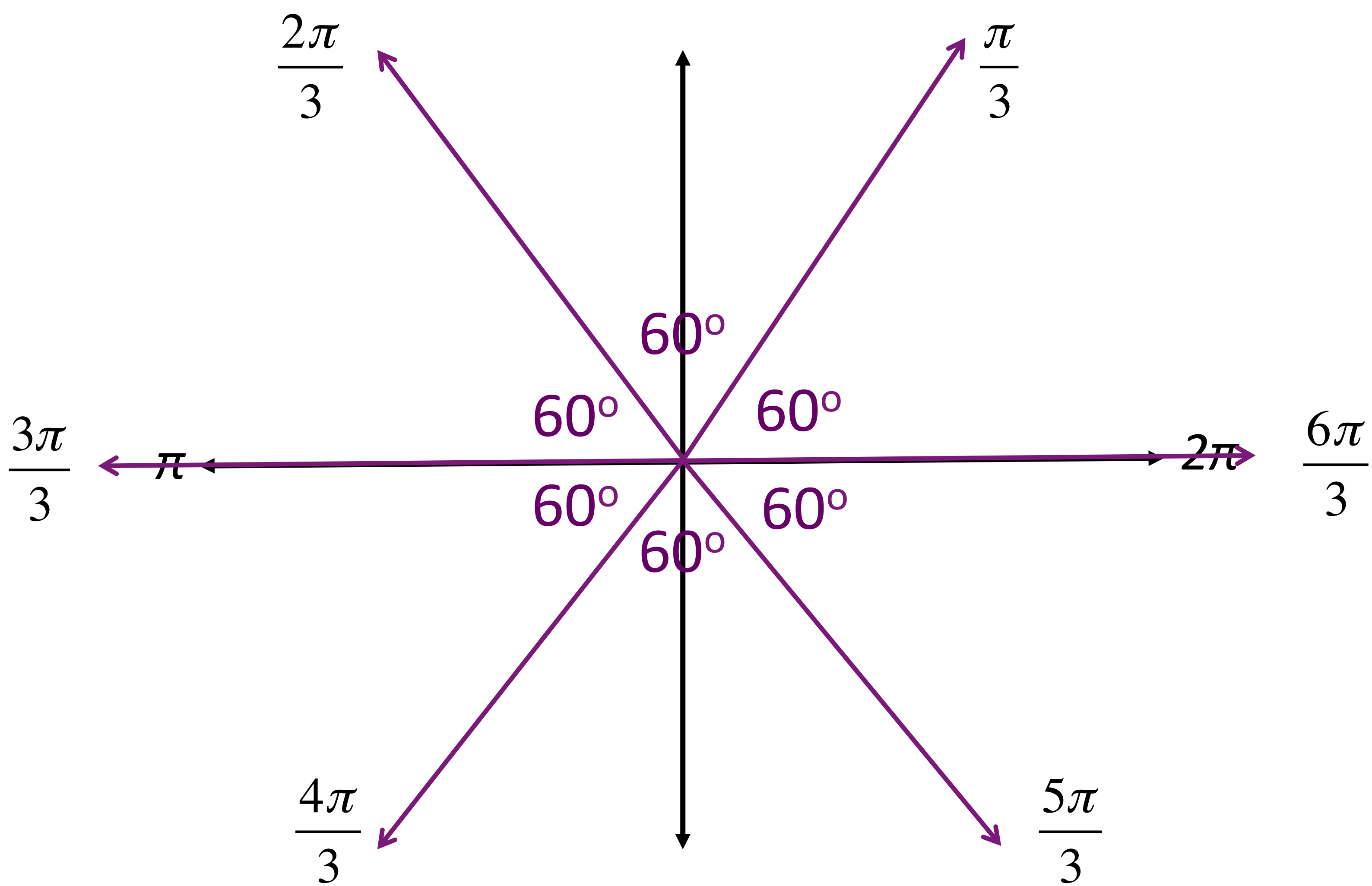
Use the reference angle to sketch a right triangle to find $\tan\frac{7\pi}{6}$.



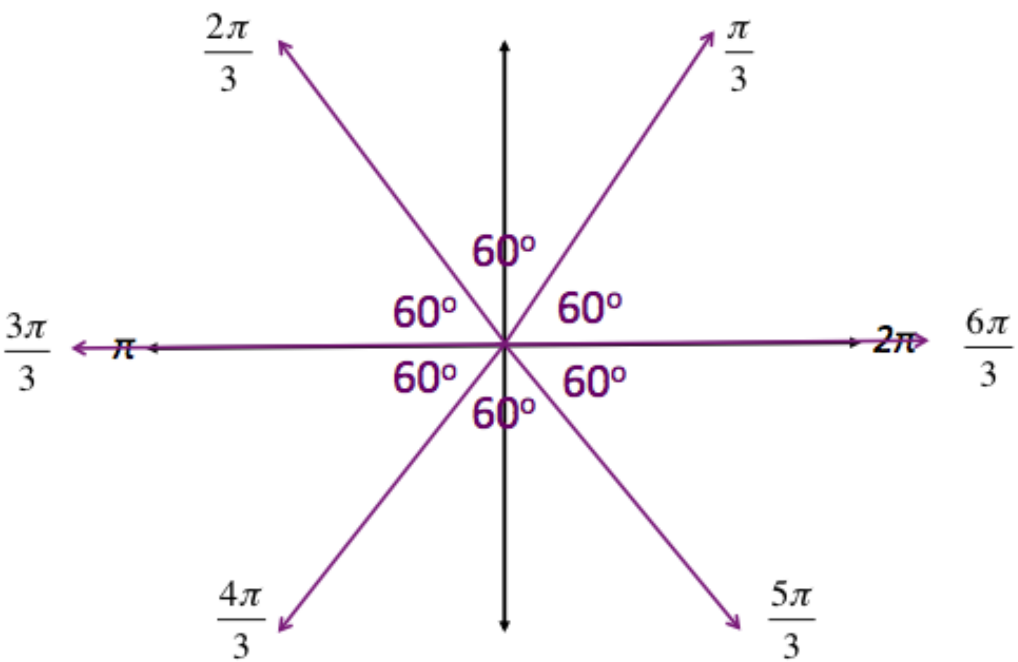


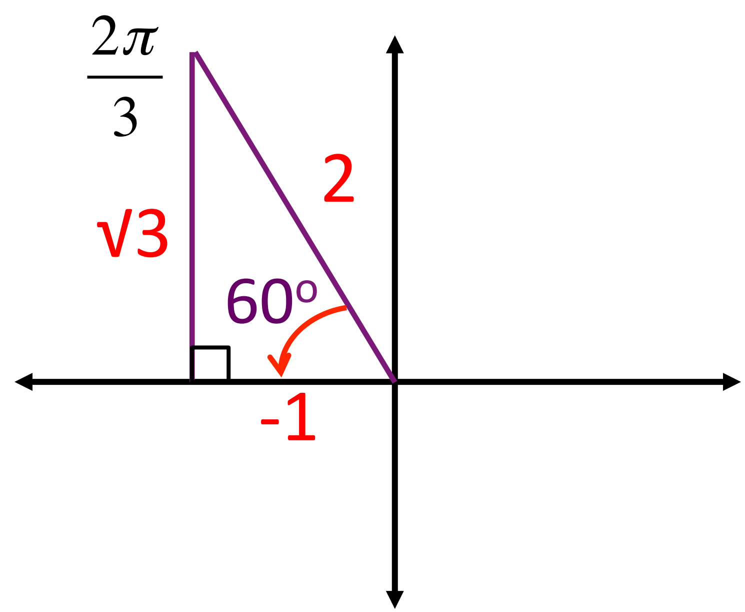
$$\tan\frac{7\pi}{6} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Split up π into thirds (counting by $\frac{\pi}{3}$)



Use the reference angle to sketch a right triangle to find $\cos\frac{2\pi}{3}$.





$$\cos\frac{2\pi}{3} = \frac{-1}{2} = -\frac{1}{2}$$

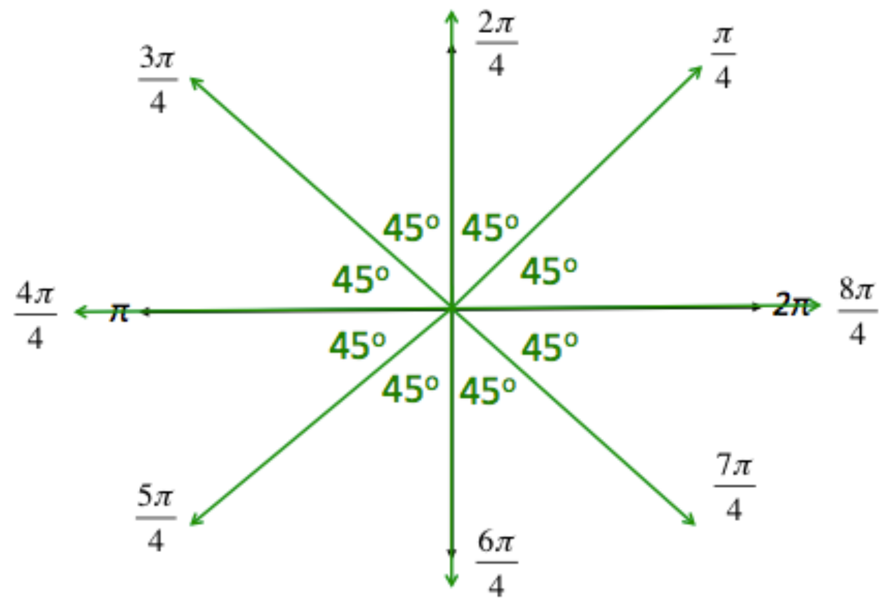
Homework

Reference Angle Worksheet (back side)

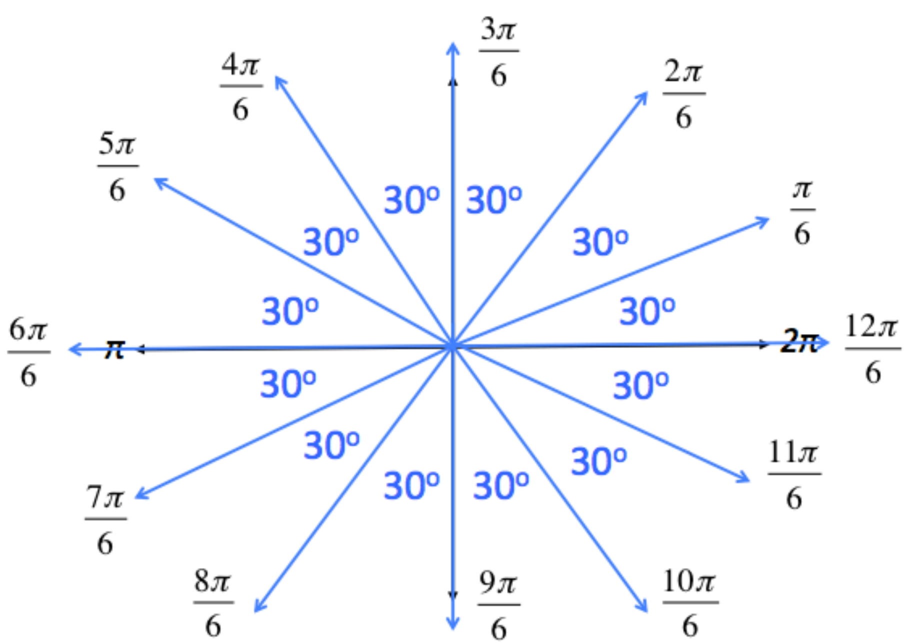
Objective

Students will be able to use their knowledge of special right triangles, degrees, radians, and reference angles to make the unit circle.

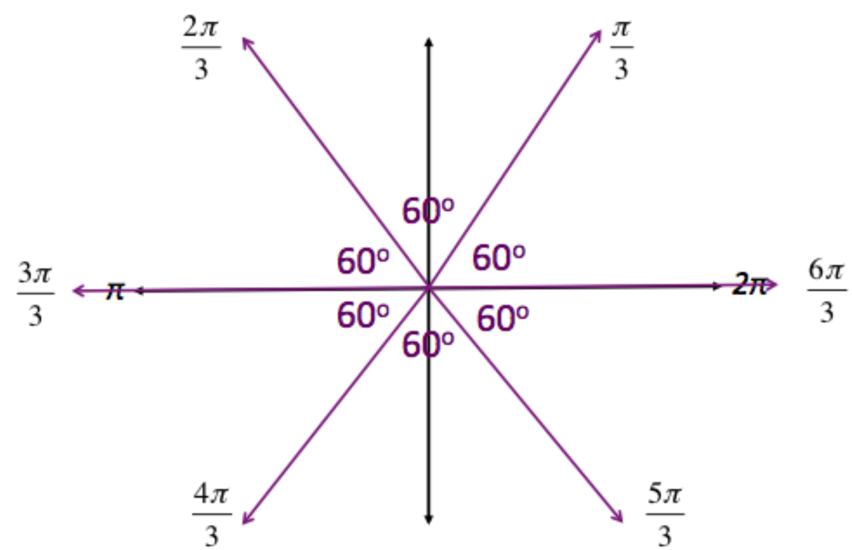
Evaluate Trigonometric Functions of Any Angle (13.1-13.3) Test on Thursday (Calculator Portion) and Friday (Non-Calculator Portion)!



2π/4
π/4
3π/4
4π/4
π
2π
8π/4
5π/4
6π/4
7π/4
45°
45°
45°
45°
45°
45°
45°
45°



3π/6
4π/6
2π/6
5π/6
π/6
6π/6
π
2π
12π/6
7π/6
11π/6
8π/6
9π/6
10π/6
30°
30°
30°
30°
30°
30°
30°
30°
30°
30°
30°
30°

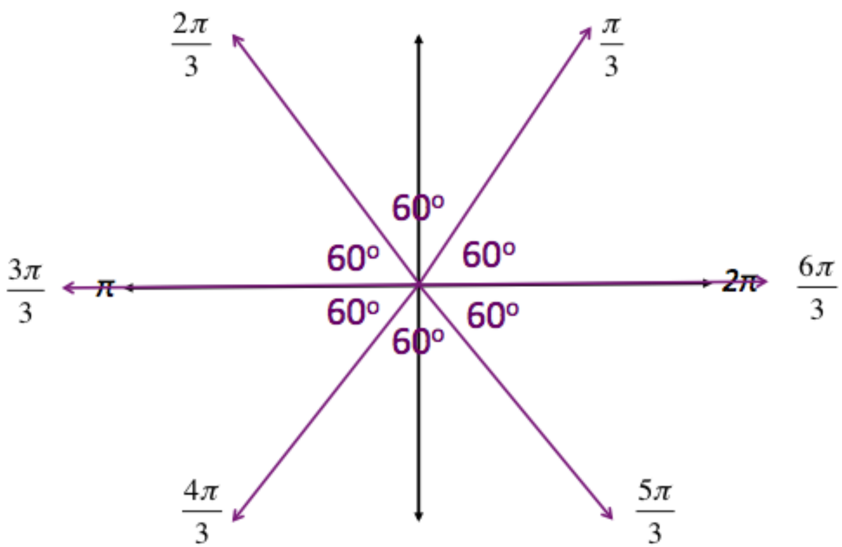


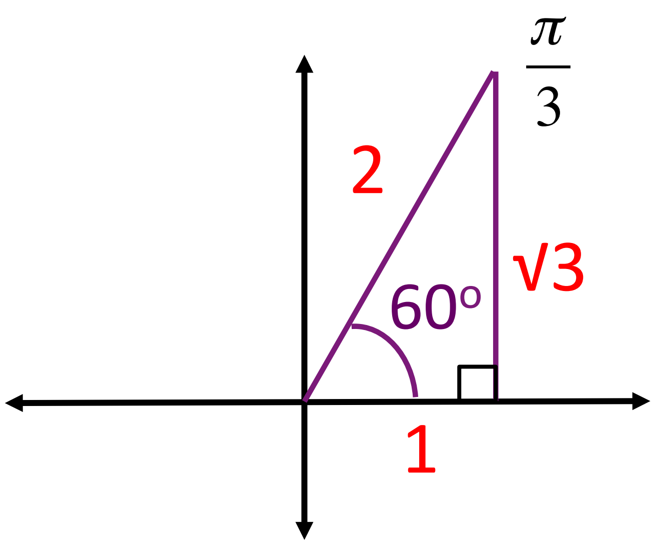
2π/3
π/3
3π/3
π
2π
6π/3
4π/3
5π/3
60°
60°
60°
60°
60°
60°

Use the reference angle to sketch a right triangle to find $\cos -\frac{5\pi}{3}$.

Find positive coterminal angle in standard position

$$\theta = -\frac{5\pi}{3} + 2\pi = -\frac{5\pi}{3} + \frac{6\pi}{3} = \frac{\pi}{3}$$



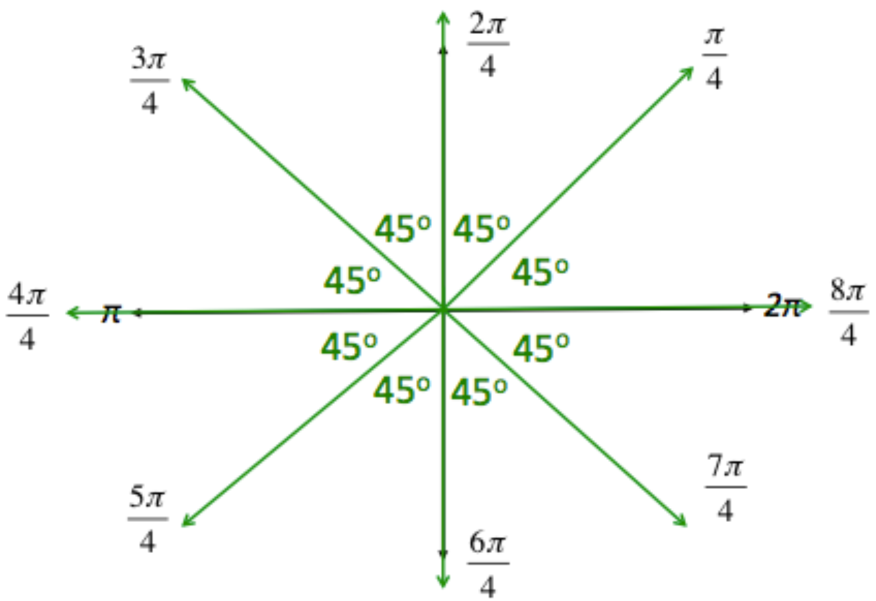


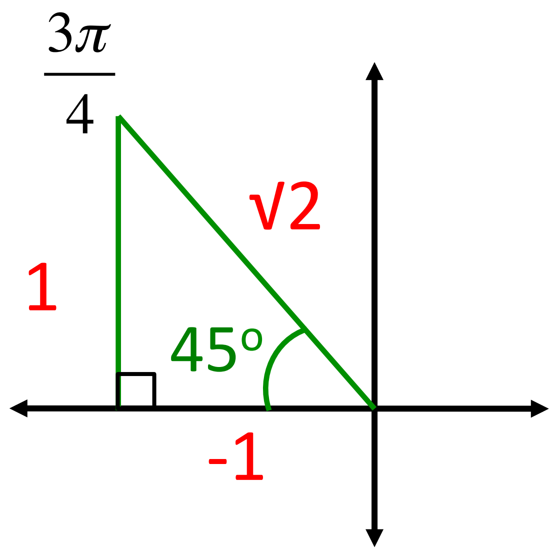
$$\cos -\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$$

Use the reference angle to sketch a right triangle to find $\csc\dfrac{11\pi}{4}$.

Find coterminal angle less than 2π in standard position

$$\theta = \frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$$

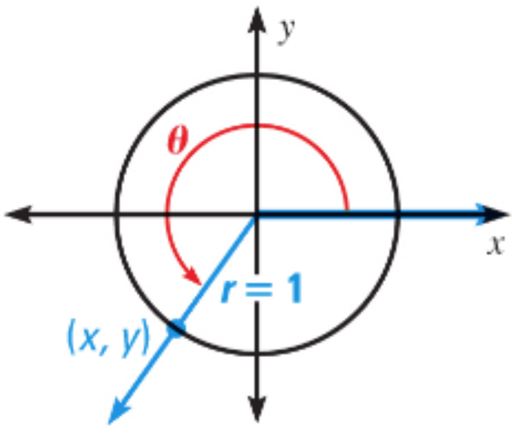




$$\csc\frac{11\pi}{4} = \csc\frac{3\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

Unit Circle

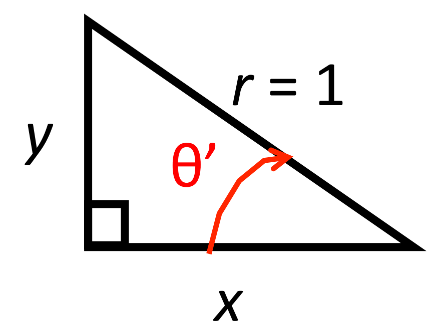
The circle $x^2 + y^2 = 1$, which has center (0, 0) and radius 1, is called the <u>unit circle</u>. The sin θ and cos θ are simply the *y*-coordinate and *x*-coordinate, respectively, of the point where the terminal side of θ intersects the unit circle.



$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$(\cos\theta, \sin\theta)$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

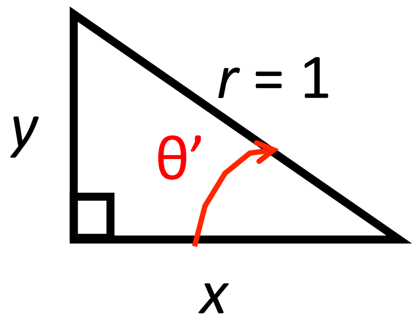


Unit Circle

The circle $x^2 + y^2 = 1$, which has center (0, 0) and radius 1, is called the unit circle.

$$\sin\theta = y \qquad \cos\theta = x$$

$$\tan\theta = \frac{y}{x} \qquad \boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$



Unit Circle

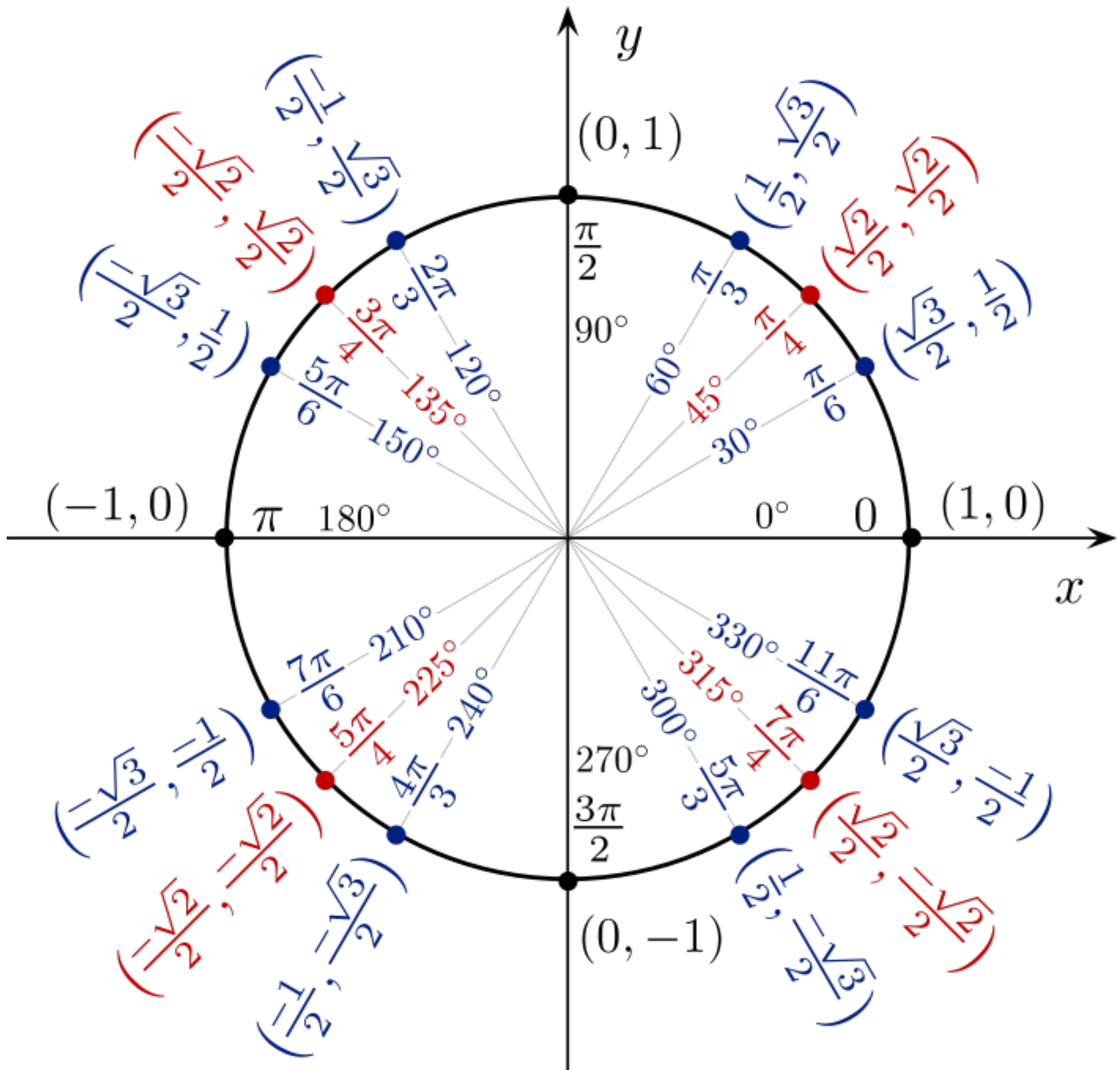
Use your knowledge of special right triangles, degrees, radians, and reference angles to make the unit circle. First complete the back portion (make each one a different color; for example: pi/6 is blue, pi/4 is green, and pi/3 is purple) and then copy it over to the first quadrant on your unit circle using the same colors and continue to fill in the rest.

Objective

Students will be able to define and use the trigonometric functions based on the unit circle.

Evaluate Trigonometric Functions of Any Angle (13.1-13.3) Test tomorrow (Calculator Portion) and Friday (Non-Calculator Portion)!

Unit Circle



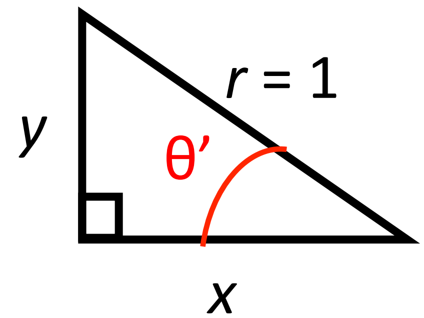
y
x
$(0,1)$
$(1,0)$
$(-1,0)$
$(0,-1)$
$\frac{\pi}{2}$
$90°$
π
$180°$
$0°$
0
$270°$
$\frac{3\pi}{2}$
$\frac{\pi}{6}$
$30°$
$\frac{\pi}{4}$
$45°$
$\frac{\pi}{3}$
$60°$
$\frac{2\pi}{3}$
$120°$
$\frac{3\pi}{4}$
$135°$
$\frac{5\pi}{6}$
$150°$
$\frac{7\pi}{6}$
$210°$
$\frac{5\pi}{4}$
$225°$
$\frac{4\pi}{3}$
$240°$
$\frac{5\pi}{3}$
$300°$
$\frac{7\pi}{4}$
$315°$
$\frac{11\pi}{6}$
$330°$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$
$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
$\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

Unit Circle

The circle $x^2 + y^2 = 1$, which has center (0, 0) and radius 1, is called the <u>unit circle</u>.

$$\sin\theta = y \qquad \cos\theta = x$$

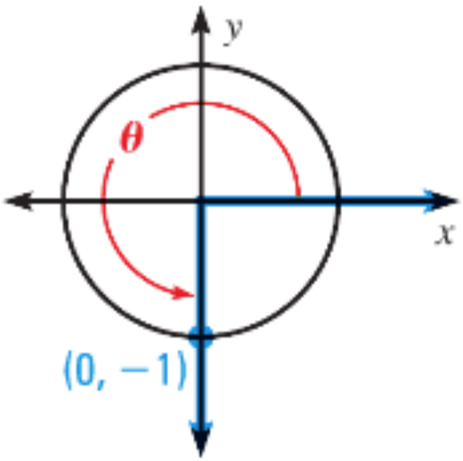
$$\tan\theta = \frac{y}{x} \qquad \boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$



Quadrantal Angle

A <u>quadrantal angle</u> is an angle in standard position whose terminal side lies on an axis.

EX: Use the unit circle to evaluate the six trigonometric functions of θ = 270º.



$$\sin\theta = \frac{y}{r} = \frac{-1}{1} = -1$$

$$\cos\theta = \frac{x}{r} = \frac{0}{1} = 0$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{0}$$

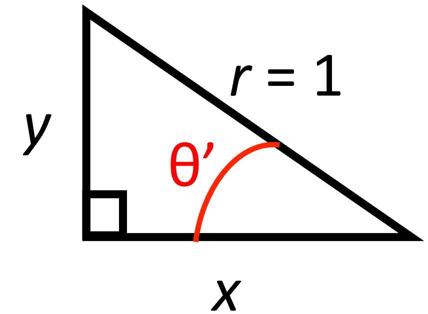
undefined

$$\csc\theta = \frac{r}{y} = \frac{1}{-1} = -1$$

$$\sec\theta = \frac{r}{x} = \frac{1}{0}$$

undefined

$$\cot\theta = \frac{x}{y} = \frac{0}{-1} = 0$$

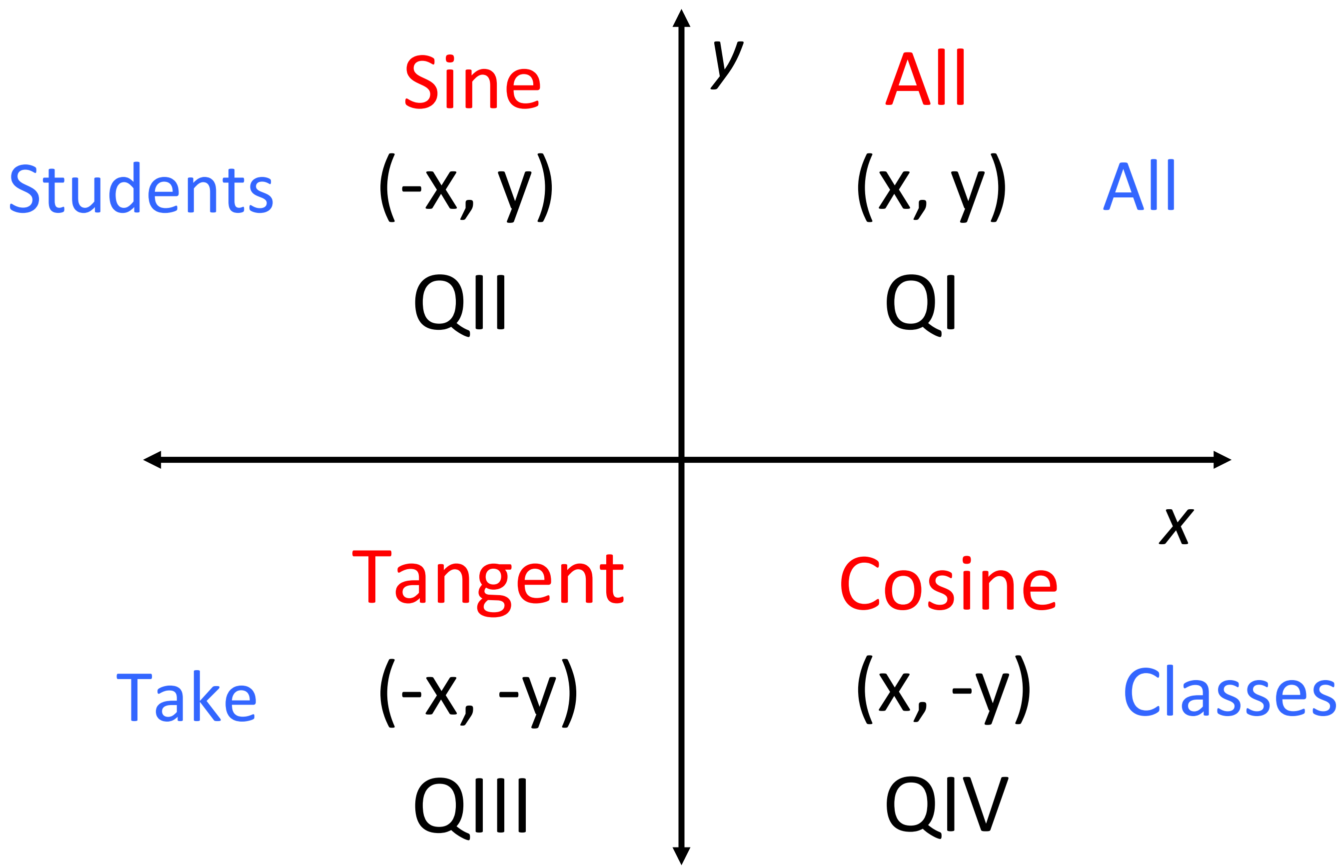


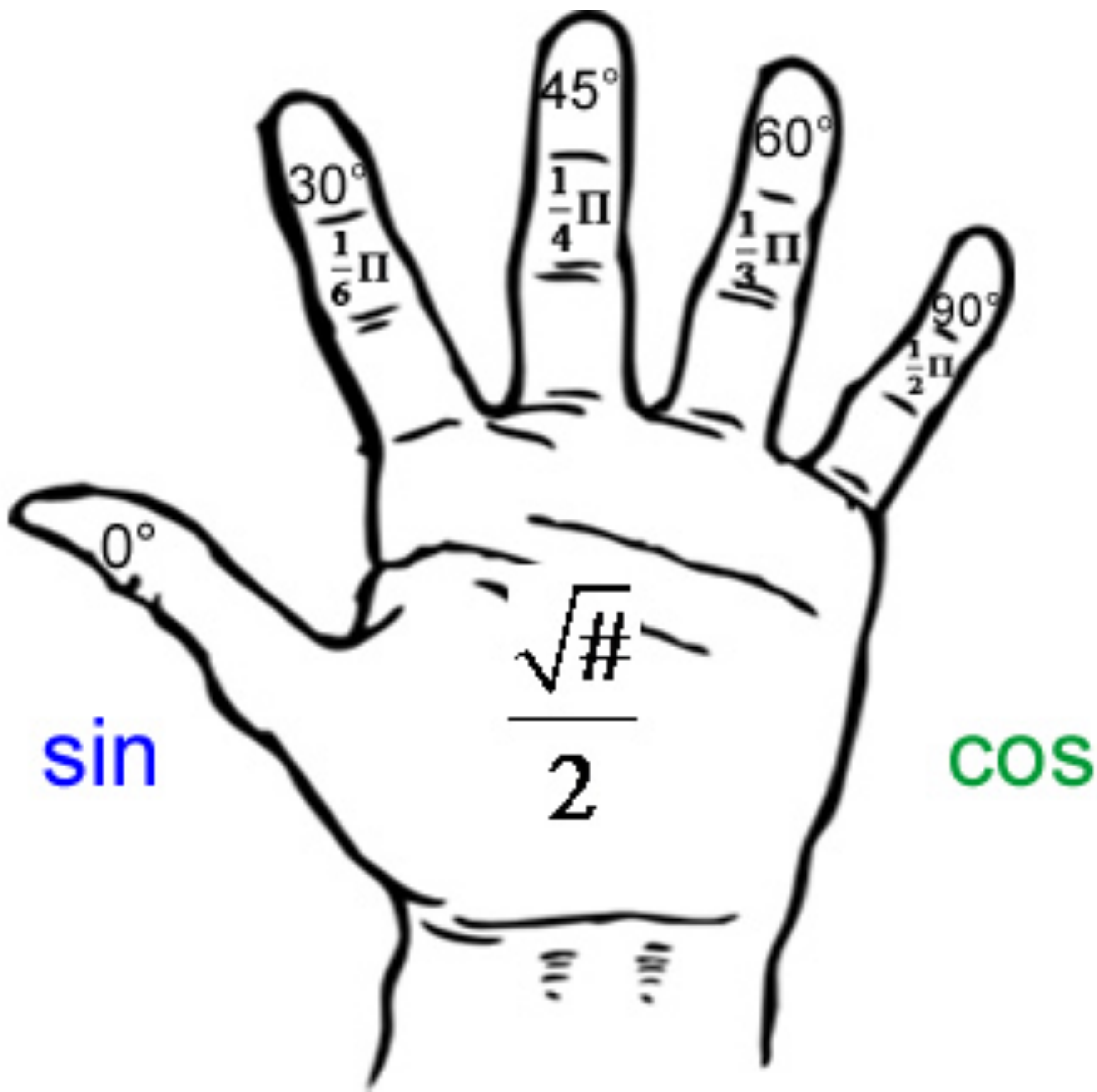
Tangent on Unit Circle

On your unit circle, find the tangent of each angle.

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

What trig functions are positive on the unit circle?





0°
30°
$\frac{1}{6}\Pi$
45°
$\frac{1}{4}\Pi$
60°
$\frac{1}{3}\Pi$
90°
$\frac{1}{2}\Pi$
$\frac{\sqrt{\#}}{2}$
sin
cos

Objective

Students will be able to evaluate inverse trigonometric functions.

Inverse Trigonometric Functions

We can also find an angle that corresponds to a given value of a trigonometric function.

What is the angle if sine is $\frac{1}{2}$?

$$\frac{\pi}{6} \qquad \frac{5\pi}{6} \qquad \frac{13\pi}{6} \qquad \frac{17\pi}{6} \qquad -\frac{7\pi}{6}$$

To obtain a unique angle θ such that $\sin\theta = \frac{1}{2}$, you must restrict the domain of the sine function.

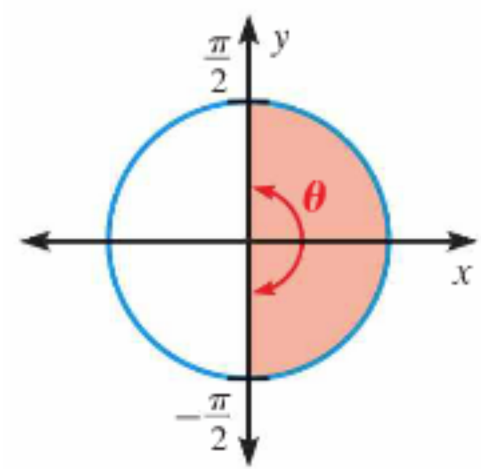
Inverse Trigonometric Functions

Domain restrictions allow the *inverse sine, inverse cosine,* and *inverse tangent* functions to be defined.

If $-1 \leq a \leq 1$, then the **inverse sine** of a is an angle θ, written $\theta = \sin^{-1} a$, where:

(1) $\sin \theta = a$

(2) $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ (or $-90° \leq \theta \leq 90°$)

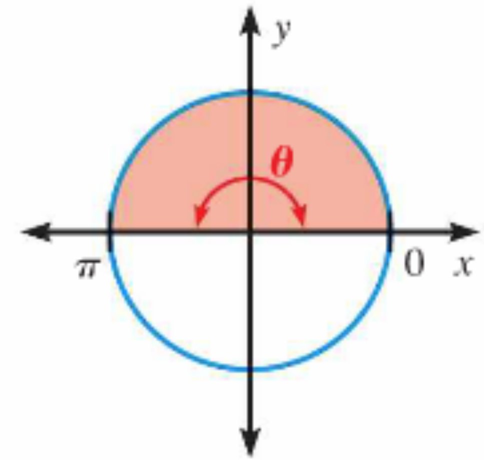


Inverse Trigonometric Functions

If $-1 \le a \le 1$, then the **inverse cosine** of a is an angle θ, written $\theta = \cos^{-1} a$, where:

(1) $\cos \theta = a$

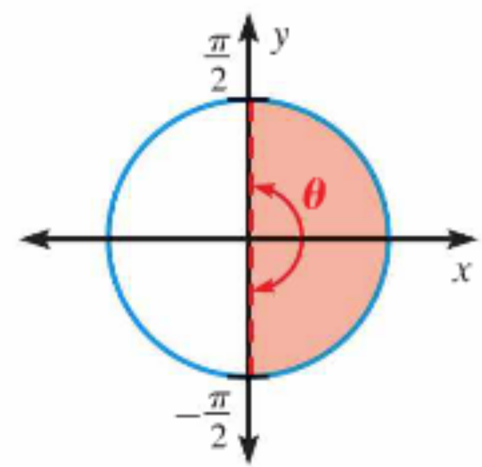
(2) $0 \le \theta \le \pi$ (or $0° \le \theta \le 180°$)



If a is any real number, then the **inverse tangent** of a is an angle θ, written $\theta = \tan^{-1} a$, where:

(1) $\tan \theta = a$

(2) $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90° < \theta < 90°$)



Evaluate the expression in both radians and degrees.

1) $\cos^{-1}\dfrac{\sqrt{3}}{2}$

Domain: $0 \le \theta \le \pi$ or $0^\circ \le \theta \le 180^\circ$

$\theta = \cos^{-1}\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{6}$ or $\theta = \cos^{-1}\dfrac{\sqrt{3}}{2} = 30^\circ$

2) $\sin^{-1} 2$

There is no angle whose sine is 2.
So $\sin^{-1} 2$ is undefined.

3) $\tan^{-1}(-\sqrt{3})$

Domain: $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$ or $-90^\circ < \theta < 90^\circ$

$\theta = \tan^{-1}(-\sqrt{3}) = -\dfrac{\pi}{3}$ or $\theta = \tan^{-1}(-\sqrt{3}) = -60^\circ$

Evaluate the expression in both radians and degrees.

1) $\sin^{-1}\dfrac{\sqrt{2}}{2}$

$$\frac{\pi}{4}, 45^\circ$$

2) $\cos^{-1}\dfrac{1}{2}$

$$\frac{\pi}{3}, 60^\circ$$

3) $\tan^{-1}(-1)$

$$-\frac{\pi}{4}, -45^\circ$$

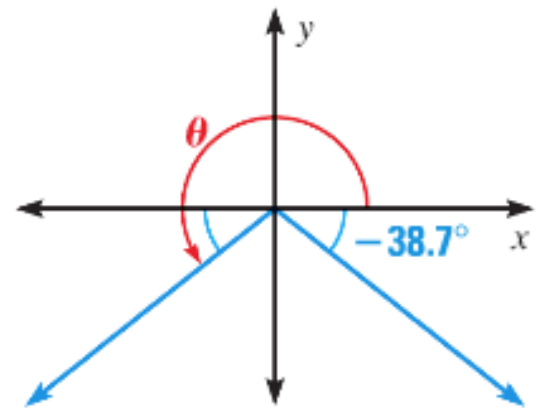
4) $\sin^{-1}\left(-\dfrac{1}{2}\right)$

$$-\frac{\pi}{6}, -30^\circ$$

Solve the equation $\sin\theta = -\dfrac{5}{8}$ where 180° < θ < 270°.

Step 1: Use a calculator to determine that in the interval -90° ≤ θ ≤ 90°, the angle whose sine is -5/8 is

$\sin^{-1}\left(-\dfrac{5}{8}\right) \approx -38.7^{\circ}$; Quadrant IV



Step 2: Find the angle in Quadrant III (where 180° < θ < 270°) that has the same sine value as the angle in Step 1.

**Needs to have same reference angle

$$\theta \approx 38.7^{\circ} + 180^{\circ} = 218.7^{\circ}$$

CHECK: $\sin 218.7^{\circ} \approx -0.625 = -\dfrac{5}{8}$ ✓

Homework

p. 878: 4-10 (evens), 21, 23

EVALUATING EXPRESSIONS **Evaluate the expression without using a calculator. Give your answer in both radians and degrees.**

4. $\tan^{-1}(-1)$ **6.** $\cos^{-1}(-2)$ **8.** $\sin^{-1}\frac{1}{2}$ **10.** $\cos^{-1}\left(-\frac{1}{2}\right)$

SOLVING EQUATIONS **Solve the equation for θ.**

21. $\sin\theta = -0.45;\ 180° < \theta < 270°$

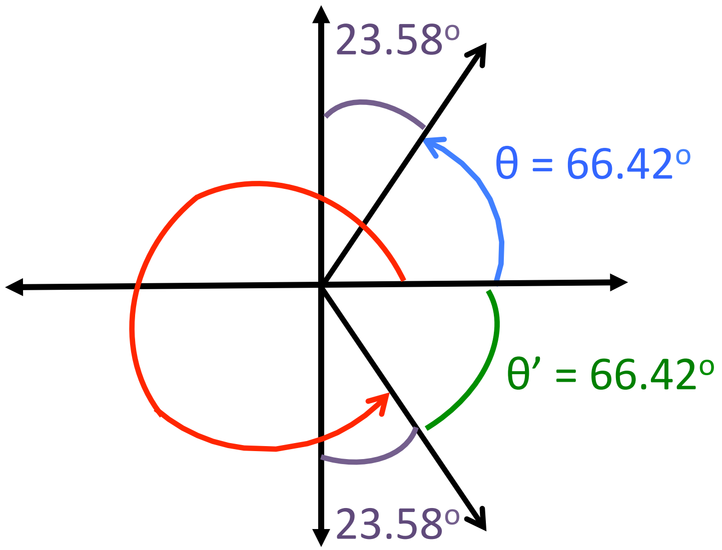
23. $\tan\theta = 3.2;\ 180° < \theta < 270°$

Objective

Students will be able to solve a trigonometric function using inverse sine, cosine, and tangent.

Solve the equation $\cos\theta = 0.4$ where $270^\circ < \theta < 360^\circ$

$$\theta = \cos^{-1}(0.4)$$
$$\approx 66.42^\circ$$

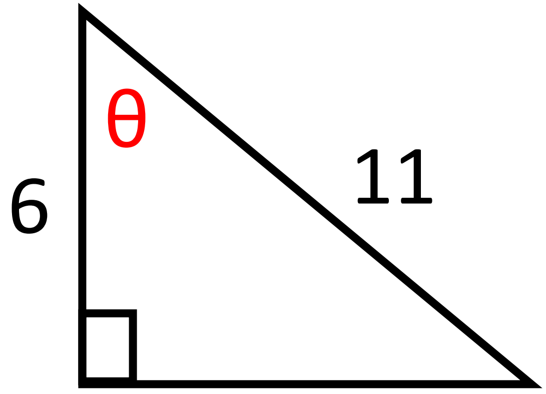


$$\theta = 270^\circ + 23.58^\circ = 293.58^\circ$$

CHECK: $\cos 293.58^\circ \approx 0.4$ ✓

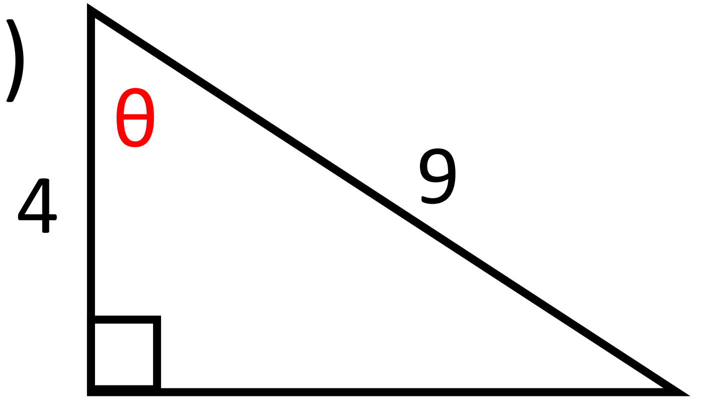
Find the measure of the angle θ.

1)



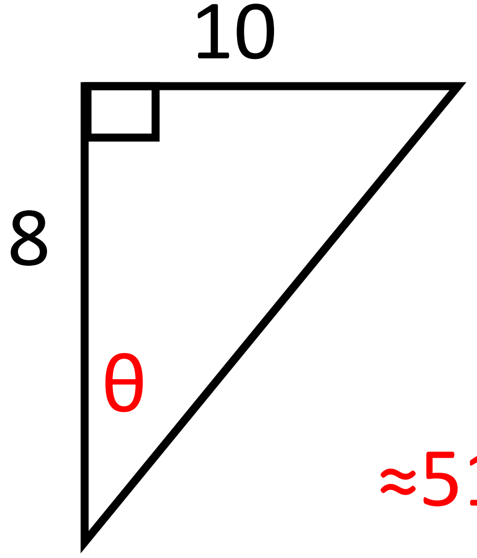
≈56.9º

2)



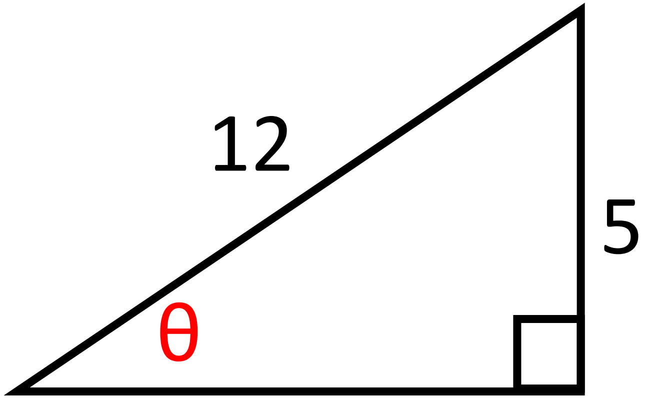
≈63.6º

3)



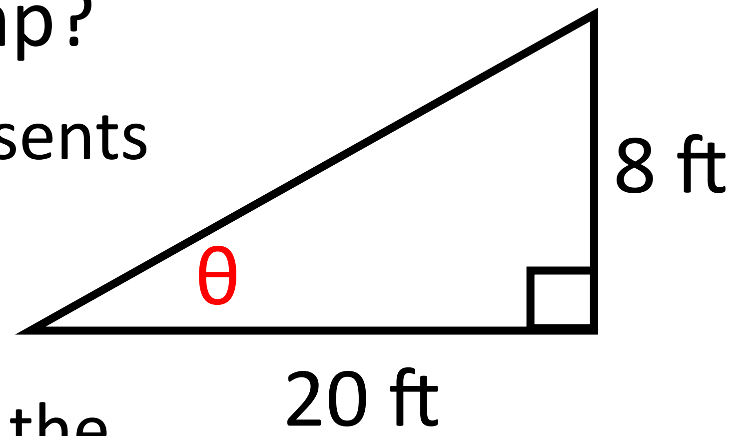
≈51.3º

4)



≈24.6º

A monster truck drives off a ramp in order to jump onto a row of cars. The ramp has a height of 8 feet and a horizontal length of 20 feet. What is the angle θ of the ramp?



Step 1: Draw a triangle that represents the ramp

Step 2: Write a trigonometric equation that involves the ratio of the ramp's height and horizontal length.

$$\tan\theta = \frac{opp}{adj} = \frac{8}{20}$$

Step 3: Use a calculator to find the measure of θ.

$$\theta = \tan^{-1}\frac{8}{20} \approx 21.8^{\circ}$$

The angle of the ramp is about 22°.

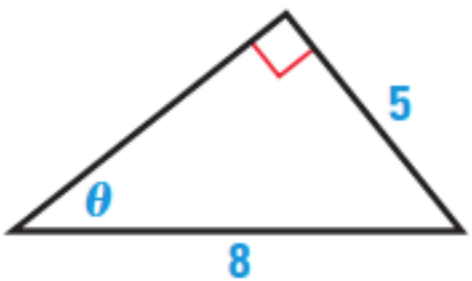
Homework   p. 878: 25, 27-29, 36

SOLVING EQUATIONS **Solve the equation for θ.**

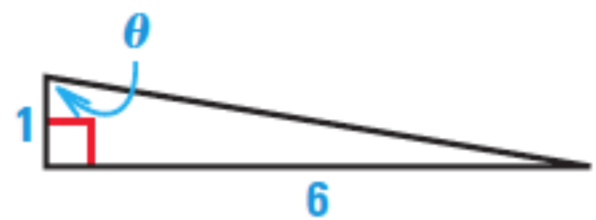
25. $\cos\theta = 0.25;\ 270° < \theta < 360°$

FINDING ANGLES **Find the measure of the angle θ.**

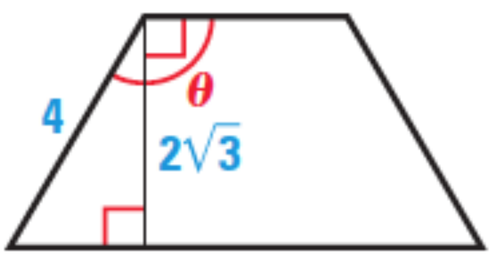
27.



28.



29.



36. ANGLE OF DESCENT An airplane is flying at an altitude of 31,000 feet when it begins its descent for landing. If the runway is 104 miles away, at what angle does the airplane descend?

Objective

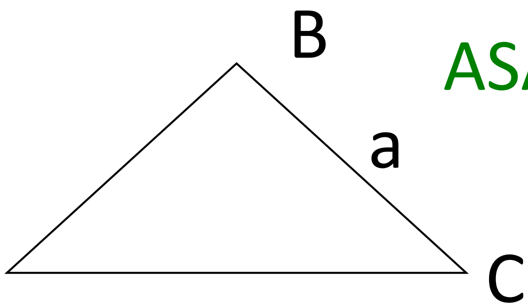
Students will be able to solve non-right triangles using Law of Sines.

Trigonometric Ratios and Functions (13.4-13.6) Test on Wednesday!

Triangles Review

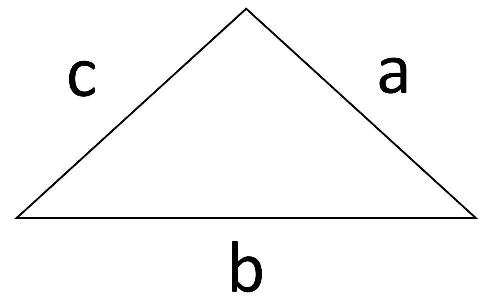
Looking at each triangle, what type of triangles are they (AAS, ASA, SAS, SSA, SSS)?

1)



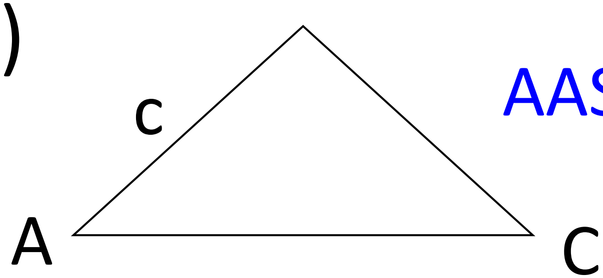
ASA

2)



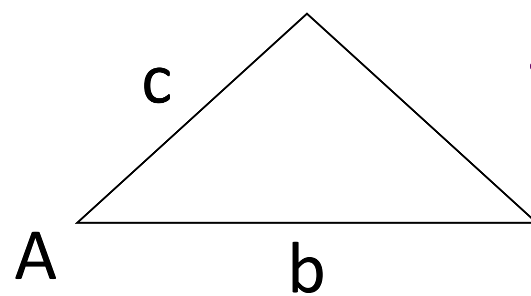
SSS

3)



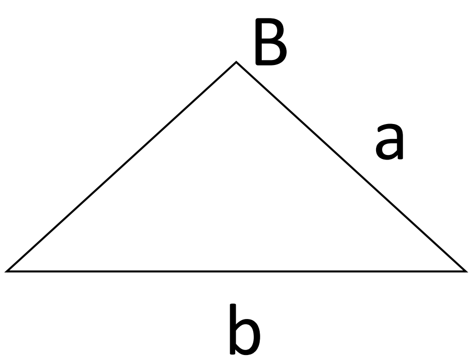
AAS

4)



SAS

5)



SSA

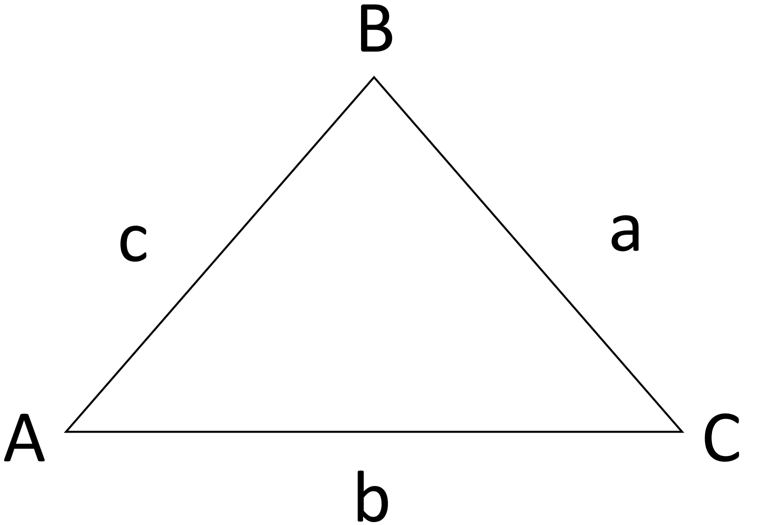
Law of Sines

To solve a triangle with no right angle, you need to know the length of at least one side and any two other parts of the triangle.

The law of sines can be used to solve triangles when two angles and the length of any side are known (AAS or ASA cases).

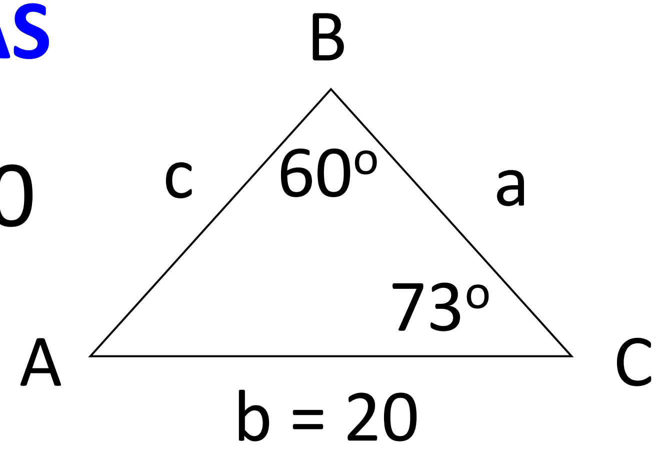
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Solve triangle ABC with the given side lengths and angles.

AAS



1) B = 60º, C = 73º, and b = 20

$$\angle A = 180^\circ - 60^\circ - 73^\circ$$

$$\boxed{\angle A = 47^\circ}$$

$$\frac{a}{\sin 47^\circ} = \frac{20}{\sin 60^\circ} \qquad a \sin 60^\circ = 20 \sin 47^\circ \qquad a = \frac{20 \sin 47^\circ}{\sin 60^\circ}$$

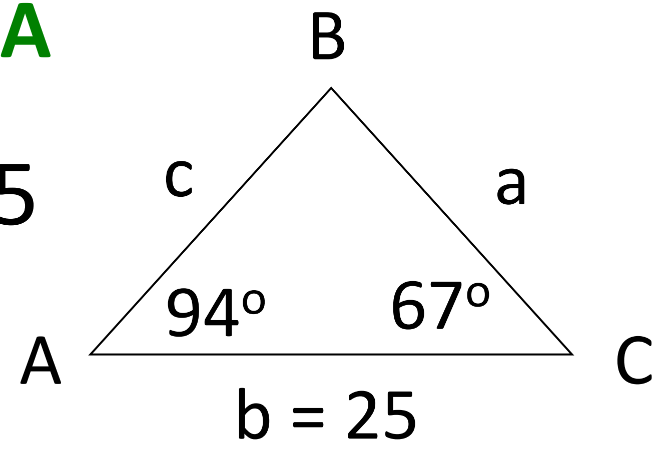
$$\boxed{a \approx 16.9}$$

$$\frac{c}{\sin 73^\circ} = \frac{20}{\sin 60^\circ} \qquad c \sin 60^\circ = 20 \sin 73^\circ \qquad c = \frac{20 \sin 73^\circ}{\sin 60^\circ}$$

$$\boxed{c \approx 22.1}$$

Solve triangle ABC with the given side lengths and angles.

ASA



2) A = 94°, C = 67°, and b = 25

$$\angle B = 180^\circ - 94^\circ - 67^\circ$$

$$\boxed{\angle B = 19^\circ}$$

$$\frac{a}{\sin 94^\circ} = \frac{25}{\sin 19^\circ} \qquad a\sin 19^\circ = 25\sin 94^\circ \qquad a = \frac{25\sin 94^\circ}{\sin 19^\circ}$$

$$\boxed{a \approx 76.6}$$

$$\frac{c}{\sin 67^\circ} = \frac{25}{\sin 19^\circ} \qquad c\sin 19^\circ = 25\sin 67^\circ \qquad c = \frac{25\sin 67^\circ}{\sin 19^\circ}$$

$$\boxed{c \approx 70.7}$$

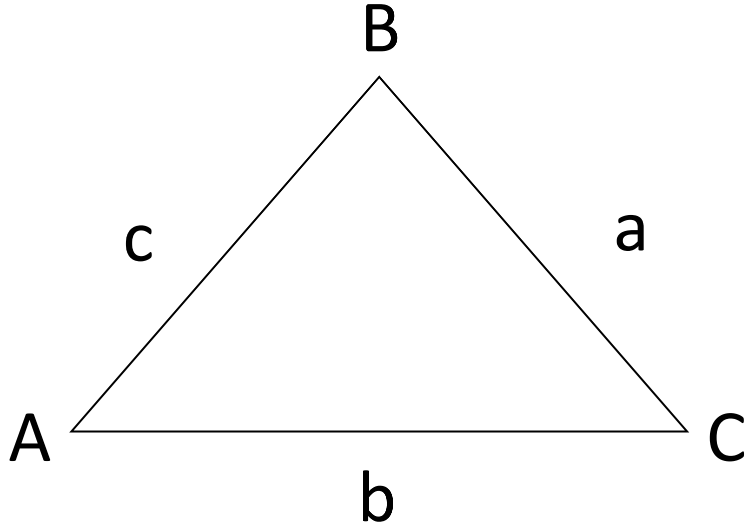
Area of a Triangle

The area of any triangle is given by one half the product of the lengths of two sides times the sine of their included angle.

$$Area = \frac{1}{2}bc\sin A$$

$$Area = \frac{1}{2}ac\sin B$$

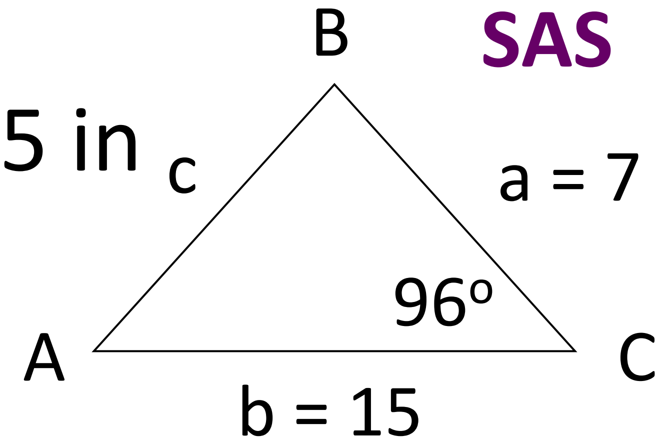
$$Area = \frac{1}{2}ab\sin C$$



****has to be SAS**

Find the area of the triangle with the given side lengths and the included angle.

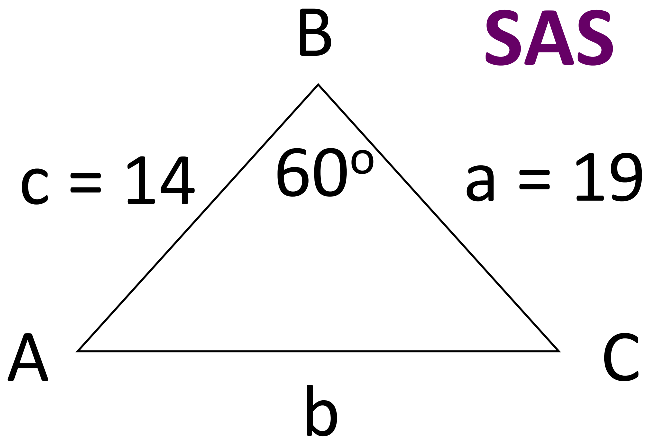
1) $C = 96^{o}$, $a = 7$ in, and $b = 15$ in



$$Area = \frac{1}{2}ab\sin C \quad = \frac{1}{2}(7)(15)\sin 96^{\circ}$$

$$\approx 52.2in^{2}$$

2) $B = 60^{o}$, $a = 19$, and $c = 14$



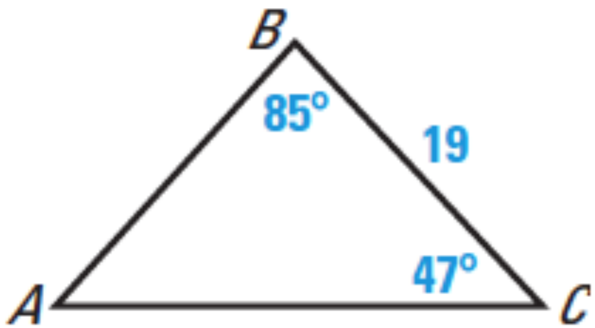
$$Area = \frac{1}{2}ac\sin B \quad = \frac{1}{2}(19)(14)\sin 60^{\circ}$$

$$\approx 115.2units^{2}$$

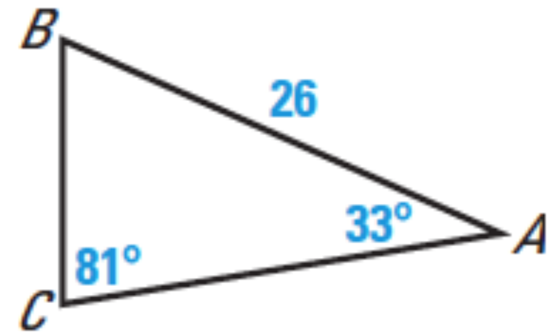
Homework   p. 886: 12, 14, 29, 39

SOLVING TRIANGLES Solve $\triangle ABC$.

12.



14.

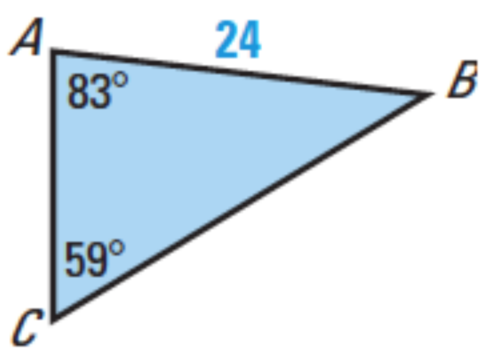


FINDING AREA Find the area of $\triangle ABC$ with the given side lengths and included angle.

29. $B = 124°, a = 9, c = 11$

FINDING AREA Find the area of $\triangle ABC$.

39.



Objective

Students will be able to solve non-right triangles using Law of Sines.

Trigonometric Ratios and Functions (13.4-13.6) Test on Wednesday!

Homework Quiz on Tuesday!

Law of Sines

Two angles and one side (AAS or ASA) determine <u>exactly one triangle</u>. Two sides and an angle opposite one of the sides (SSA) may determine no triangle, one triangle, or two triangles.

<u>Possible triangles for SSA</u>:

**the letters do not need to be A, a, and b...it just has to be SSA

If A is obtuse...

-No triangle ($a \leq b$)

-One triangle ($a > b$)

If A is acute...

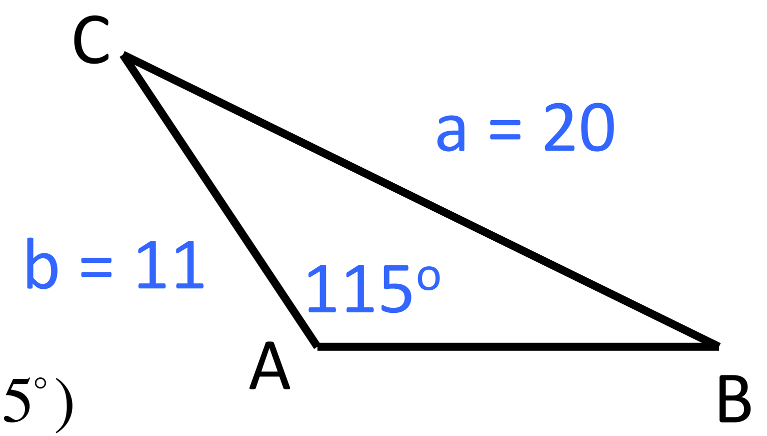
-No triangle ($h > a$)

-Two triangles ($h < a < b$)

-One triangle ($h = a$ or $a > b$)

** $h = b \sin A$

1) Solve triangle ABC with A = 115°, a = 20, and b = 11. **SSA**



A is obtuse and $a > b$; one triangle

To find B, use Law of Sines:

$$\frac{\sin B}{11} = \frac{\sin 115^\circ}{20}$$

$$20 \sin B = 11 \sin 115^\circ$$

$$\sin B = \frac{11 \sin(115^\circ)}{20}$$

$$\sin B \approx 0.4985$$

$$\angle B = \sin^{-1} 0.4985$$

$$\boxed{\angle B \approx 29.9^\circ}$$

$$\angle C = 180^\circ - 115^\circ - 29.9^\circ$$

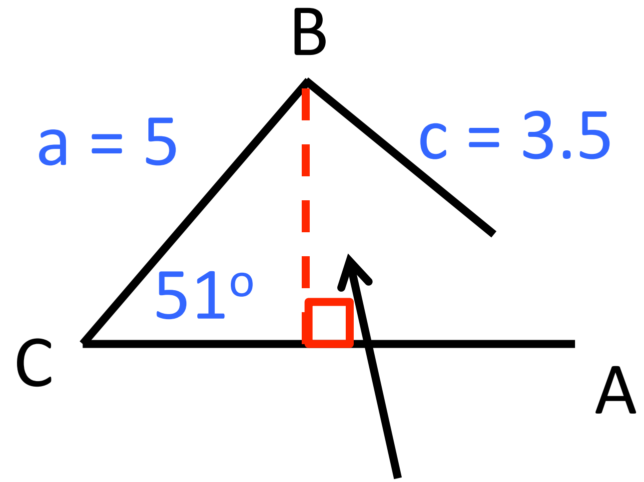
$$\boxed{\angle C = 35.1^\circ}$$

$$\frac{c}{\sin 35.1^\circ} = \frac{20}{\sin 115^\circ} \qquad c = \frac{20 \sin 35.1^\circ}{\sin 115^\circ}$$

$$\boxed{c \approx 12.7}$$

2) Solve triangle ABC with $C = 51^{\circ}$, $c = 3.5$, and $a = 5$.

SSA



C is acute and *c* < *a*; two triangles or no triangles

You need to see if *h* < *c* in order to see if we can even create a triangle.

$$h = a \sin C$$

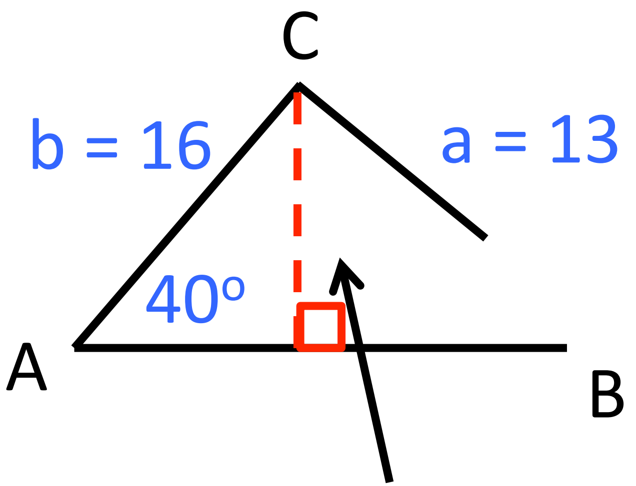
$$= 5 \sin 51^{\circ}$$

$$\approx 3.9$$

h > *c*; no triangle exists

3) Solve triangle ABC with $A = 40^\circ$, $a = 13$, and $b = 16$.

SSA



A is acute and $a < b$; two triangles or no triangles

You need to see if $h < a$ in order to see if we can even create a triangle.

$$h = b \sin A$$
$$= 16 \sin 40^\circ$$
$$\approx 10.3$$

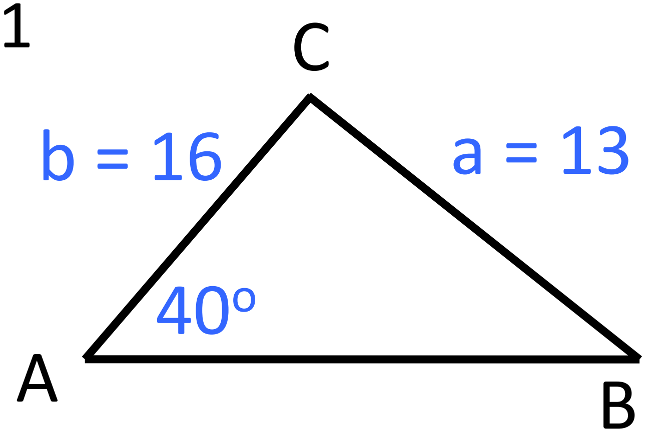
$h < a < b$; two triangles exists

Solve triangle ABC with $A = 40^{\circ}$, $a = 13$, and $b = 16$.

$$\frac{\sin B}{16} = \frac{\sin 40^{\circ}}{13}$$

There are two angles B between 0° and 180° for which $\sin B \approx 0.7911$

Triangle 1:



$$13\sin B = 16\sin 40^{\circ}$$

$$\angle B = \sin^{-1} 0.7911$$

$$\sin B = \frac{16\sin 40^{\circ}}{13}$$

$$\boxed{\angle B \approx 52.3^{\circ}}$$

$$\sin B \approx 0.7911$$

$$\angle C = 180^{\circ} - 40^{\circ} - 52.3^{\circ}$$

$$\boxed{\angle C = 87.7^{\circ}}$$

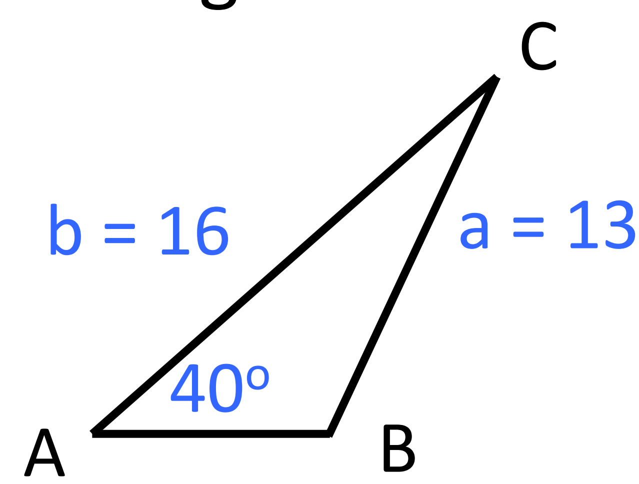
$$\frac{c}{\sin 87.7^{\circ}} = \frac{13}{\sin 40^{\circ}}$$

$$c = \frac{13\sin 87.7^{\circ}}{\sin 40^{\circ}}$$

$$\boxed{c \approx 20.2}$$

Solve triangle ABC with *A* = 40º, *a* = 13, and *b* = 16.

Triangle 2:



$$\sin B \approx 0.7911 \quad \angle B = \sin^{-1} 0.7911$$

$$\angle B \approx 52.3^\circ$$

The obtuse angle has 52.3º as a reference angle, so its measure is 180º – 52.3º = 127.7º

$$\boxed{\angle B \approx 127.7^\circ}$$

$$\angle C = 180^\circ - 40^\circ - 127.7^\circ \qquad \frac{c}{\sin 12.3^\circ} = \frac{13}{\sin 40^\circ} \qquad c = \frac{13 \sin 12.3^\circ}{\sin 40^\circ}$$

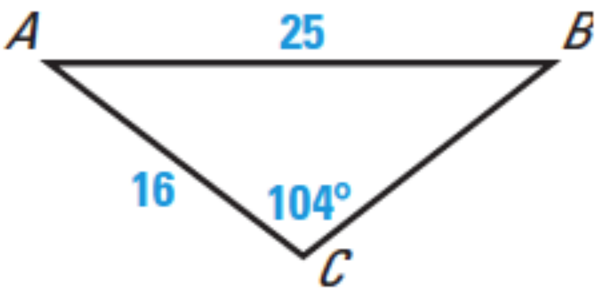
$$\boxed{\angle C = 12.3^\circ}$$

$$\boxed{c \approx 4.3}$$

Homework p. 886: 13, 21, 24, 40

SOLVING TRIANGLES Solve $\triangle ABC$.

13.



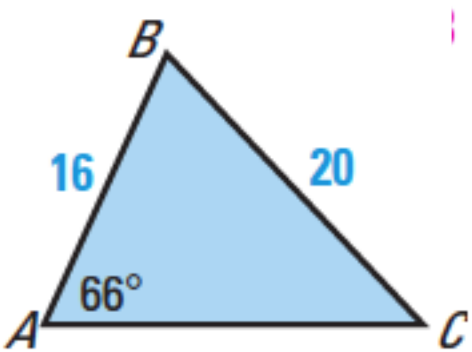
SOLVING TRIANGLES Solve $\triangle ABC$. (*Hint:* Some of the "triangles" have no solution and some have two solutions.)

21. $A = 38°, a = 19, b = 25$

24. $C = 98°, c = 29, a = 33$

FINDING AREA Find the area of $\triangle ABC$.

40.



Objective

Students will be able to solve non-right triangles using Law of Cosines.

Trigonometric Ratios and Functions (13.4-13.6) Test on Wednesday!

Homework Quiz <u>tomorrow</u>!

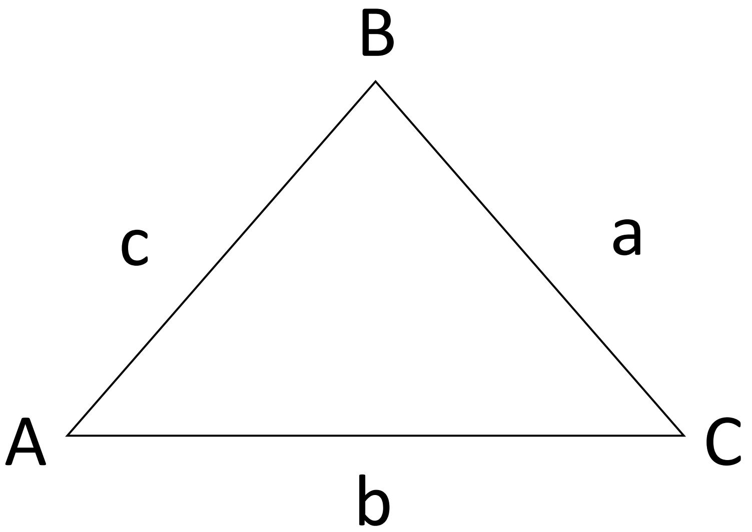
Law of Cosines

The law of cosines can be used to solve triangles when two sides and the included angle are known (SAS), or when all three sides are known (SSS).

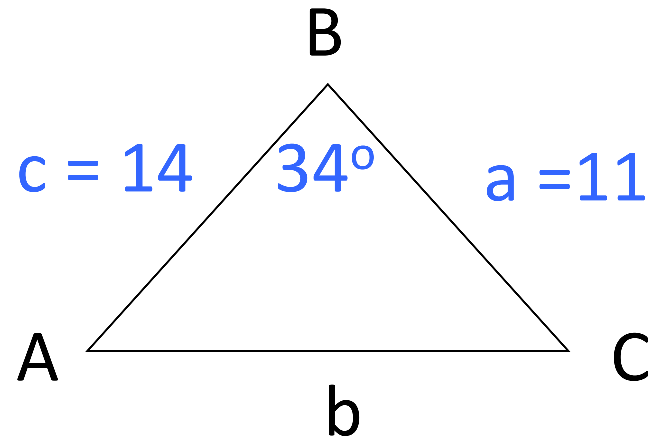
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



Solve triangle ABC with the given side lengths and angles. **SAS**



1) $B = 34°$, $c = 14$, and $a = 11$

Use law of cosines to find side length *b*.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 11^2 + 14^2 - 2(11)(14)\cos 34^\circ \approx 61.7$$

$$b \approx \sqrt{61.7} \qquad \boxed{b \approx 7.85}$$

$$\frac{\sin A}{11} = \frac{\sin 34^\circ}{7.85} \qquad 7.85\sin A = 11\sin 34^\circ$$

$$\sin A = \frac{11\sin 34^\circ}{7.85}$$

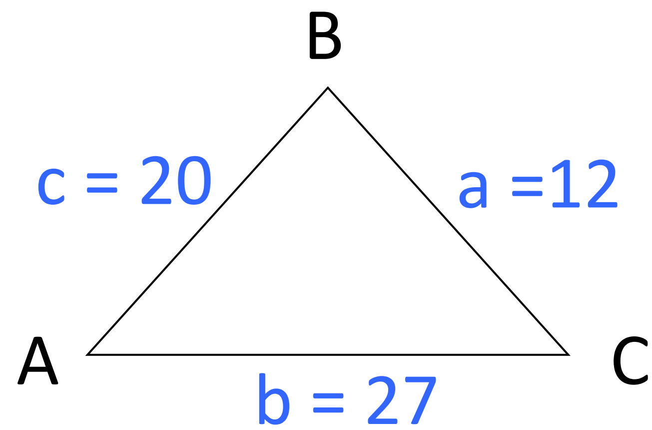
$$\sin A \approx 0.7836$$

$$\angle A = \sin^{-1} 0.7836$$

$$\boxed{\angle A \approx 51.6^\circ}$$

$$\angle C = 180^\circ - 34^\circ - 51.6^\circ \qquad \boxed{\angle C = 94.4^\circ}$$

Solve triangle ABC with the given side lengths and angles. **SSS**



2) b = 27, c = 20, and a = 12

**If given SSS, always find the angle opposite the largest side first

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$27^2 = 12^2 + 20^2 - 2(12)(20)\cos B$$

$$\frac{27^2 - 12^2 - 20^2}{-2(12)(20)} = \cos B$$

$$-0.3854 \approx \cos B$$

$$\angle B \approx \cos^{-1}(-0.3854)$$

$$\boxed{\angle B \approx 112.7^\circ}$$

$$\frac{\sin A}{12} = \frac{\sin 112.7^\circ}{27} \qquad \sin A = \frac{12\sin 112.7^\circ}{27}$$

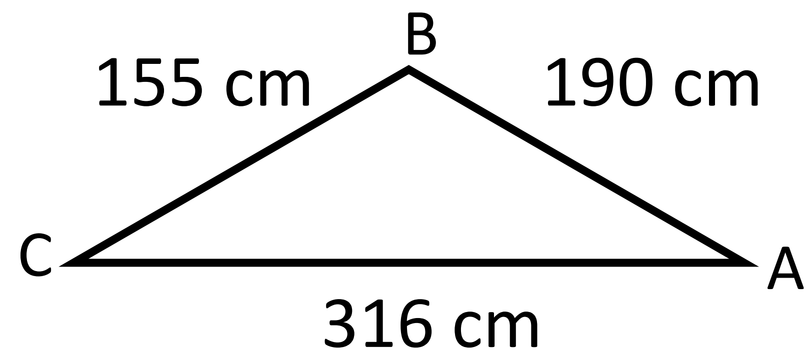
$$\sin A \approx 0.4100$$

$$\angle A = \sin^{-1} 0.4100$$

$$\angle C = 180^\circ - 112.7^\circ - 24.2^\circ \qquad \boxed{\angle C = 43.1^\circ} \qquad \boxed{\angle A \approx 24.2^\circ}$$

Scientists can use a set of footprints to calculate an organism's *step angle*, which is a measure of walking efficiency. The closer the step angle is to 180°, the more efficiently the organism walked. The triangle shows a set of footprints for a dinosaur.

Find the step angle *B*.



SSS

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$316^2 = 155^2 + 197^2 - 2(155)(197)\cos B$$

$$\frac{316^2 - 155^2 - 197^2}{-2(155)(197)} = \cos B$$

$$-.6062 \approx \cos B$$

$$B \approx \cos^{-1}(-.6062) \approx 127.3^\circ$$

The step angle *B* is about 127.3°.

Heron's Area Formula

The area of the triangle with side lengths of *a*, *b*, and *c* is

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$.

(*s* is the semiperimeter, or half perimeter, of the triangle)

****has to be SSS**

Find the area of the triangle with side lengths *a* = 8, *b* = 11, and *c* = 5. **SSS**

1) Find the semiperimeter *s*.

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(8+11+5) = 12$$

2) Use Heron's formula to find the area of triangle ABC.

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-8)(12-11)(12-5)}$$

$$= \sqrt{12(4)(1)(7)} = \sqrt{336}$$

$$\approx 18.3 units^2$$

Homework

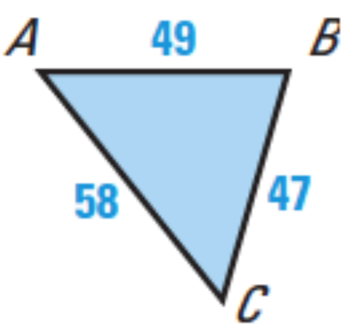
p. 892: 18, 19, 23, 31

SOLVING TRIANGLES **Solve $\triangle ABC$.**

18. $a = 23, b = 24, c = 20$ **19.** $C = 96°, a = 35, b = 43$

FINDING AREA **Find the area of $\triangle ABC$.**

23.



FINDING AREA **Find the area of $\triangle ABC$ with the given side lengths.**

31. $a = 51, b = 51, c = 43$