Objectives

Students will be able to identify and simplify complex numbers and solve quadratic equations with complex solutions by square roots.

Students will be able to add and subtract complex numbers.

Not all quadratic equations have real-number solutions. For example, x^2 = -1 has no real-number solutions because the square of any real number x is never a negative number.

Solution to this is a system of numbers using the imaginary unit i, defined as $i = \sqrt{-1}$.

The Square Root of a Negative Number

Property	Example
1. If *r* is a positive real number, then $\sqrt{-r} = i\sqrt{r}$	$\sqrt{-3} = i\sqrt{3}$

Solve $2x^2 + 11 = -37$.

$2x^2 + 11 = -37$	Write original equation.
$2x^2 = -48$	Subtract 11 from each side.
$x^2 = -24$	Divide each side by 2.
$x = \pm\sqrt{-24}$	Take square roots of each side
$x = \pm i\sqrt{24}$	Write in terms of *i*
$x = \pm 2i\sqrt{6}$	Simplify radical

The solutions are $2i\sqrt{6}$ and $-2i\sqrt{6}$

Solve the following equations.

1) $x^2 - 8 = -36$

$\underline{+}\ 2i\sqrt{7}$

2) $3x^2 - 7 = -31$

$\underline{+}\ 2i\sqrt{2}$

3) $5x^2 + 33 = 3$

$\underline{+}\ i\sqrt{6}$

Complex Numbers

A complex number written in standard form is a number $a + bi$ where a and b are real numbers.

$$a + bi$$

real part imaginary part

Example: $-3 + 15i$

real part imaginary part

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Treat *i* like it is a variable. Combine your like terms together. If subtracting two complex numbers, distribute in the negative and and combine your like terms.

Write the expression as a complex number in standard form.

1) $(8 - i) + (5 + 4i)$

$13 + 3i$

2) $(7 - 6i) - (3 - 6i)$

4

3) $10 - (6 + 7i) + 4i$

$4 - 3i$

Homework

p. 279: 3-17 **(odds)**

SOLVING QUADRATIC EQUATIONS **Solve the equation.**

3. $x^2 = -28$

4. $r^2 = -624$

5. $z^2 + 8 = 4$

6. $s^2 - 22 = -112$

7. $2x^2 + 31 = 9$

8. $9 - 4y^2 = 57$

9. $6t^2 + 5 = 2t^2 + 1$

10. $3p^2 + 7 = -9p^2 + 4$

11. $-5(n - 3)^2 = 10$

ADDING AND SUBTRACTING **Write the expression as a complex number in standard form.**

12. $(6 - 3i) + (5 + 4i)$

13. $(9 + 8i) + (8 - 9i)$

14. $(-2 - 6i) - (4 - 6i)$

15. $(-1 + i) - (7 - 5i)$

16. $(8 + 20i) - (-8 + 12i)$

17. $(8 - 5i) - (-11 + 4i)$

Objective

Students will be able to multiply and divide complex numbers.

$$i = \sqrt{-1}$$

$$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot \sqrt{-1} = -\sqrt{-1} = -i$$

$$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$$

$$i^5 = i^4 \cdot i = 1 \cdot \sqrt{-1} = \sqrt{-1} = i$$

$$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$$

Do you notice a pattern?

The Square Root of a Negative Number

Property

1. If *r* is a positive real number, then $\sqrt{-r} = i\sqrt{r}$

2. By Property (1), it follows that $(i\sqrt{r})^2 = -r$

Example

$$\sqrt{-3} = i\sqrt{3}$$

$$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$$

Multiplying Complex Numbers

To multiply complex numbers, use the distributive property or the FOIL method just as you do when multiplying real numbers or algebraic expressions.

Examples:

1) $4i(-6 + i)$

$-4 - 24i$

2) $(9 - 2i)(-4 + 7i)$

$-22 + 71i$

Complex Conjugates

Two complex number of the form $a + bi$ and $a - bi$ are called <u>complex conjugates</u>.

The product of complex conjugates is *always* a real number.

Example: $(2 + 4i)(2 - 4i) = 4 - 8i + 8i - 16i^2$

$$= 4 - 16(-1) = 4 + 16 = 20$$

Want to use this to write the quotient of two complex numbers in standard form.

Dividing Complex Numbers

Write the quotient $\dfrac{7+5i}{1-4i}$ in standard form.

**Very similar to rationalizing your denominator with radicals

$$\frac{7+5i}{1-4i}\cdot\frac{1+4i}{1+4i}=\frac{7+28i+5i+20i^2}{1+4i-4i-16i^2}=\frac{7+33i+20(-1)}{1-16(-1)}$$

multiply numerator and denominator by complex conjugate

FOIL

Simplify and use i^2 = -1

$$=\frac{-13+33i}{17}\qquad=-\frac{13}{17}+\frac{33}{17}i$$

Simplify

Write in standard form

Write the expressions as a complex number in standard form.

1) $\dfrac{5}{1+i}$

$\dfrac{5}{2}-\dfrac{5}{2}i$

2) $\dfrac{5+2i}{3-2i}$

$\dfrac{11}{13}+\dfrac{16}{13}i$

Homework

p. 279: 23-33 **(odds)**

MULTIPLYING AND DIVIDING **Write the expression as a complex number in standard form.**

22. $6i(3 + 2i)$

23. $-i(4 - 8i)$

24. $(5 - 7i)(-4 - 3i)$

25. $(-2 + 5i)(-1 + 4i)$

26. $(-1 - 5i)(-1 + 5i)$

27. $(8 - 3i)(8 + 3i)$

28. $\dfrac{7i}{8 + i}$

29. $\dfrac{6i}{3 - i}$

30. $\dfrac{-2 - 5i}{3i}$

31. $\dfrac{4 + 9i}{12i}$

32. $\dfrac{7 + 4i}{2 - 3i}$

33. $\dfrac{-1 - 6i}{5 + 9i}$

Objectives

Students will be able to plot and find the absolute value of complex numbers.

Students will be able to solve quadratic equations with complex solutions (by completing the square and by the quadratic formula).

Complex Plane

Just as every real number corresponds to a point on the real number line, every complex number corresponds to a point in the complex plane.

The complex plane has a horizontal axis called the *real axis* and a vertical axis called the *imaginary axis*.

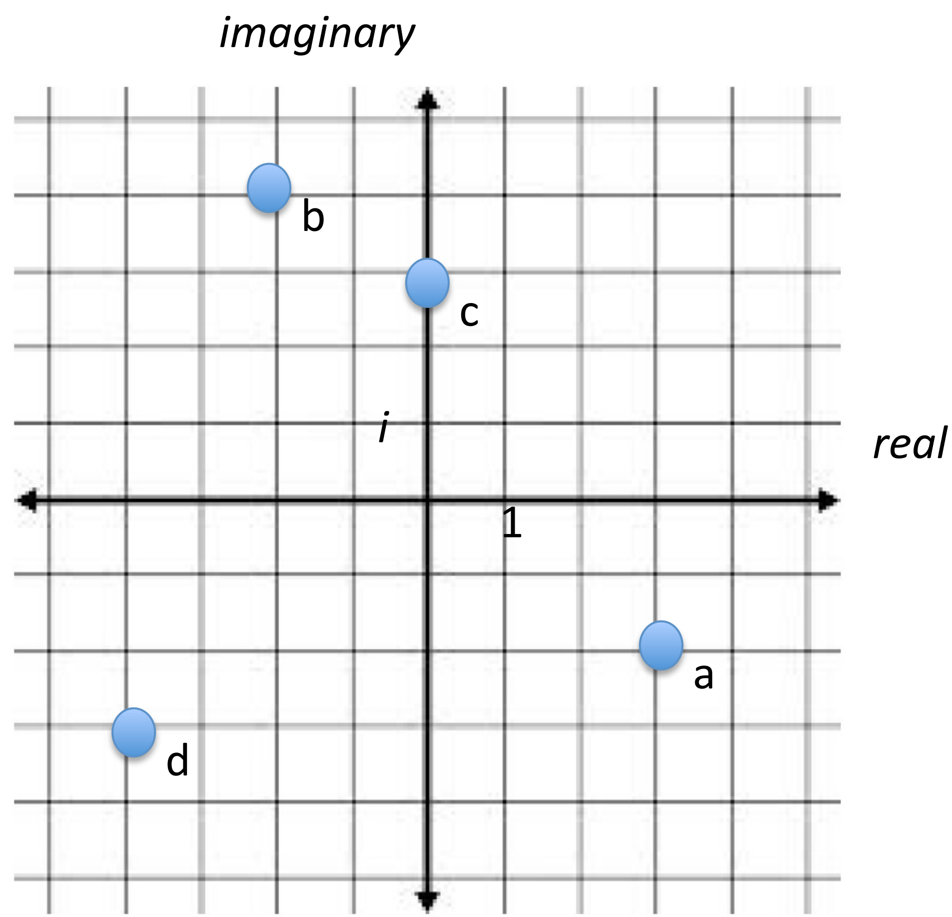
Plot complex numbers

a) $3 - 2i$

b) $-2 + 4i$

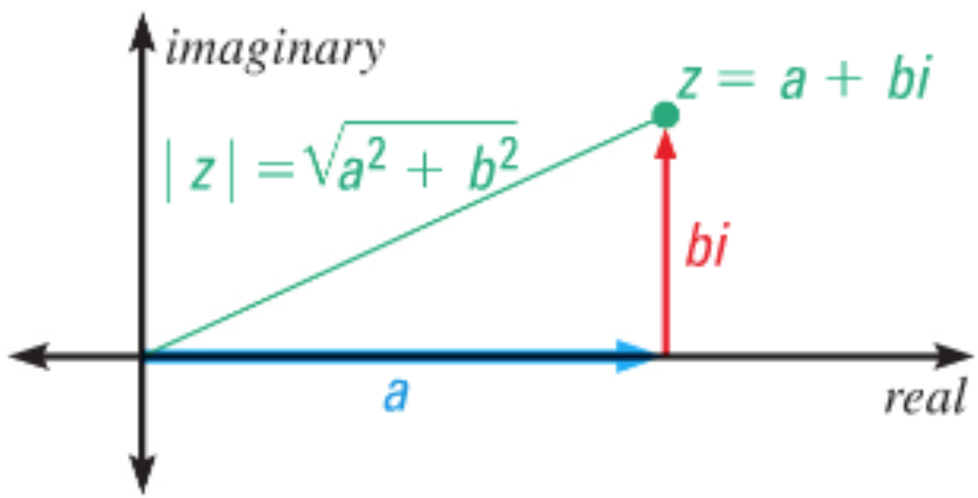
c) $3i$

d) $-4 - 3i$



Absolute Value of a Complex Number

The **absolute value** of a complex number $z = a + bi$, denoted $|z|$, is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$. This is the distance between z and the the origin in the complex plane.



Find the absolute value of:

1) $\left|-4+3i\right| = \sqrt{(-4)^2+3^2} = \sqrt{25} = 5$

2) $\left|-3i\right| = \left|0+(-3i)\right| = \sqrt{0^2+(-3)^2} = \sqrt{9} = 3$

Completing the Square

Solve $2x^2 + 8x + 14 = 0$ by completing the square.

$2x^2 + 8x + 14 = 0$ $\quad$ $2x^2 + 8x = -14$ $\quad$ $2(x^2 + 4x) = -14$

$2(x^2 + 4x + \underline{\quad\quad}) = -14 + 2(\underline{\quad\quad})$

$2(x^2 + 4x + 4) = -14 + 2(4)$ $\quad$ $2(x + 2)^2 = -6$

$(x + 2)^2 = -3$ $\quad$ $x + 2 = \pm \sqrt{-3}$ $\quad$ $x + 2 = \pm i\sqrt{3}$

$$x = -2 \pm i\sqrt{3}$$

The Quadratic Formula

Let a, b, and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are:

discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What happens when we get a negative discriminant?

Use the quadratic formula to solve $-x^2 + 4x = 5$.

$$-x^2 + 4x - 5 = 0$$

a = -1, b = 4, c = -5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-5)}}{2(-1)} = \frac{-4 \pm \sqrt{16 - 20}}{-2}$$

$$= \frac{-4 \pm \sqrt{-4}}{-2} = \frac{-4 \pm 2i}{-2} = \frac{-4}{-2} \pm \frac{2}{-2} i \quad = 2 \pm i$$

Solve $5x^2 + 20x + 21 = 0$ by completing the square.

$$-2 \pm \frac{\sqrt{5}}{5}i$$

Use the quadratic formula to solve $7x - 5x^2 - 4 = 2x + 3$.

$$\frac{1}{2} \pm \frac{\sqrt{115}}{10}i$$

Homework

p. 279: 39, 42, 43

Plot the numbers in the same complex plane.

39. $5 - 5i$

Find the absolute value of the complex number.

42. $4 + 3i$ **43.** $-3 + 10i$

p. 288: 27, 31, 32

Solve the equation by completing the square.

27. $x^2 - 2x + 25 = 0$

31. $3s^2 + 6s + 9 = 0$ **32.** $7t^2 + 28t + 56 = 0$

p. 296: 5, 8, 10, 14, 15

EQUATIONS IN STANDARD FORM **Use the quadratic formula to solve the equation.**

5. $t^2 + 8t + 19 = 0$ **8.** $5p^2 - 10p + 24 = 0$ **10.** $6u^2 + 4u + 11 = 0$

EQUATIONS NOT IN STANDARD FORM **Use the quadratic formula to solve the equation.**

14. $x^2 + 6x = -15$ **15.** $s^2 = -14 - 3s$