

Objectives

Students will be able to identify and simplify complex numbers and solve quadratic equations with complex solutions by square roots.

Students will be able to add and subtract complex numbers.

Not all quadratic equations have real-number solutions. For example, $x^2 = -1$ has no real-number solutions because the square of any real number x is never a negative number.

Solution to this is a system of numbers using the imaginary unit i , defined as $i = \sqrt{-1}$.

The Square Root of a Negative Number

Property

1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$

Example

$$\sqrt{-3} = i\sqrt{3}$$

Solve $2x^2 + 11 = -37$.

$$2x^2 + 11 = -37$$

Write original equation.

$$2x^2 = -48$$

Subtract 11 from each side.

$$x^2 = -24$$

Divide each side by 2.

$$x = \pm \sqrt{-24}$$

Take square roots of each side

$$x = \pm i\sqrt{24}$$

Write in terms of i

$$x = \pm 2i\sqrt{6}$$

Simplify radical

The solutions are $2i\sqrt{6}$ and $-2i\sqrt{6}$

Solve the following equations.

1) $x^2 - 8 = -36$

$\pm 2i\sqrt{7}$

2) $3x^2 - 7 = -31$

$\pm 2i\sqrt{2}$

3) $5x^2 + 33 = 3$

$\pm i\sqrt{6}$

Complex Numbers

A complex number written in standard form is a number $a + bi$ where a and b are real numbers.


$$a + bi$$


Diagram illustrating the components of a complex number $a + bi$. The expression $a + bi$ is shown. Below it, the text "real part" is connected to a by a blue double-headed arrow, and the text "imaginary part" is connected to bi by a blue double-headed arrow.

Example: $-3 + 15i$




Diagram illustrating the components of the example complex number $-3 + 15i$. The expression $-3 + 15i$ is shown. Below it, the text "real part" is connected to -3 by a blue double-headed arrow, and the text "imaginary part" is connected to $15i$ by a blue double-headed arrow.

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Treat i like it is a variable. Combine your like terms together. If subtracting two complex numbers, distribute in the negative and and combine your like terms.

Write the expression as a complex number in standard form.

1) $(8 - i) + (5 + 4i)$

$13 + 3i$

2) $(7 - 6i) - (3 - 6i)$

4

3) $10 - (6 + 7i) + 4i$

$4 - 3i$

Homework

p. 279: 3-17 (odds)

SOLVING QUADRATIC EQUATIONS Solve the equation.

3. $x^2 = -28$

4. $r^2 = -624$

5. $z^2 + 8 = 4$

6. $s^2 - 22 = -112$

7. $2x^2 + 31 = 9$

8. $9 - 4y^2 = 57$

9. $6t^2 + 5 = 2t^2 + 1$

10. $3p^2 + 7 = -9p^2 + 4$

11. $-5(n - 3)^2 = 10$

ADDING AND SUBTRACTING Write the expression as a complex number in standard form.

12. $(6 - 3i) + (5 + 4i)$

13. $(9 + 8i) + (8 - 9i)$

14. $(-2 - 6i) - (4 - 6i)$

15. $(-1 + i) - (7 - 5i)$

16. $(8 + 20i) - (-8 + 12i)$

17. $(8 - 5i) - (-11 + 4i)$

Objective

Students will be able to multiply and divide complex numbers.

$$i = \sqrt{-1}$$

$$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot \sqrt{-1} = -\sqrt{-1} = -i$$

$$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$$

$$i^5 = i^4 \cdot i = 1 \cdot \sqrt{-1} = \sqrt{-1} = i$$

$$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$$

Do you notice a pattern?

The Square Root of a Negative Number

Property

1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$

2. By Property (1), it follows that $(i\sqrt{r})^2 = -r$

Example

$$\sqrt{-3} = i\sqrt{3}$$

$$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$$

Multiplying Complex Numbers

To multiply complex numbers, use the distributive property or the FOIL method just as you do when multiplying real numbers or algebraic expressions.

Examples:

$$1) \quad 4i(-6 + i)$$

$$-4 - 24i$$

$$2) \quad (9 - 2i)(-4 + 7i)$$

$$-22 + 71i$$

Complex Conjugates

Two complex number of the form $a + bi$ and $a - bi$ are called complex conjugates.

The product of complex conjugates is *always* a real number.

$$\begin{aligned}\text{Example: } (2 + 4i)(2 - 4i) &= 4 - 8i + 8i - 16i^2 \\ &= 4 - 16(-1) = 4 + 16 = 20\end{aligned}$$

Want to use this to write the quotient of two complex numbers in standard form.

Dividing Complex Numbers

Write the quotient $\frac{7+5i}{1-4i}$ in standard form.

****Very similar to rationalizing your denominator with radicals**

$$\frac{7+5i}{1-4i} \cdot \frac{1+4i}{1+4i} = \frac{7+28i+5i+20i^2}{1+4i-4i-16i^2} = \frac{7+33i+20(-1)}{1-16(-1)}$$

multiply numerator
and denominator by
complex conjugate

FOIL

Simplify and use $i^2 = -1$

$$= \frac{-13+33i}{17}$$

Simplify

$$= -\frac{13}{17} + \frac{33}{17}i$$

Write in standard form

Write the expressions as a complex number in standard form.

$$1) \frac{5}{1+i}$$

$$\frac{5}{2} - \frac{5}{2}i$$

$$2) \frac{5+2i}{3-2i}$$

$$\frac{11}{13} + \frac{16}{13}i$$

Homework

p. 279: 23-33 (odds)

MULTIPLYING AND DIVIDING Write the expression as a complex number in standard form.

22. $6i(3 + 2i)$

23. $-i(4 - 8i)$

24. $(5 - 7i)(-4 - 3i)$

25. $(-2 + 5i)(-1 + 4i)$

26. $(-1 - 5i)(-1 + 5i)$

27. $(8 - 3i)(8 + 3i)$

28. $\frac{7i}{8 + i}$

29. $\frac{6i}{3 - i}$

30. $\frac{-2 - 5i}{3i}$

31. $\frac{4 + 9i}{12i}$

32. $\frac{7 + 4i}{2 - 3i}$

33. $\frac{-1 - 6i}{5 + 9i}$