Objective

Students will be able to name the degree of a polynomial expression and identify different types of polynomials.

Quiz on Thursday over Evaluating and Graphing Polynomial Functions

Polynomial

Polynomial is a monomial or a sum of monomials; all exponents are whole numbers and the coefficients are all real numbers

A polynomial function is written in standard form if its terms are written in descending order of exponents from left to right:

degree

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

leading coefficient

constant term

Common Polynomial Functions

Degree	Type	Standard form	Example
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^4 + 2x - 1$

Decide whether the function is a polynomial. If so, write in standard form and state its degree, type, and leading coefficient. If not, explain why.

1) $g(x) = 3x^2 - \sqrt{5}x - x^3 + \sqrt{2}$

Yes, $g(x) = -x^3 + 3x^2 - \sqrt{5}x + \sqrt{2}$; it has degree 3 (cubic) and a leading coefficient of -1

2) $f(x) = 4x^4 + 3x^{-2} - x^{1/2}$

No, $3x^{-2}$ and $x^{1/2}$ have exponents that are not whole numbers.

Use direct substitution to find the value of *f(2)* when

$$f(x) = -3x^3 + x^2 - 12x - 5$$

$$f(2) = -3(2)^3 + (2)^2 - 12(2) - 5$$

$$= -3(8) + 4 - 24 - 5$$

$$= -24 - 25$$

$$= -49$$

f(2) = -49

We can also evaluate polynomials by <u>synthetic substitution</u>.

Use synthetic substitution to evaluate
$f(x) = -2x^3 + x^2 - 6$ when x = 3.

write coefficients of *f(x)* in standard form

x-value →	3	-2	1	0	-6	← coefficients
			-6	-15	-45	
		-2	-5	-15	-51	

1) bring down the leading coefficient

2) multiply leading coefficient by x-value

3) Add

f(3) = -51

Use synthetic substitution to evaluate

$f(x) = 5x^4 - 2x^3 - 6x$ when x = 2.

$$
\begin{array}{r|rrrrr}
2 & 5 & -2 & 0 & -6 & 0 \\
  &   & 10 & 16 & 32 & 52 \\
\hline
  & 5 & 8 & 16 & 26 & 52
\end{array}
$$

f(2) = 52

Homework

p. 341: 3-17 (odds)

POLYNOMIAL FUNCTIONS **Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.**

3. $f(x) = 8 - x^2$ **5.** $g(x) = \pi x^4 + \sqrt{6}$ **7.** $h(x) = -\frac{5}{2}x^3 + 3x - 10$

DIRECT SUBSTITUTION **Use direct substitution to evaluate the polynomial function for the given value of *x*.**

9. $f(x) = 5x^3 - 2x^2 + 10x - 15;\ x = -1$

11. $g(x) = 4x^3 - 2x^5;\ x = -3$

13. $h(x) = x + \frac{1}{2}x^4 - \frac{3}{4}x^3 + 10;\ x = -4$

SYNTHETIC SUBSTITUTION **Use synthetic substitution to evaluate the polynomial function for the given value of *x*.**

15. $f(x) = 5x^3 - 2x^2 - 8x + 16;\ x = 3$

17. $g(x) = x^3 + 8x^2 - 7x + 35;\ x = -6$

Objective

Students will be able to identify end behavior and find the zeros for polynomials functions in order to sketch the graph of a polynomial function.

Quiz on Thursday over Evaluating and Graphing Polynomial Functions

End Behavior of Polynomial Functions

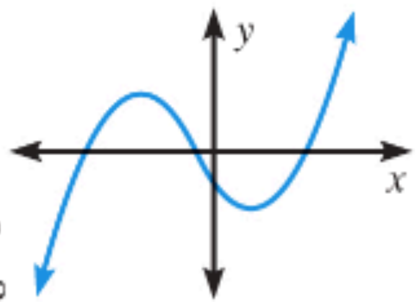
The <u>end behavior</u> of a function's graph is the behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$)

For the graph of a polynomial function, the end behavior is determined by the function's degree and the sign of its leading coefficient.

End Behavior of Polynomial Functions

Degree: odd
Leading coefficient: positive

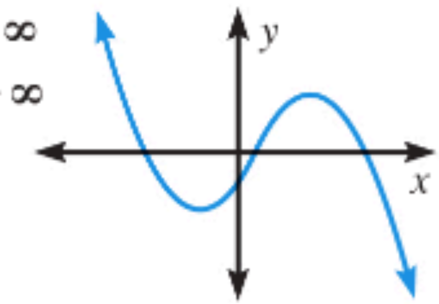
$f(x) \to +\infty$
as $x \to +\infty$



$f(x) \to -\infty$
as $x \to -\infty$

Degree: odd
Leading coefficient: negative

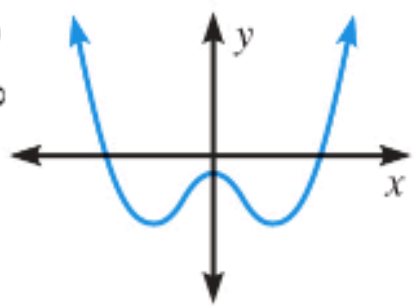
$f(x) \to +\infty$
as $x \to -\infty$



$f(x) \to -\infty$
as $x \to +\infty$

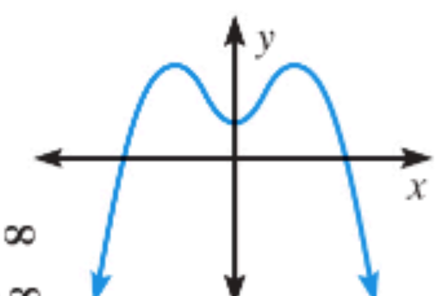
Degree: even
Leading coefficient: positive

$f(x) \to +\infty$
as $x \to -\infty$



$f(x) \to +\infty$
as $x \to +\infty$

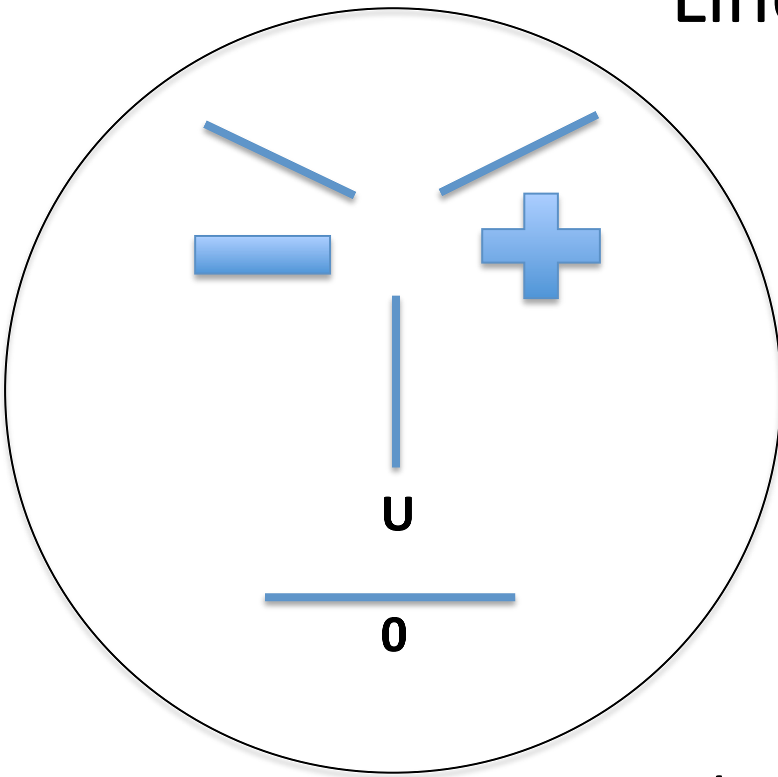
Degree: even
Leading coefficient: negative



$f(x) \to -\infty$
as $x \to -\infty$

$f(x) \to -\infty$
as $x \to +\infty$

Remember Slope Man?

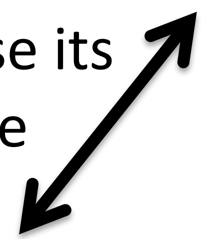


Lines: Have a degree of one (odd)

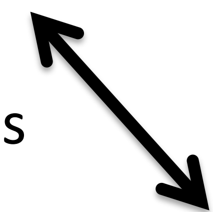
$$y = mx + b$$

slope represents the leading coefficient

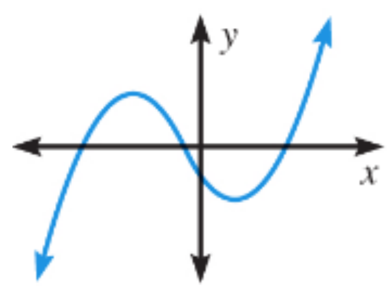
<u>L.C:</u> positive because its positive slope



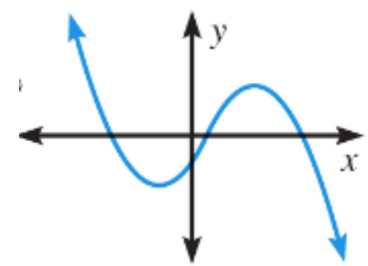
<u>L.C:</u> negative because its negative slope



Similarly,



Degree: odd
L.C: positive

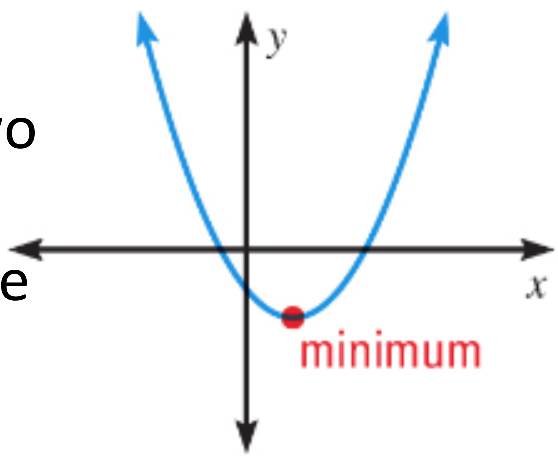


Degree: odd
L.C: negative

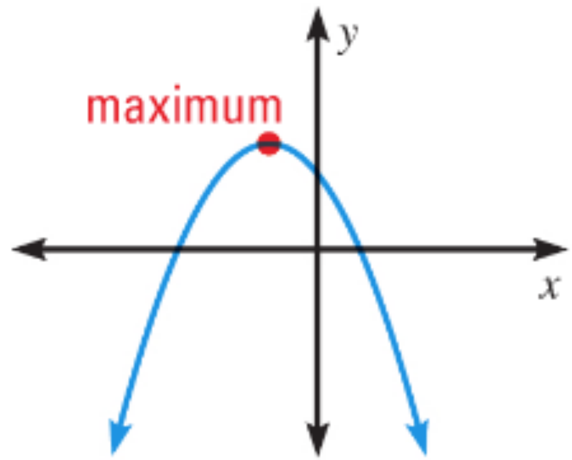
Quadratics and Even Degree

Degree: two
(even)
L.C: positive
Opens Up

end behavior
toward +∞



a is positive



a is negative

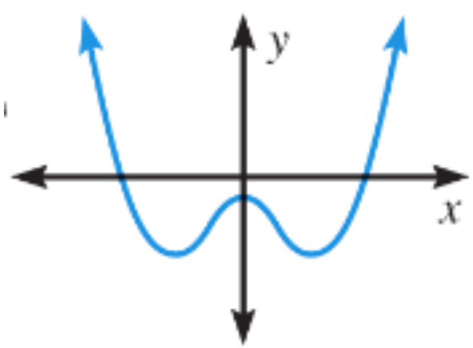
Degree: two
(even)
L.C: negative
Opens Down

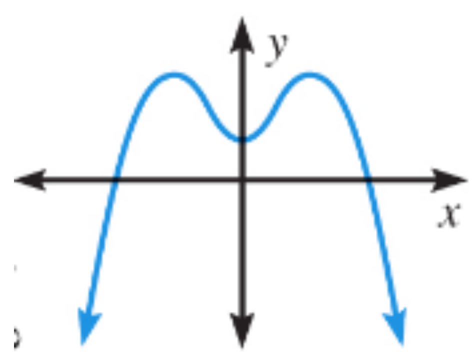
end behavior
toward -∞

Similarly,

Degree: even
L.C: positive
Opens Up

end behavior
toward +∞





Degree: even
L.C: negative
Opens Down

end behavior
toward -∞

Multiplicity of a Polynomial Function

<u>Multiplicity</u> of a polynomial function is the number of times that a zero's associated factor appears in the polynomial

Example: $f(x) = (x - 3)(x + 2)^2$

the zero of 3 has a multiplicity of one
the zeros of -2 has a multiplicity of two

even multiplicity bounces off and odd multiplicity goes through the x-axis

Sketch the graph of the polynomial:

Ex 1: $g(x) = (x+4)(x+1)(x-2)$

Degree: 3 Leading coefficient: 1

End behavior: f(x) $\longrightarrow$ -∞ as x $\longrightarrow$ -∞
and f(x) $\longrightarrow$ +∞ as x $\longrightarrow$ +∞

x-intercept(s): -4, -1, 2 y-intercept: -8

multiplicity: -4, -1, 2 have multiplicities of one

Sketch the graph of the polynomial:

Ex 2: $h(x) = -(x+3)^2(x-1)$

Degree: 3

Leading coefficient: -1

End behavior: f(x) $\longrightarrow$ $+\infty$ as x $\longrightarrow$ $-\infty$
and f(x) $\longrightarrow$ $-\infty$ as x $\longrightarrow$ $+\infty$

x-intercept(s): -3, 1

y-intercept: 9

multiplicity: -3 has a multiplicity of two
1 has a multiplicity of one

Ex 1: Sketch the graph of the polynomial:

$g(x) = (x+4)(x+1)(x-2)$

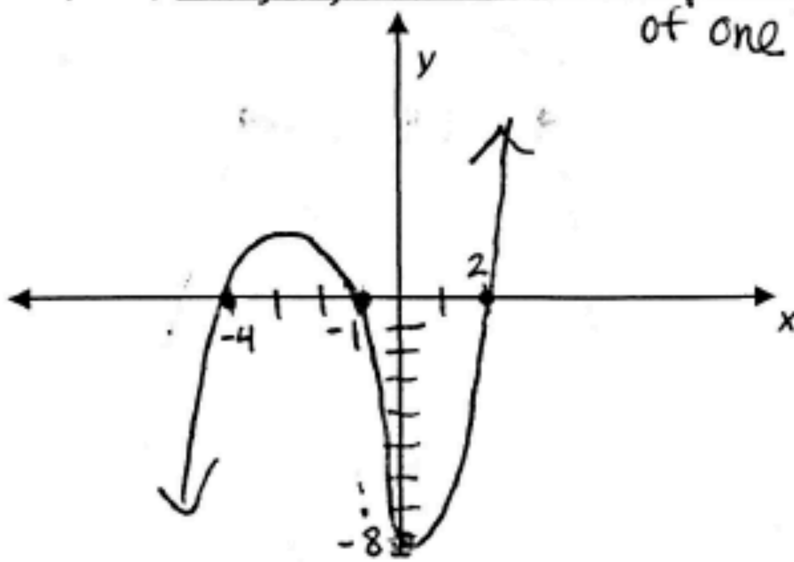
·Degree: 3 ·Leading coefficient: 1

·End behavior: $f(x) \to -\infty$ as $x \to -\infty$

and $f(x) \to +\infty$ as $x \to +\infty$

·x-int(s): −4, −1, 2 y-int: −8

·multiplicity: −4, −1, 2 have multiplicity of one



Ex 2: Sketch the graph of the polynomial:

$h(x) = -(x+3)^2(x-1)$

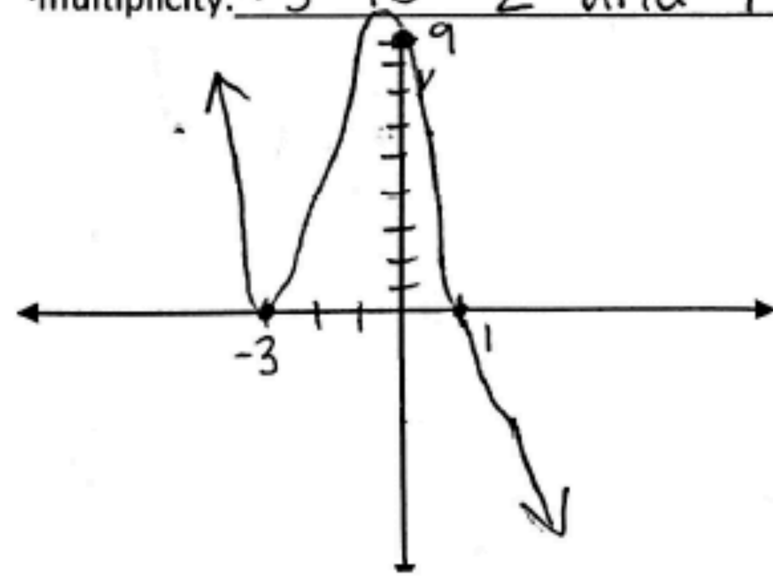
·Degree: 3 ·Leading coefficient: −1

·End behavior: $f(x) \to +\infty$ as $x \to -\infty$

and $f(x) \to -\infty$ as $x \to +\infty$

·x-int(s): −3, 1 y-int: 9

·multiplicity: −3 is 2 and 1 is 1



Homework

Finish examples on 5.2/5.8 Sketch Polynomial Functions in Factored Form Note Sheet

Objective

Students will be able to sketch the graph of a polynomial function using the end behavior, x-intercepts, and y-intercept of the function.

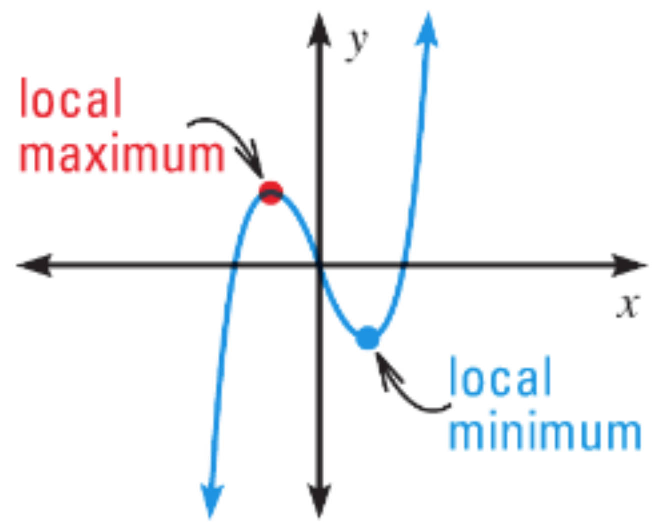
Quiz on Thursday over Evaluating and Graphing Polynomial Functions

NO GRAPHING CALCULATORS for Quiz

Turning Points

Another important characteristic of graphs of polynomial functions is that they have turning points corresponding to local maximum and minimum values.

The y-coordinate of a turning point is a local maximum/ minimum of the function if the point is higher (lower) than all nearby points.



Turning Points of Polynomial Functions

1) The graph of every polynomial function of degree n has at most $n - 1$ turning points.

2) If a polynomial function has n distinct real zeros, then its graph has exactly $n - 1$ turning points.

How many turning points to the following polynomials have? Are they local maximum(s) or local minimum(s)?

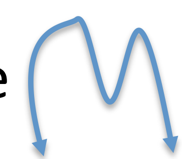
1) $f(x) = x^3 - 3x^2 + 6$

degree: 3; odd   leading coefficient: 1; positive

two turning points; a local max and a local min

2) $g(x) = 10x - 6x^3 + 3x^2 - x^4 - 3$

degree: 4; even   leading coefficient: -1; negative



three turning points; two local maxs and a local min

Sketch the graph of the polynomial:

7) $y=(x-2)^2(x-1)(x+3)^2$

Degree: 5 Leading coefficient: 1

four turning points, two local mins and two local maxs

End behavior: f(x)$\longrightarrow$ -∞ as x $\longrightarrow$ -∞

and f(x) $\longrightarrow$ +∞ as x $\longrightarrow$ +∞

x-intercept(s): 2, 1, -3 y-intercept: -36

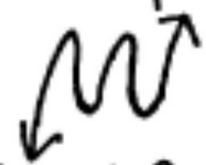
multiplicity: 2 and -3 have a multiplicity of two

1 has a multiplicity of one

7) $y = (x-2)^2(x-1)(x+3)^2$

Degree: 5 (odd)

L.C: 1 (positive)



4 turning points

x-ints: 2, 1, -3

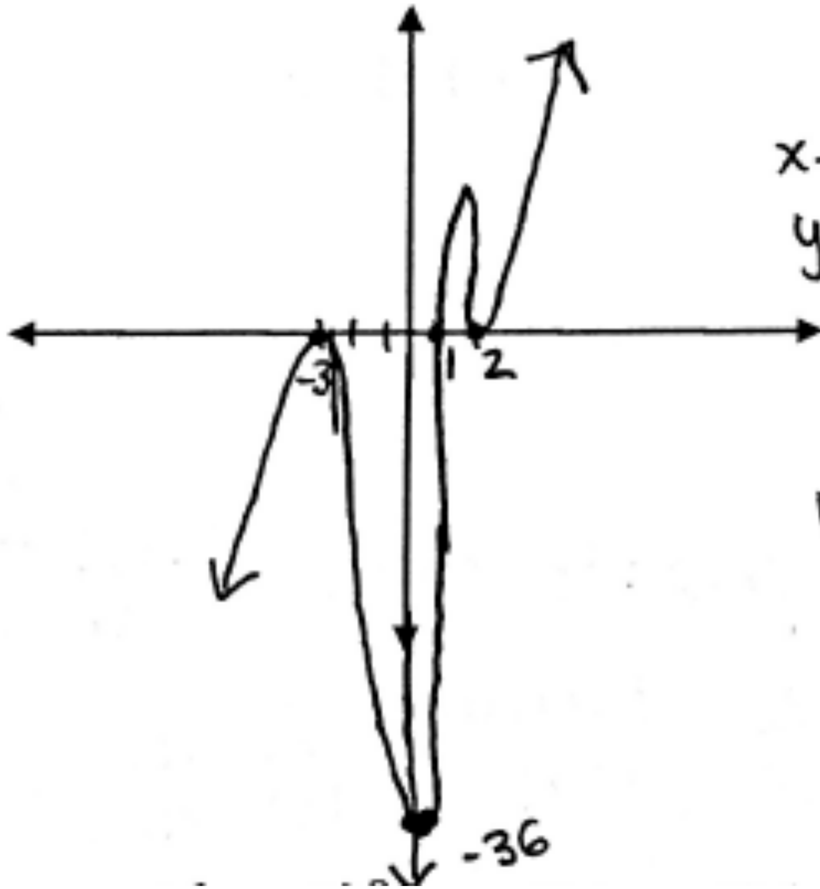
y-int:

$(-2)^2(-1)(3)^2=$

$4(-1)(9)=$

-36

bounce at 2 and -3



Homework

Finish practice problems on 5.8 Analyze Graphs of Polynomial Functions/ Sketch Polynomial Functions in Factored Form Note Sheet

Study for Quiz tomorrow!

Objective

Students will be able to add, subtract, and multiply polynomials.

Adding and Subtracting Polynomials

To add or subtract polynomials, add or subtract the coefficients of like terms. You can use a vertical or horizontal format.

1) Add $2x^3 - 5x^2 + 3x - 9$ and $x^3 + 6x^2 + 11$ in a vertical format.

$$\begin{array}{rrrrr} & 2x^3 & - 5x^2 & + 3x & - 9 \\ + & x^3 & + 6x^2 & & + 11 \\ \hline & 3x^3 & + x^2 & + 3x & + 2 \end{array}$$

2) Add $3y^3 - 2y^2 - 7y$ and $-4y^2 + 2y - 5$ in a horizontal format.

$$(3y^3 - 2y^2 - 7y) + (-4y^2 + 2y - 5)$$

$$= 3y^3 - 2y^2 - 4y^2 - 7y + 2y - 5$$

$$= 3y^3 - 6y^2 - 5y - 5$$

Subtracting Polynomials

1) Subtract $3x^3 + 2x^2 - x + 7$ from $8x^3 - x^2 - 5x + 1$ in a vertical format.

$$\begin{array}{r} 8x^3 - x^2 - 5x + 1 \\ -\underline{(3x^3 + 2x^2 - x + 7)} \end{array}$$

$$\begin{array}{rr} & 8x^3 - x^2 - 5x + 1 \\ + & \underline{-3x^3 - 2x^2 + x - 7} \\ & 5x^3 - 3x^2 - 4x - 6 \end{array}$$

**Align like terms, then add the opposite of the subtracted polynomial

2) Subtract $5z^2 - z + 3$ from $4z^2 + 9z - 12$ in a horizontal format.

**Write the opposite of the subtracted polynomial, then add like terms

$$(4z^2 + 9z - 12) - (5z^2 - z + 3) = 4z^2 + 9z - 12 - 5z^2 + z - 3$$

$$= 4z^2 - 5z^2 + 9z + z - 12 - 3 \qquad = -z^2 + 10z - 15$$

Find the sum or difference of the following problems.

1) $(t^2 - 6t + 2) + (5t^2 - t - 8)$

$6t^2 - 7t - 6$

2) $(8z - 3 + 9z^3) - (z^3 - 13z^2 - 4)$

$8z^3 + 13z^2 + 8z + 1$

Multiplying Polynomials Vertically and Horizontally

To multiply two polynomials, you multiply each term of the first polynomial by each term of the second polynomial.

1) Multiply $-2x^2 + 3x - 6$ and $x - 2$ in a vertical format.

$$\begin{array}{rr} & -2x^2 + 3x - 6 \\ \times & x - 2 \\ \hline \text{Multiply } -2x^2 + 3x - 6 \text{ by } -2 & 4x^2 - 6x + 12 \\ \text{Multiply } -2x^2 + 3x - 6 \text{ by } x \quad + & -2x^3 + 3x^2 - 6x \\ \hline \text{Combine like terms} & -2x^3 + 7x^2 - 12x + 12 \end{array}$$

2) Multiply $z + 3$ and $3z^2 - 2z + 4$ in a horizontal format.

$$(z + 3)(3z^2 - 2z + 4) = (z + 3)3z^2 - (z + 3)2z + (z + 3)4$$

$$= 3z^3 + 9z^2 - 2z^2 - 6z + 4z + 12 = 3z^3 + 7z^2 - 2z + 12$$

**It is like FOILing (but with three terms)

Multiply $x - 5$, $x + 1$, and $x + 3$ in a horizontal format.

$$\underbrace{(x-5)(x+1)}_{\text{FOIL}}(x+3) = (x^2 - 5x + x - 5)(x + 3)$$

$$= (x^2 - 4x - 5)(x + 3)$$

$$= x^3 + 3x^2 - 4x^2 - 12x - 5x - 15$$

$$= x^3 - x^2 - 17x - 15$$

Product Patterns

Sum and Difference	Example
$(a+b)(a-b)=a^2-b^2$	$(x+4)(x-4)=x^2-16$

Square of a Binomial	Example
$(a+b)^2=a^2+2ab+b^2$	$(y+3)^2=y^2+6y+9$
$(a-b)^2=a^2-2ab+b^2$	$(3z^2-5)^2=9z^4-30z^2+25$

Cube of a Binomial	Example
$(a+b)^3=a^3+3a^2b+3ab^2+b^3$	$(x+2)^3=x^3+6x^2+12x+8$
$(a-b)^3=a^3-3a^2b+3ab^2-b^3$	$(p-3)^3=p^3-9p^2+27p-27$

Use Special Product Patterns to Solve the Following Problems:

1) $(3t + 4)(3t - 4)$

sum and difference

$9t^2 - 16$

2) $(8x - 3)^2$

square of a binomial

$64x^2 - 48x + 9$

3) $(pq + 5)^3$

cube of a binomial

$p^3q^3 - 15p^2q^2 + 75pq + 125$

Homework p. 349: 5-11 (odds), 17-21 (odds), 29, 39, 44

ADDING AND SUBTRACTING POLYNOMIALS **Find the sum or difference.**

5. $(4y^2 + 9y - 5) - (4y^2 - 5y + 3)$

7. $(3s^3 + s) + (4s^3 - 2s^2 + 7s + 10)$

9. $(5c^2 + 7c + 1) + (2c^3 - 6c + 8)$

11. $(5b - 6b^3 + 2b^4) - (9b^3 + 4b^4 - 7)$

MULTIPLYING POLYNOMIALS **Find the product of the polynomials.**

17. $5x^2(6x + 2)$

19. $(3z + 1)(z - 3)$

21. $(2a - 3)(a^2 - 10a - 2)$

MULTIPLYING THREE BINOMIALS **Find the product of the binomials.**

29. $(x + 1)(x - 7)(x + 3)$

SPECIAL PRODUCTS **Find the product.**

39. $(w - 9)^2$

44. $(7x - y)^3$

Objective

Students will be able to factor polynomial equations.

We have learned how to factor the following types of quadratic expressions.

Type	Example
General trinomial	$2x^2 - 3x - 20 = (2x + 5)(x - 4)$
Perfect square trinomial	$x^2 + 8x + 16 = (x + 4)^2$
Difference of two squares	$9x^2 - 1 = (3x + 1)(3x - 1)$
Common monomial factor	$8x^2 + 20x = 4x(2x + 5)$

Factoring Polynomials

You can also factor polynomials with degree greater than 2.

A factorable polynomial with integer coefficients is factored completely if it is written as a product of unfactorable polynomials with integer coefficients.

Examples:

$2(x + 1)(x – 4)$ and $5x^2(x^2 – 3)$ are factored completely

$3x(x^2 – 4)$ is not factored completely because $x^2 – 4$ can be factored as $(x + 2)(x – 2)$; $3x(x + 2)(x – 2)$ is factored completely

Factor the following polynomials completely.

1) $x^3 + 2x^2 - 15x$

$= x(x^2 + 2x - 15)$ Factor common monomial (greatest common factor)

$= x(x + 5)(x - 3)$ Factor trinomial

2) $2y^5 - 18y^3$

$= 2y^3(y^2 - 9)$ Factor greatest common factor

$= 2y^3(y - 3)(y + 3)$ Difference of two squares

3) $4z^4 - 16z^3 + 16z^2$

$= 4z^2(z^2 - 4z + 4)$ Factor greatest common factor

$= 4z^2(z - 2)^2$ Perfect square trinomial

Special Factoring Patterns

Sum of Two Cubes	Example
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$8x^3 + 27 = (2x)^3 + 3^3$
	$= (2x + 3)(4x^2 - 6x + 9)$

Difference of Two Cubes	Example
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$64x^3 - 1 = (4x)^3 - 1^3$
	$= (4x - 1)(16x^2 + 4x + 1)$

S.O.A.P

ame

pposite

lways

ositive

Factor the following polynomials completely.

1) $x^3 + 64$

$= x^3 + 4^3$ Sum of two cubes

$= (x + 4)(x^2 - 4x + 16)$

2) $16z^5 - 250z^2$

$= 2z^2(8z^3 - 125)$ Factor greatest common factor

$= 2z^2[(2z)^3 - 5^3]$ Difference of two cubes

$= 2z^2(2z - 5)(4z^2 + 10z + 25)$

Homework

p. 356: 3-13 (odds), 16

MONOMIAL FACTORS **Factor the polynomial completely.**

3. $14x^2 - 21x$

5. $c^3 + 9c^2 + 18c$

7. $3y^5 - 48y^3$

9. ★ **MULTIPLE CHOICE** What is the complete factorization of $2x^7 - 32x^3$?

Ⓐ $2x^3(x + 2)(x - 2)(x^2 + 4)$

Ⓑ $2x^3(x^2 + 2)(x^2 - 2)$

Ⓒ $2x^3(x^2 + 4)^2$

Ⓓ $2x^3(x + 2)^2(x - 2)^2$

SUM OR DIFFERENCE OF CUBES **Factor the polynomial completely.**

11. $y^3 - 64$

13. $125n^3 + 216$

16. $192w^3 - 3$

Objective

Students will be able to factor polynomial equations.

Factoring By Grouping

For some polynomials, you can factor by grouping pairs of terms that have a common monomial factor. The pattern for factoring by grouping is shown below:

$$ra + rb + sa + sb = r(a + b) + s(a + b)$$

$$= (r + s)(a + b)$$

Factor the polynomial $x^3 - 3x^2 - 16x + 48$ completely.

$x^3 - 3x^2 - 16x + 48$ Factor by grouping

$(x^3 - 3x^2) + (- 16x + 48)$

$= x^2(x - 3) - 16(x - 3)$ Factor greatest common factor

$= (x^2 - 16)(x - 3)$ Factor greatest common factor (Distributive property)

$= (x - 4)(x + 4)(x - 3)$ Difference of two squares

Quadratic Form

An expression of the form $au^2 + bu + c$, where u is any expression in x, is said to be in <u>quadratic form</u>. We can use the factoring techniques that we learned previously to help us factor such expressions.

Example: $x^4 + 2x^2 + 1$ is in quadratic form (it is "quadratic-like"

Factor the following polynomials completely:

1) $16x^4 - 81$ $= (4x^2)^2 - 9^2$ Write as a difference of two squares

$= (4x^2 - 9)(4x^2 + 9)$ Difference of two squares

$= (2x - 3)(2x + 3)(4x^2 + 9)$ Difference of two squares

2) $2p^8 + 10p^5 + 12p^2$

$= 2p^2(p^6 + 5p^3 + 6)$ Factor greatest common factor

$= 2p^2(p^3 + 3)(p^3 + 2)$ Factor trinomial in quadratic form

Homework

p. 357: 19-29 (odds)

FACTORING BY GROUPING **Factor the polynomial completely.**

19. $y^3 - 7y^2 + 4y - 28$

21. $3m^3 - m^2 + 9m - 3$

23. $4c^3 + 8c^2 - 9c - 18$

QUADRATIC FORM **Factor the polynomial completely.**

25. $a^4 + 7a^2 + 6$

27. $32z^5 - 2z$

29. $15x^5 - 72x^3 - 108x$

Objective

Students will be able to factor and solve polynomial equations.

Polynomial and Polynomial Functions (5.2 - 5.5, 5.8) Test on Thursday!

Solving Polynomial Equations

After we factor, we are going to use the zero product property to solve higher-degree polynomial equations.

What are the solutions of the equation $3x^5 + 15x = 18x^3$?

$3x^5 + 15x = 18x^3$	
$3x^5 - 18x^3 + 15x = 0$	Write in standard form
$3x(x^4 - 6x^2 + 5) = 0$	Factor greatest common factor
$3x(x^2 - 1)(x^2 - 5) = 0$	Factor trinomial
$3x(x - 1)(x + 1)(x^2 - 5) = 0$	Difference of two squares
$x = 0, 1, -1, \sqrt{5}, -\sqrt{5}$	Zero product property

What are the solutions of the equation $x^3 - 2x^2 + 2x = 4$?

$x^3 - 2x^2 + 2x = 4$

$x^3 - 2x^2 + 2x - 4 = 0$ Write in standard form

$(x^3 - 2x^2) + (2x - 4) = 0$ Factor by grouping

$x^2(x - 2) + 2(x - 2) = 0$ Factor greatest common factor

$(x - 2)(x^2 + 2) = 0$ Factor greatest common factor

$x = 2, i\sqrt{2}, -i\sqrt{2}$ Zero product property

What are the solutions of the equation $x^3 - 1 = 0$?

$x^3 - 1 = 0$ $(x)^3 - 1^3 = 0$ Difference of cubes

$(x - 1)(x^2 + x + 1) = 0$ Zero product property

unfactorable, use quadratic formula to solve

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$x = 1, \frac{-1 \pm i\sqrt{3}}{2}$$

Homework

p. 357: 32-38 (evens), 39 (find both real and imaginary solutions)

SOLVING EQUATIONS **Find the real-number solutions of the equation.**

AND imaginary solutions, if possible

32. $y^3 - 5y^2 = 0$

34. $g^3 + 3g^2 - g - 3 = 0$

36. $4w^4 + 40w^2 - 44 = 0$

38. $5b^3 + 15b^2 + 12b = -36$

39. $x^6 - 4x^4 - 9x^2 + 36 = 0$