Objective

Students will be able to use rational exponents and their properties (along with radicals) to evaluate and simplify expressions.

Complex Numbers and Properties of Exponents Quest on Thursday!

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

Radical form

Rational Exponent Form

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, \; a \neq 0$$

Evaluate the following in both rational exponent and radical form.

Rational Exponent Form: Radical Form:

1) $16^{3/2}$

$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$

$16^{3/2} = (16^{1/2})^3 = ((4^2)^{1/2})^3 = 4^3 = 64$

2) $27^{-2/3}$

$$27^{-2/3} = \left(\sqrt[3]{27}\right)^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$27^{-2/3} = \left(27^{1/3}\right)^{-2} = \left((3^3)^{1/3}\right)^{-2} = \left(3^1\right)^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

3) $4^{5/2}$

$4^{5/2} = (\sqrt{4})^5 = 2^5 = 32$

$4^{5/2} = (4^{1/2})^5 = ((2^2)^{1/2})^5 = (2^1)^5 = 2^5 = 32$

Properties of Rational Exponents

Let *a* and *b* be real numbers and let *m* and *n* be rational numbers.

	Property	**Example**
1.	$a^m \cdot a^n =$	$5^{1/2} \cdot 5^{3/2} =$
2.	$(a^m)^n =$	$(3^{5/2})^2 =$
3.	$(ab)^m =$	$(16 \cdot 9)^{1/2} =$
4.	$a^{-m} =$	$36^{-1/2} =$
5.	$\dfrac{a^m}{a^n} =$	$\dfrac{4^{5/2}}{4^{1/2}} =$
6.	$\left(\dfrac{a}{b}\right)^m =$	$\left(\dfrac{27}{64}\right)^{1/3} =$

Use the properties of rational exponents to simplify the following expressions.

1) $\left(6^{1/2} \cdot 4^{1/3}\right)^2 = \left(6^{1/2}\right)^2 \cdot \left(4^{1/3}\right)^2 = 6^{2/2} \cdot 4^{2/3}$

$= 6^1 \cdot 4^{2/3} \quad = 6 \cdot 4^{2/3}$

2) $\left(\dfrac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\dfrac{42}{6}\right)^{1/3}\right]^2 = \left(7^{1/3}\right)^2 = 7^{2/3}$

Properties of Radicals

Product property of radicals:

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient property of radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$$

Use the properties of radicals to simplify the following expressions:

1) $\sqrt[3]{12}\cdot\sqrt[3]{18} = \sqrt[3]{12\cdot 18} = \sqrt[3]{216} = \sqrt[3]{6^3} = 6$

2) $\dfrac{\sqrt{3}}{\sqrt{75}}\cdot\dfrac{\sqrt{75}}{\sqrt{75}} = \dfrac{\sqrt{225}}{75} = \dfrac{15}{75} = \dfrac{1}{5}$

3) $\dfrac{\sqrt[3]{375}}{\sqrt[3]{3}} = \sqrt[3]{\dfrac{375}{3}} = \sqrt[3]{125} = \sqrt[3]{5^3} = 5$

p. 417: 3-6, 9, 12

Homework

MATCHING EXPRESSIONS Match the expression in rational exponent notation with the equivalent expression in radical notation.

3. $2^{1/3}$
4. $2^{3/2}$
5. $2^{2/3}$
6. $2^{1/2}$

A. $(\sqrt{2})^3$
B. $\sqrt{2}$
C. $\sqrt[3]{2}$
D. $(\sqrt[3]{2})^2$

USING RATIONAL EXPONENT NOTATION Rewrite the expression using rational exponent notation.

9. $(\sqrt[3]{10})^7$

USING RADICAL NOTATION Rewrite the expression using radical notation.

12. $7^{1/3}$

p. 424: 1, 4, 8, 16

1. **VOCABULARY** Are $2\sqrt{5}$ and $2\sqrt[3]{5}$ like radicals? *Explain* why or why not.

Simplify the expression.

4. $(6^{2/3})^{1/2}$
8. $\left(\frac{7^3}{4^3}\right)^{-1/3}$
16. $\sqrt[3]{16} \cdot \sqrt[3]{4}$

Objective

Students will be able to use properties of exponents and rational exponents to evaluate and simplify expressions.

Complex Numbers and Properties of Exponents Quest on <u>Friday</u>!

Simplify the following expressions:

4) $$\sqrt{\frac{m^4}{n^8}} = \frac{\sqrt{m^4}}{\sqrt{n^8}} = \frac{(m^4)^{1/2}}{(n^8)^{1/2}} = \frac{m^{4/2}}{n^{8/2}} = \frac{m^2}{n^4}$$

5) $$\sqrt[3]{32a^6b^4c^9d^2} = \sqrt[3]{2^3 \cdot 4a^3a^3b^3bc^3c^3c^3d^2}$$

$$= 2aabccc\sqrt[3]{4bd^2} = 2a^2bc^3\sqrt[3]{4bd^2}$$

6) $$\frac{14xy^{1/3}}{2x^{1/2}z^{-6}} = 7x^{1-1/2}y^{1/3}z^6 = 7x^{1/2}y^{1/3}z^6$$

Simplify the following expressions:

7) $\sqrt[3]{\dfrac{x^{15}}{y^{6}}}$ $\qquad \dfrac{x^{5}}{y^{2}}$

8) $\dfrac{\sqrt[3]{x^{5}y^{2}} \cdot \sqrt[3]{xy^{2}}}{\sqrt{25x^{16}}}$ $\qquad \dfrac{y\sqrt[3]{y}}{5x^{6}}$