Objective

Students will be able to find the nth root of a radical and evaluate rational expressions with and without a calculator.

*n*th Roots

For an integer n greater than 1, if $b^n = a$, then b is an nth root of a. An nth root of a is written as $\sqrt[n]{a}$ where n is the index of the radical.

You can also write an nth root of a as a power of a.

Recall: $\sqrt{a} = a^{1/2}$ $\sqrt[3]{a} = a^{1/3}$ $\sqrt[4]{a} = a^{1/4}$

In general, $\sqrt[n]{a} = a^{1/n}$ for any integer n greater than 1.

Real *n*th Roots of *a*

Let n be an integer ($n > 1$) and let a be a real number.

n is an even integer.

a is negative	$a < 0$	No real nth roots.
	$a = 0$	One real nth root: $\sqrt[n]{0} = 0$
a is positive	$a > 0$	Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$

n is an odd integer.

$a < 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$	One real nth root: $\sqrt[n]{0} = 0$
$a > 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$

Find the indicated real *n*th root(s) of *a*.

1) $n = 3$, $a = -216$

Because $n = 3$ is odd, and $a = -216 < 0$, -216 has one real cube root. Because $(-6)^3 = -216$, you can write $\sqrt[3]{-216} = -6$ or $(-216)^{1/3} = -6$

2) $n = 4$, $a = 81$

Because $n = 4$ is even, and $a = 81 > 0$, 81 has two real fourth roots. Because $3^4 = 81$ and $(-3)^4 = 81$, you can write $\sqrt[4]{81} = \pm 3$ or

$81^{1/4} = \pm 3$

Recall: Rational Exponents

Let $a^{1/n}$ be an *n*th root of *a*, and let *m* be a positive integer.

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^{1/n}\right)^m} = \frac{1}{\left(\sqrt[n]{a}\right)^m},\ a \neq 0$$

radical

rational exponent

$$\sqrt[n]{a} = a^{1/n}$$

Recall: Evaluate the following in both rational exponent and radical form.

Rational Exponent Form:

Radical Form:

*only need to use the principal root

1) $16^{3/2}$

$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$

$16^{3/2} = (16^{1/2})^3 = ((4^2)^{1/2})^3 = 4^3 = 64$

$$27^{-2/3} = \left(\sqrt[3]{27}\right)^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

2) $27^{-2/3}$

$$27^{-2/3} = \left(27^{1/3}\right)^{-2} = \left((3^3)^{1/3}\right)^{-2} = \left(3^1\right)^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Approximate Roots with a Calculator

	Expression	Keystrokes	Display
a.	$9^{1/5}$	9 [^] [(] 1 [÷] 5 [)] [ENTER]	1.551845574
b.	$12^{3/8}$	12 [^] [(] 3 [÷] 8 [)] [ENTER]	2.539176951
c.	$(\sqrt[4]{7})^3 = 7^{3/4}$	7 [^] [(] 3 [÷] 4 [)] [ENTER]	4.303517071

Find the indicated real nth root(s) of a.

1) $n = 4$, $a = 625$

± 5

2) $n = 3$, $a = -64$

-4

Evaluate the expression without using a calculator.

3) $4^{5/2}$

32

4) $81^{3/4}$

27

5) $1^{7/8}$

1

6) $9^{-1/2}$

1/3

Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.

7) $64^{-2/3}$

0.06

8) $(\sqrt[3]{-30})^2$

9.65

Homework p. 417: 15-17, 21-31 (odds), 46

FINDING *N*TH ROOTS **Find the indicated real *n*th root(s) of *a*.**

15. $n = 2, a = 64$ **16.** $n = 3, a = -27$ **17.** $n = 4, a = 0$

EVALUATING EXPRESSIONS **Evaluate the expression without using a calculator.**

21. $\sqrt[6]{64}$ **22.** $8^{1/3}$ **23.** $16^{3/2}$ **24.** $\sqrt[3]{-125}$

25. $27^{2/3}$ **26.** $(-243)^{1/5}$ **27.** $(\sqrt[3]{8})^{-2}$ **28.** $(\sqrt[3]{-64})^4$

29. $(\sqrt[4]{16})^{-7}$ **30.** $25^{3/2}$ **31.** $64^{-2/3}$ **32.** $\frac{1}{81^{-3/4}}$

46. ★ **MULTIPLE CHOICE** Which expression has the greatest value?

(A) $27^{3/5}$ (B) $5^{3/2}$ (C) $\sqrt[3]{81}$ (D) $(\sqrt[3]{2})^8$

Objective

Students will be able to find the nth root of a radical and evaluate rational expressions with and without a calculator.

Solve the equations using *n*th roots

1) $4x^5 = 128$

$$x^5 = 32$$ Divide each side by 4

$$x = \sqrt[5]{32}$$ Take fifth root of each side

$$x = 2$$ Simplify

2) $(x - 3)^4 = 21$

$$x - 3 = \pm\sqrt[4]{21}$$ Take fourth roots of each side

$$x = 3 \pm \sqrt[4]{21}$$ Add 3 to each side

$x = \sqrt[4]{21} + 3$ or $x = -\sqrt[4]{21} + 3$ Write solutions separately

$x \approx 5.14$ or $x \approx 0.86$ Use a calculator

Solve the equation. Round the result to two decimal places when appropriate.

1) $x^3 = 64$

4

2) $3x^2 = 108$

± 6

3) $(1/2)x^5 = 512$

4

4) $(x - 2)^3 = -14$

-0.41

5) $(x + 5)^4 = 16$

-7, -3

Homework p. 417: 48-55

ERROR ANALYSIS *Describe* **and correct the error in solving the equation.**

48.

$$x^3 = 27$$
$$x = \sqrt[3]{27}$$
$$x = 9$$

49.

$$x^4 = 81$$
$$x = \sqrt[4]{81}$$
$$x = 3$$

SOLVING EQUATIONS **Solve the equation. Round the result to two decimal places when appropriate.**

50. $x^3 = 125$

51. $5x^3 = 1080$

52. $x^6 + 36 = 100$

53. $(x - 5)^4 = 256$

54. $x^5 = -48$

55. $7x^4 = 56$

Objective

Students will be able to add, subtract, multiply, and divide radical expressions.

Properties of Rational Exponents

Let *a* and *b* be real numbers and let *m* and *n* be rational numbers.

Property	Example
1. $a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
2. $(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
3. $(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Use the properties of rational exponents to simplify the following expressions.

1) $\left(4^5 \cdot 3^5\right)^{-1/5} = \left[(4\cdot 3)^5\right]^{-1/5} = \left[(12)^5\right]^{-1/5} = (12)^{-5/5}$

$$= (12)^{-1} = \frac{1}{12}$$

2) $\left(\dfrac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\dfrac{42}{6}\right)^{1/3}\right]^2 = \left(7^{1/3}\right)^2 = 7^{2/3}$

Properties of Radicals

Product property of radicals:

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient property of radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$$

Use the properties of radicals to simplify the following expressions:

1) $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$

2) $\dfrac{\sqrt{3}}{\sqrt{75}} \cdot \dfrac{\sqrt{75}}{\sqrt{75}} = \dfrac{\sqrt{225}}{75} = \dfrac{15}{75} = \dfrac{1}{5}$

3) $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\dfrac{80}{5}} = \sqrt[4]{16} = 2$

Use the properties of radicals to simplify the following expressions:

4) $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5} = \sqrt[3]{27} \cdot \sqrt[3]{5} = 3\sqrt[3]{5}$

Factor out perfect cube

Product Property

5) $$\frac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}} = \frac{\sqrt[5]{28}}{\sqrt[5]{32}} = \frac{\sqrt[5]{28}}{2}$$

Make denominator a perfect fifth power

Like Radicals

Radical expressions with the same index and radicand are like radicals. To add or subtract like radicals, use the distributive property.

1) $\sqrt[4]{10} + 7\sqrt[4]{10} \quad = (1+7)\sqrt[4]{10} \quad = 8\sqrt[4]{10}$

2) $2(8^{1/5}) + 10(8^{1/5}) \;\; = (2+10)(8^{1/5}) \;\; = 12(8^{1/5})$

3) $\sqrt[3]{54} - \sqrt[3]{2} \quad = \sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2} \;\; = 3\sqrt[3]{2} - \sqrt[3]{2}$

Factor out perfect cube

$= (3-1)\sqrt[3]{2} \qquad = 2\sqrt[3]{2}$

Homework p. 424: 15-29 (odds), 35, 37

PROPERTIES OF RADICALS **Simplify the expression.**

15. $\sqrt{20} \cdot \sqrt{5}$ **17.** $\sqrt[4]{8} \cdot \sqrt[4]{8}$ **19.** $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}}$ **21.** $\dfrac{\sqrt[4]{36} \cdot \sqrt[4]{9}}{\sqrt[4]{4}}$

23. ★ **MULTIPLE CHOICE** What is the simplest form of the expression $3\sqrt[4]{32} \cdot (-6\sqrt[4]{5})$?

(A) $\sqrt[4]{10}$ (B) $-18\sqrt[4]{10}$ (C) $-36\sqrt[4]{10}$ (D) $36\sqrt[8]{10}$

SIMPLEST FORM **Write the expression in simplest form.**

25. $\sqrt[6]{256}$ **27.** $5\sqrt[4]{64} \cdot 2\sqrt[4]{8}$ **29.** $\dfrac{3}{\sqrt[4]{144}}$

COMBINING RADICALS AND ROOTS **Simplify the expression.**

35. $\frac{1}{8}\sqrt[4]{7} + \frac{3}{8}\sqrt[4]{7}$ **37.** $-6\sqrt[7]{2} + 2\sqrt[7]{256}$

Objective

Students will be able to simplify expressions without using a calculator.

Simplify the following expressions without a calculator.

To do so, write the prime factorization of your *a* value to help (factor tree), then rewrite expression as a product of prime factors so that if possible, you can eliminate the root.

1) $\sqrt[5]{3125} = \sqrt[5]{5^5} = 5$

2) $\sqrt[7]{512} = \sqrt[7]{2^9} = \sqrt[7]{2^7 \cdot 2^2} = \sqrt[7]{2^7} \cdot \sqrt[7]{2^2}$

$= 2 \cdot \sqrt[7]{2^2} = 2\sqrt[7]{4}$

Simplify the following expressions without a calculator.

3) $\sqrt[4]{30000} = \sqrt[4]{2^4 \cdot 3 \cdot 5^4} = \sqrt[4]{2^4} \cdot \sqrt[4]{3} \cdot \sqrt[4]{5^4}$

$= 2 \cdot \sqrt[4]{3} \cdot 5 \quad = 10\sqrt[4]{3}$

4) $\sqrt[5]{5832} = \sqrt[5]{2^3 \cdot 3^6} = \sqrt[5]{2^3 \cdot 3^5 \cdot 3^1}$

$= \sqrt[5]{2^3} \cdot \sqrt[5]{3^5} \cdot \sqrt[5]{3} = \sqrt[5]{8} \cdot 3 \cdot \sqrt[5]{3}$

$= 3\sqrt[5]{3 \cdot 8} \quad = 3\sqrt[5]{24}$

Simplify the following expressions without a calculator.

5) $\sqrt[3]{987840} = \sqrt[3]{2^6 \cdot 3^2 \cdot 5^1 \cdot 7^3}$

$= \sqrt[3]{2^3 \cdot 2^3 \cdot 3^2 \cdot 5^1 \cdot 7^3}$

$= \sqrt[3]{2^3} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{3^2} \cdot \sqrt[3]{5} \cdot \sqrt[3]{7^3}$

$= 2 \cdot 2 \cdot \sqrt[3]{9} \cdot \sqrt[3]{5} \cdot 7 \;\; = 28\sqrt[3]{45}$

Homework

Simplify the following expressions without a calculator.

1) $\sqrt[5]{864}$ 2) $\sqrt[4]{18225}$ 3) $\sqrt[3]{63000}$

p. 469: 5-12

Evaluate the expression without using a calculator.

5. $8^{5/3}$

6. $16^{-3/2}$

7. $(\sqrt[3]{-27})^2$

8. $(\sqrt[3]{64})^{-4}$

9. $\sqrt[3]{88}$

10. $\sqrt[5]{16} \cdot \sqrt[5]{8}$

11. $\sqrt{\frac{12}{49}}$

12. $\frac{\sqrt[3]{24}}{\sqrt[3]{9}}$

Objective

Students will be able to simplify rational expressions involving variables.

You will have an opportunity to earn some points back on your 5.6, 5.7, & 5.9 Quiz tomorrow after school in room 303.

Variable Expressions

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	Example
When *n* is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When *n* is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Absolute value is not needed when all variables are assumed to be positive.

Simplify the following expressions. Assume all variables are positive.

1) $\sqrt[3]{64y^6} = \sqrt[3]{4^3y^3y^3} = \sqrt[3]{4^3}\cdot\sqrt[3]{y^3}\cdot\sqrt[3]{y^3}$

$= 4\cdot y\cdot y \quad = 4y^2$

2) $\left(27p^3q^{12}\right)^{1/3} = 27^{1/3}(p^3)^{1/3}(q^{12})^{1/3} \quad = 3p^{3/3}q^{12/3}$

$= 3pq^4$

3) $\sqrt[4]{\frac{m^4}{n^8}} = \frac{\sqrt[4]{m^4}}{\sqrt[4]{n^8}} = \frac{m}{\sqrt[4]{n^4\cdot n^4}} = \frac{m}{\sqrt[4]{n^4}\cdot\sqrt[4]{n^4}} = \frac{m}{n\cdot n} = \frac{m}{n^2}$

**Never leave radicals or rational exponents in the denominator

Simplify the following expressions. Assume all variables are positive.

4) $\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^5b^5b^4c^5}$

$= \sqrt[5]{a^5b^5b^5c^5} \cdot \sqrt[5]{4a^3b^4} = abbc \cdot \sqrt[5]{4a^3b^4}$

$= ab^2c\sqrt[5]{4a^3b^4}$

5) $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^6 = 7x^{1/4}y^{1/3}z^6$

**Never leave radicals or rational exponents in the denominator

Perform the indicated operation. Assume all variables are positive.

1) $\frac{1}{5}\sqrt{w}+\frac{3}{5}\sqrt{w} \quad =\left(\frac{1}{5}+\frac{3}{5}\right)\sqrt{w} \quad =\left(\frac{4}{5}\right)\sqrt{w}$

2) $3xy^{1/4}-8xy^{1/4} \quad =(3-8)\,xy^{1/4} \quad =-5xy^{1/4}$

3) $12\sqrt[3]{2z^5}-z\sqrt[3]{54z^2} \quad =12\sqrt[3]{2z^3z^2}-z\sqrt[3]{27\cdot 2z^2}$

$=12z\sqrt[3]{2z^2}-3z\sqrt[3]{2z^2} \quad =(12-3)\,z\sqrt[3]{2z^2} \quad =9z\sqrt[3]{2z^2}$

Homework

p. 425: 47, 49, **53-59 (odds)**, 60, 63, 64

VARIABLE EXPRESSIONS **Simplify the expression. Assume all variables are positive.**

47. $\dfrac{x^{2/5}y}{xy^{-1/3}}$

49. $\left(\sqrt[3]{x^2} \cdot \sqrt[6]{x^4}\right)^{-3}$

SIMPLEST FORM **Write the expression in simplest form. Assume all variables are positive.**

53. $\sqrt[4]{12x^2y^6z^{12}}$

55. $\sqrt{x^2yz^3} \cdot \sqrt{x^3z^5}$

57. $\sqrt[3]{\dfrac{x^3}{y^4}}$

59. $\dfrac{\sqrt[4]{x^6}}{\sqrt[7]{x^5}}$

COMBINING VARIABLE EXPRESSIONS **Perform the indicated operation. Assume all variables are positive.**

60. $3\sqrt[5]{x} + 9\sqrt[5]{x}$

63. $(x^4y)^{1/2} + (xy^{1/4})^2$

64. $x\sqrt{9x^3} - 2\sqrt{x^5}$

**Never leave radicals or rational exponents in the denominator

Objective

Students will be able to solve radical expressions.

Solving Radical Equations

Example of a radical equation: $\sqrt[3]{2x+7}=3$

To solve a radical equation, follow these steps:

Step 1: **Isolate** the radical on one side of the equation, if necessary.

Step 2: **Raise** each side of the equation to the same power to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3: **Solve** the polynomial equation using techniques you learned in previous chapters.

Check your solutions!

Solve $\sqrt[3]{2x+7}=3$.

$\left(\sqrt[3]{2x+7}\right)^3=3^3$	Cube each side to eliminate the radical
$2x+7=27$	Simplify
$2x=20$	Subtract 7 from each side
$x=10$	Divide each side by 2

CHECK x = 10 in the original equation:

$\sqrt[3]{2(10)+7}=3$	Substitute 10 for x
$\sqrt[3]{27}=3$	Simplify
$3=3$ ✔	

Solve the following equations. Check your solutions.

1) $\sqrt[3]{x} - 9 = -1$

512

2) $\sqrt{x + 25} = 4$

-9

3) $2\sqrt[3]{x - 3} = 4$

11

Solve $(x+2)^{3/4} - 1 = 7$.

$(x+2)^{3/4} = 8$ Add 1 to each side

$\left[(x+2)^{3/4}\right]^{4/3} = 8^{4/3}$ Raise each side to the power 4/3

$x+2 = (8^{1/3})^4$ Apply properties of exponents

$x+2 = (2)^4$ Simplify

$x+2 = 16$ Simplify

$x = 14$ Subtract 2 from each side

CHECK x = 14 in the original equation:

$(14+2)^{3/4} - 1 = 7$ $\quad (16)^{3/4} - 1 = 7$ $\quad \left(16^{1/4}\right)^3 - 1 = 7$

$(2)^3 - 1 = 7$ $\quad 8 - 1 = 7$ $\quad 7 = 7$ ✔

Solve the following equations. Check your solutions.

1) $3x^{3/2}=375$

25

4) $\left(x+3\right)^{5/2}=32$

1

2) $-2x^{3/4}=-16$

16

5) $\left(x-5\right)^{5/3}=243$

32

3) $-\frac{2}{3}x^{1/5}=-2$

243

6) $\left(x+2\right)^{1/3}+3=7$

62

Homework

p. 456: 3, 7, 11, 13, 17, 21, 27, 29

EQUATIONS WITH SQUARE ROOTS **Solve the equation. Check your solution.**

3. $\sqrt{5x+1} = 6$ **7.** $-2\sqrt{24x} + 13 = -11$ **11.** $\sqrt{-2x+3} - 2 = 10$

EQUATIONS WITH CUBE ROOTS **Solve the equation. Check your solution.**

13. $\sqrt[3]{x} - 10 = -3$ **17.** $-5\sqrt[3]{8x} + 12 = -8$ **21.** $-4\sqrt[3]{x+10} + 3 = 15$

EQUATIONS WITH RATIONAL EXPONENTS **Solve the equation. Check your solution.**

27. $\frac{1}{7}(x+9)^{3/2} = 49$ **29.** $\left(\frac{1}{3}x - 11\right)^{1/2} = 5$

Objective

Students will be able to solve radical expressions.

How many solutions do you get when you take an even root of an expression or you solve a quadratic function?

With two solutions, one may not work! If it does not make the equation true, then it is an extraneous solution. You must *always* check each apparent solution in the original equation.

Solve $x+1=\sqrt{7x+15}$.

FOIL

$$(x+1)^2=(\sqrt{7x+15})^2$$ Square each side

$$x^2+2x+1=7x+15$$ Expand left side and simplify right side

$$x^2-5x-14=0$$ Write in standard form

$$(x-7)(x+2)=0$$ Factor

$(x-7)=0$ or $(x+2)=0$ Zero-product property

$x=7$ or $x=-2$ Solve for x

CHECK x = 7 and x = -2 in the original equation:

$$x = 7:$$

$$7+1=\sqrt{7(7)+15}$$

$$8=\sqrt{49+15}$$

$$8=\sqrt{64}$$

$$8=8 \quad ✔$$

$$x = -2:$$

$$-2+1=\sqrt{7(-2)+15}$$

$$-1=\sqrt{-14+15}$$

$$-1=\sqrt{1}$$

$$-1\neq 1$$

The only solution is 7 (-2 is an extraneous solution).

Solve the following equations. Check for extraneous solutions.

1) $\sqrt{10x+9} = x+3$

0, 4

2) $\sqrt{44-2x} = x-10$

14

FOIL

Solve $\sqrt{x+2}+1=\sqrt{3-x}$.

$(\sqrt{x+2}+1)^2=(\sqrt{3-x})^2$	Square each side
$x+2+2\sqrt{x+2}+1=3-x$	Expand left side and simplify right side
$2\sqrt{x+2}=-2x$	Isolate radical expression
$\sqrt{x+2}=-x$	Divide each side by 2
$(\sqrt{x+2})^2=(-x)^2$	Square each side again
$x+2=x^2$	Simplify
$0=x^2-x-2$	Write in standard form
$0=(x-2)(x+1)$	Factor
$x=2$ or $x=-1$	Zero-product property to solve for x

CHECK x = 2 and x = -1 in the original equation:

$$x = 2:$$

$$\sqrt{2+2} + 1 = \sqrt{3-2}$$
$$\sqrt{4} + 1 = \sqrt{1}$$
$$2 + 1 = 1$$
$$3 \neq 1$$

$$x = -1:$$

$$\sqrt{-1+2} + 1 = \sqrt{3-(-1)}$$
$$\sqrt{1} + 1 = \sqrt{4}$$
$$1 + 1 = 2$$
$$2 = 2 \quad ✔$$

The only solution is -1 (2 is an extraneous solution).

Homework

p.456: 23, 31, 34, 35, 44, 45, 46

EQUATIONS WITH RATIONAL EXPONENTS **Solve the equation. Check your solution.**

23. $2x^{2/3} = 32$

31. $(3x + 43)^{2/3} + 22 = 38$

SOLVING RADICAL EQUATIONS **Solve the equation. Check for extraneous solutions.**

34. $x - 6 = \sqrt{3x}$

35. $x - 10 = \sqrt{9x}$

44. ★ **SHORT RESPONSE** *Explain* how you can tell that $\sqrt{x + 4} = -5$ has no solution without solving it.

EQUATIONS WITH TWO RADICALS **Solve the equation. Check for extraneous solutions.**

45. $\sqrt{4x + 1} = \sqrt{x + 10}$

46. $\sqrt[3]{12x - 5} - \sqrt[3]{8x + 15} = 0$

Objective

Students will be able to perform function operations.

$$+ \; - \; \times \; \div$$

What types of functions have we studied so far?

Hint: there are three types so far….

Power Function

A power function is written in the form

$$y = ax^{b}$$

where *a* is a real number and *b* is a rational number.

Operations on Functions

Let *f* and *g* be any two functions. A new function *h* can be defined by performing any of the four basic operations on *f* and *g*.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$h(x) = f(x) + g(x)$	$h(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$h(x) = \dfrac{f(x)}{g(x)}$	$h(x) = \dfrac{5x}{x + 2}$

The domain of *h* consists of the *x*-values that are in the domains of both *f* and *g*. Additionally, the domain of the quotient does not include *x*-values for which $g(x) = 0$.

Add and Subtract Functions

Let $f(x) = 4x^{1/2}$ and $g(x) = -9x^{1/2}$.

Find the following:

1) $f(x) + g(x)$ $= 4x^{1/2} + (-9x^{1/2}) = [4 + (-9)]x^{1/2}$

$= -5x^{1/2}$

2) $f(x) - g(x)$ $= 4x^{1/2} - (-9x^{1/2})$ $= [4 - (-9)]x^{1/2}$

$= 13x^{1/2}$

3) The domains of $f + g$ and $f - g$

The functions f and g each have the same domain: all nonnegative real numbers. So, the domain of $f + g$ and $f - g$ also consist of all nonnegative real numbers.

Let $f(x) = -2x^{2/3}$ and $g(x) = 7x^{2/3}$.

Find the following:

1) $f(x) + g(x)$ $= -2x^{2/3} + 7x^{2/3}$

$= 5x^{2/3}$

2) $f(x) - g(x)$ $= -2x^{2/3} - 7x^{2/3}$

$= -9x^{2/3}$

3) The domains of $f + g$ and $f - g$

all real numbers; all real numbers

Multiply and Divide Functions

Let $f(x) = 6x$ and $g(x) = x^{3/4}$.

Find the following:

1) $f(x) \cdot g(x) = (6x)(x^{3/4}) = 6x^{(1+3/4)} = 6x^{7/4}$

2) $\dfrac{f(x)}{g(x)} = \dfrac{6x}{x^{3/4}} = 6x^{(1-3/4)} = 6x^{1/4}$

3) The domains of $f \bullet g$ and $f \div g$

The domain of f consists of all real numbers, and the domain of g consists of all nonnegative real numbers. So, the domain of $f \bullet g$ consists of all nonnegative real numbers. Because $g(0) = 0$, the domain of $f \div g$ is restricted to all positive real numbers.

Let $f(x) = 3x$ and $g(x) = x^{1/5}$.

Find the following:

1) $f(x) \cdot g(x) \quad = (3x)(x^{1/5}) \quad = 3x^{6/5}$

2) $\dfrac{f(x)}{g(x)} \quad = \dfrac{3x}{x^{1/5}} \quad = 3x^{4/5}$

3) The domains of $f \bullet g$ and $f \div g$

all real numbers; all real numbers except x = 0

Homework p.432: 3, 6, 7, 8, 13, 14, 16, 17

ADD AND SUBTRACT FUNCTIONS Let $f(x) = -3x^{1/3} + 4x^{1/2}$ and $g(x) = 5x^{1/3} + 4x^{1/2}$. Perform the indicated operation and state the domain.

3. $f(x) + g(x)$
6. $g(x) + g(x)$
7. $f(x) - g(x)$
8. $g(x) - f(x)$

MULTIPLY AND DIVIDE FUNCTIONS Let $f(x) = 4x^{2/3}$ and $g(x) = 5x^{1/2}$. Perform the indicated operation and state the domain.

13. $g(x) \cdot f(x)$
14. $f(x) \cdot f(x)$
16. $\dfrac{f(x)}{g(x)}$
17. $\dfrac{g(x)}{f(x)}$

Objective

Students will be able to perform function operations and compositions.

Review of Domain

If there is an even root, your x cannot be

____negative______.

If there is a denominator, your denominator cannot be equal to _____zero_____.

If there is an x in the denominator (where it is multiplied by something or just has an exponent), your x cannot be _____zero_____.

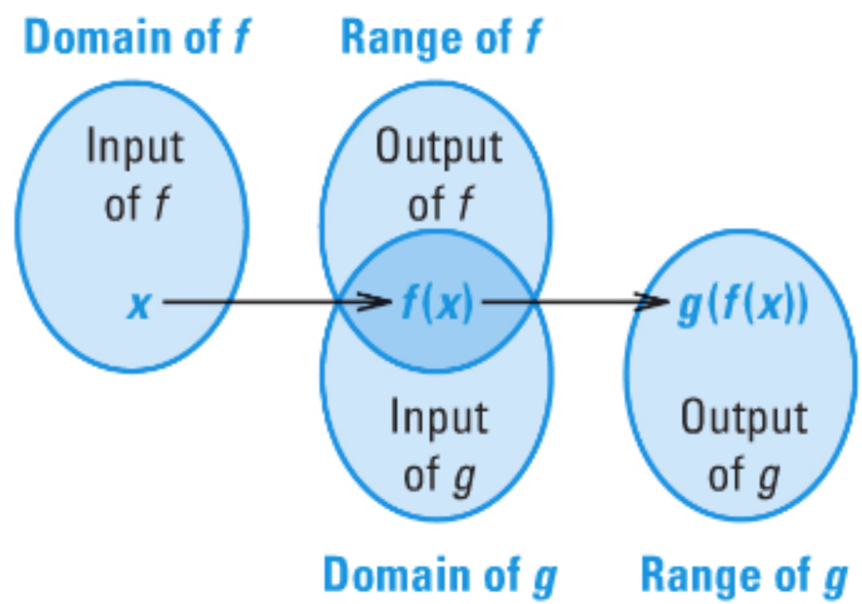
Composition of Functions

Another operation that can be performed with two functions is *composition*.

The composition of a function *g* with a function *f* is:

$$h(x) = g(f(x))$$

The domain of *h* is the set of all *x*-values such that *x* is in the domain of *f* and *f(x)* is in the domain of *g*.



Let $f(x) = 2x - 7$ and $g(x) = x^2 + 4$. What is the value of $g(f(3))$?

To evaluate $g(f(3))$, you first must find $f(3)$.

**With all compositions of functions, start on the inside and work your way out

$f(3) = 2(3) - 7 = -1 \longleftarrow$ Now, plug in -1 into $g(x)$

So $g(f(3)) = g(-1) = (-1)^2 + 4 = 5$

The value of $g(f(3))$ is 5.

Let $f(x) = 3x - 8$ and $g(x) = 2x^2$.

Find the following:

**With all compositions of functions, start on the inside and work your way out

1) $g(f(5))$

98

2) $f(g(5))$

142

3) $f(f(5))$

13

4) $g(g(5))$

5000

What is the difference between $3x^{-1}$ and $(3x)^{-1}$?

$$3x^{-1} = \frac{3}{x} \qquad (3x)^{-1} = \frac{1}{3x}$$

Let $f(x) = 4x^{-1}$ and $g(x) = 5x - 2$.

Find the following:

**With all compositions of functions, start on the inside and work your way out

***Take the inside function and plug it into *x* of the outside function (put parentheses around it)

1) $f(g(x)) = f(g(x)) = f(5x-2) = 4(5x-2)^{-1}$

$$= \frac{4}{5x-2}$$

2) $g(f(x)) = g(f(x)) = g(4x^{-1}) = 5(4x^{-1}) - 2$

$= 20x^{-1} - 2 \quad = \dfrac{20}{x} - 2$

Let $f(x) = 4x^{-1}$ and $g(x) = 5x - 2$.

Find the following:

**With all compositions of functions, start on the inside and work your way out

***Take the inside function and plug it into *x* of the outside function

3) $f(f(x)) = f(f(x)) = f(4x^{-1}) = 4(4x^{-1})^{-1}$

$= 4(4^{-1}x) = 4^0x = x$

4) The domain of each composition

**With domain, you need to look at the domain of the inside function and the final function of the composition

$f(g(x))$: all real numbers except x = 2/5

$g(f(x))$: all real numbers except x = 0

$f(f(x))$: all real numbers except x = 0

Homework

p.432: 21, 23, 25, 29, 31, 32, 45

EVALUATE COMPOSITIONS OF FUNCTIONS Let $f(x) = 3x + 2$, $g(x) = -x^2$, and $h(x) = \frac{x-2}{5}$. Find the indicated value.

21. $g(f(2))$

23. $g(h(8))$

25. $f(f(7))$

FIND COMPOSITIONS OF FUNCTIONS Let $f(x) = 3x^{-1}$, $g(x) = 2x - 7$, and $h(x) = \frac{x+4}{3}$. Perform the indicated operation and state the domain.

29. $g(f(x))$

31. $g(h(x))$

32. $h(g(x))$

45 on next slide!

45. **MULTI-STEP PROBLEM** An online movie store is having a sale. You decide to open a charge account and buy four DVDs.

a. Use composition of functions to find the sale price of $85 worth of DVDs when the $15 discount is applied before the 10% discount.

b. Use composition of functions to find the sale price of $85 worth of DVDs when the 10% discount is applied before the $15 discount.

c. Which order of discounts gives you a better deal? *Explain.*

Objective

Students will be able to find inverse functions and verify that functions are inverses.

A dollar was rolled up in a rubber band and then placed into a a plastic bag and then put into a small box which was put into a large box and then wrapped with wrapping paper. Write down the steps I need to take to get the dollar.

Inverse Relations

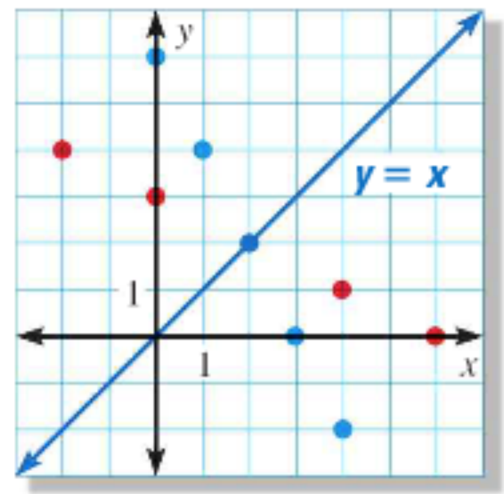
An <u>inverse relation</u> interchanges the input and output values of the original relation. This means that the domain and range are also interchanged.

Original relation

x	0	1	2	3	4
y	6	4	2	0	−2

Inverse relation

x	6	4	2	0	−2
y	0	1	2	3	4



The graph of an inverse relation is a reflection of the graph of the original relation. The line of reflection is $y = x$.

Finding Inverse Relations

To find the inverse of a relation given by an equation in x and y, switch the roles of x and y and solve for y.

Find an equation for the inverse of the relation $y = 3x – 5$.

$y = 3x – 5$	Write original relation.
$x = 3y – 5$	Switch x and y
$x + 5= 3y$	Add 5 to each side
$1/3x + 5/3= y$	Solve for y.

$y = \frac{1}{3}x + \frac{5}{3}$ is the inverse relation of $y = 3x – 5$

Inverse Functions

In the previous example, both the original relation and the inverse relation happen to be functions. In such cases, the two functions are called <u>inverse functions</u>.

Functions *f* and *g* are inverses of each other provided:

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

The function *g* is denoted by f^{-1}, read as "*f* inverse."

**The symbol -1 in f^{-1} is <u>not</u> to be interpreted as an exponent!!

Verify that $f(x) = 3x - 5$ and $f^{-1}(x) = \frac{1}{3}x + \frac{5}{3}$ are inverse functions.

Step 1: Show that

$$f(f^{-1}(x)) = x$$

$$f(f^{-1}(x)) = f\left(\frac{1}{3}x + \frac{5}{3}\right)$$

$$= 3\left(\frac{1}{3}x + \frac{5}{3}\right) - 5$$

$$= \left(\frac{3}{3}x + \frac{15}{3}\right) - 5$$

$$= x + 5 - 5$$

$$= x \quad ✓$$

Step 2: Show that

$$f^{-1}(f(x)) = x$$

$$f^{-1}(f(x)) = f^{-1}(3x - 5)$$

$$= \frac{1}{3}(3x - 5) + \frac{5}{3}$$

$$= \left(\frac{3}{3}x - \frac{5}{3}\right) + \frac{5}{3}$$

$$= x \quad ✓$$

Find the inverse of the given function. Then verify that your result and the original function are inverses.

1) $f(x) = x + 4$ $\quad f^{-1}(x) = x - 4$

$$f(f^{-1}(x)) = f(x-4) = (x-4)+4 = x$$

$$f^{-1}(f(x)) = f^{-1}(x+4) = (x+4)-4 = x$$

2) $f(x) = 2x - 1$ $\quad f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

$$f(f^{-1}(x)) = f\left(\frac{1}{2}x+\frac{1}{2}\right) = 2\left(\frac{1}{2}x+\frac{1}{2}\right)-1 = \left(\frac{2}{2}x+\frac{2}{2}\right)-1 = (x+1)-1 = x$$

$$f^{-1}(f(x)) = f^{-1}\left(2x-1\right) = \frac{1}{2}(2x-1)+\frac{1}{2} = \left(\frac{2}{2}x-\frac{1}{2}\right)+\frac{1}{2} = \left(x-\frac{1}{2}\right)+\frac{1}{2} = x$$

*When writing the inverse of the function, replace *f*(*x*) with *y* as your first step and replace *y* with $f^{-1}(x)$ as your last step!

Homework

p. 442: 3, 5, 7, 10, 16, 17, 19

INVERSE RELATIONS **Find an equation for the inverse relation.**

3. $y = 4x - 1$

5. $y = 7x - 6$

7. $y = 12x + 7$

10. $y = -\frac{2}{3}x + 2$

VERIFYING INVERSE FUNCTIONS **Verify that *f* and *g* are inverse functions.**

16. $f(x) = 2x + 3, g(x) = \frac{1}{2}x - \frac{3}{2}$

17. $f(x) = \frac{1}{4}x^3, g(x) = (4x)^{1/3}$

19. $f(x) = 4x + 9, g(x) = \frac{1}{4}x - \frac{9}{4}$

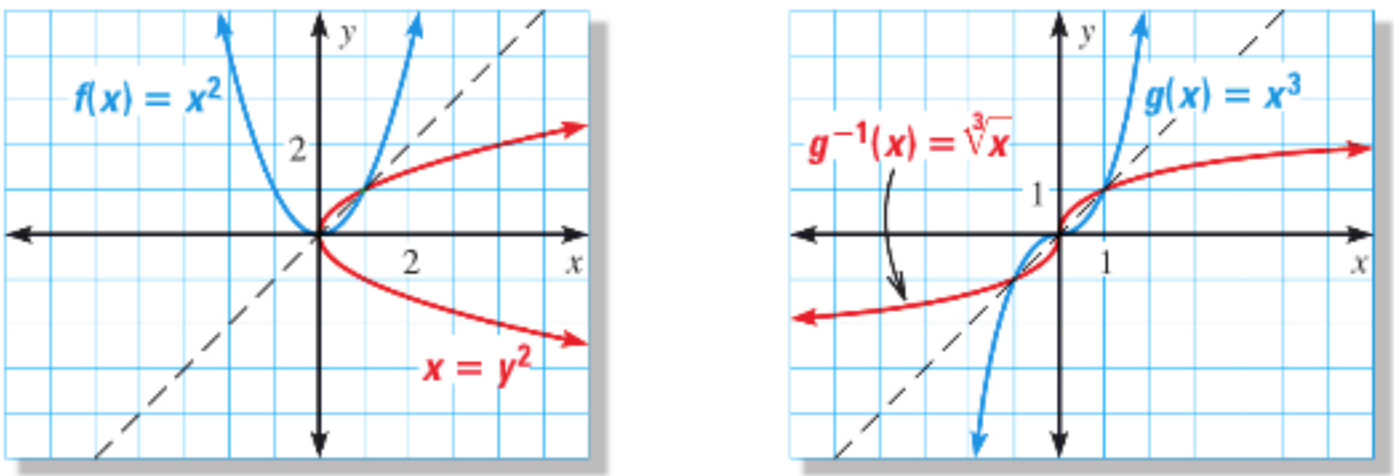
Objective

Students will be able to find inverse relations and decide if the inverse is actually a function.

Rational Exponents and Radical Functions Test on Wednesday!

Inverses of Nonlinear Functions

The graphs of the power functions $f(x) = x^2$ and $g(x) = x^3$ are shown below along with their reflections in the line $y = x$. Notice that the inverse of $g(x) = x^3$ is a function, but that the inverse of $f(x) = x^2$ is *not* a function.

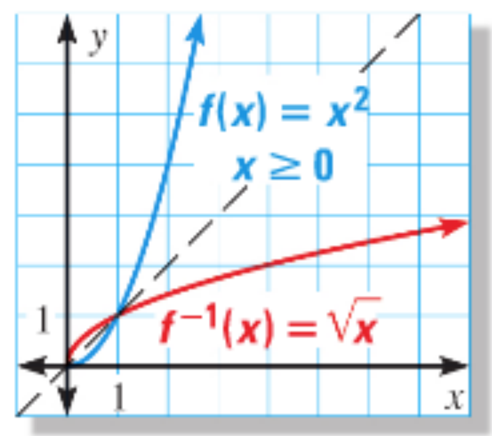


If the domain of $f(x) = x^2$ is *restricted* to only nonnegative real numbers, then the inverse of f is a function.

Find the inverse of $f(x) = x^2$, $x \geq 0$. Then graph f and f^{-1}.

$f(x) = x^2$	Write original relation.
$y = x^2$	Replace $f(x)$ with y.
$x = y^2$	Switch x and y.
$\pm\sqrt{x} = y$	Take square roots of each side to solve for y.

The domain of f is restricted to nonnegative values of x. So, the range of f^{-1} must also be restricted to nonnegative values, and therefore the inverse is $f^{-1}(x) = \sqrt{x}$.
(If the domain was restricted to $x \leq 0$, you would choose $f^{-1}(x) = -\sqrt{x}$.)

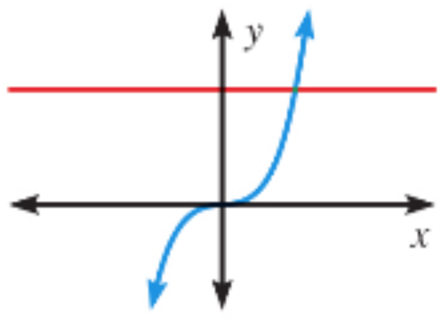


Horizontal Line Test

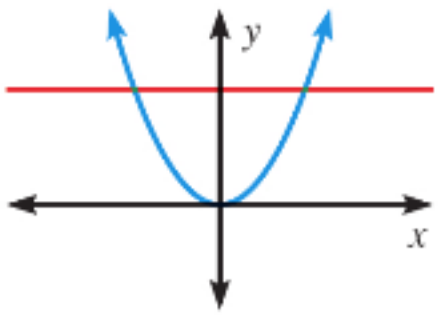
You can use the graph of a function *f* to determine whether the inverse of *f* is a function by applying the horizontal line test.

The inverse of a function *f* is also a function if and only if no horizontal line intersects the graph of *f* more than once.

Inverse is a function

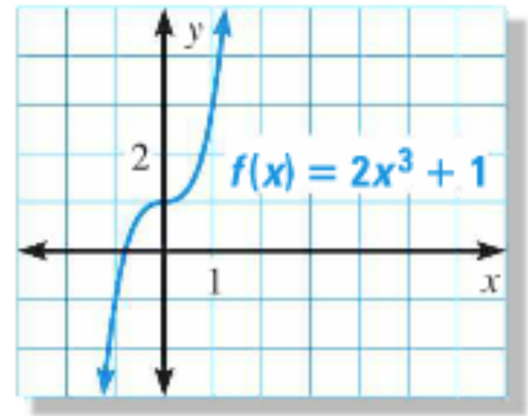


Inverse is not a function



Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

Graph the function f. Notice that no horizontal line intersects the graph more than once. So, the inverse of f is itself a function.



To find an equation for f^{-1}:

$f(x) = 2x^3 + 1$ Write original relation.

$y = 2x^3 + 1$ Replace $f(x)$ with y.

$x = 2y^3 + 1$ Switch x and y.

$x - 1 = 2y^3$ Subtract 1 from each side.

$\frac{x-1}{2} = y^3$ Divide each side by 2.

$\sqrt[3]{\frac{x-1}{2}} = y$ Take cube root of each side.

$$f^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$$

Graph the function *f*. Then use the horizontal line test to determine whether the inverse of *f* is a function.

1) $f(x) = x^2 + x - 2$

inverse is a not function

2) $f(x) = x^3 + 2$

inverse is a function

3) $f(x) = |x| + 3$

inverse is a not function

Elastic bands can be used in exercising to provide a range of resistance. A band's resistance *R* (in pounds) can be modeled by $R = \frac{3}{8}L - 5$

where *L* is the total length of the stretched band (in inches).

$$L = \frac{8}{3}R + \frac{40}{3}$$

1) Find the inverse of the model.

**With real world problems, keep variables where they are and solve for the other!

2) Use the inverse function to find the length at which the band provides 19 pounds of resistance.

64 inches

Homework

6.4 Worksheet Practice B- odd problems only!