Objective

Students will be able to graph exponential growth functions.

Exponential Functions

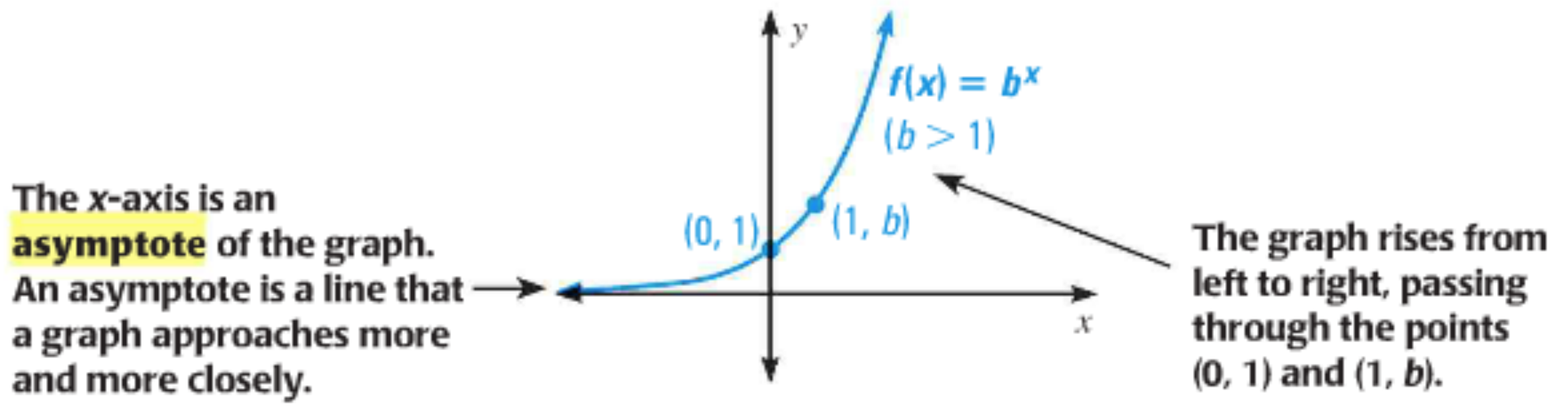
An <u>exponential function</u> has the form $\boldsymbol{y = ab^x}$ where $a \neq 0$ and the base b is a positive number other than 1.

What do you think makes a function an exponential *growth* function?
Think about the equations from the M&M's activity.

If $a > 0$ and $b > 1$, then the function $y = ab^x$ is an exponential growth function, and b is called the growth factor.

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b.



The domain of $f(x) = b^x$ is $(-\infty, \infty)$, the range is $(0, \infty)$, and the asymptote is the line $y = 0$.

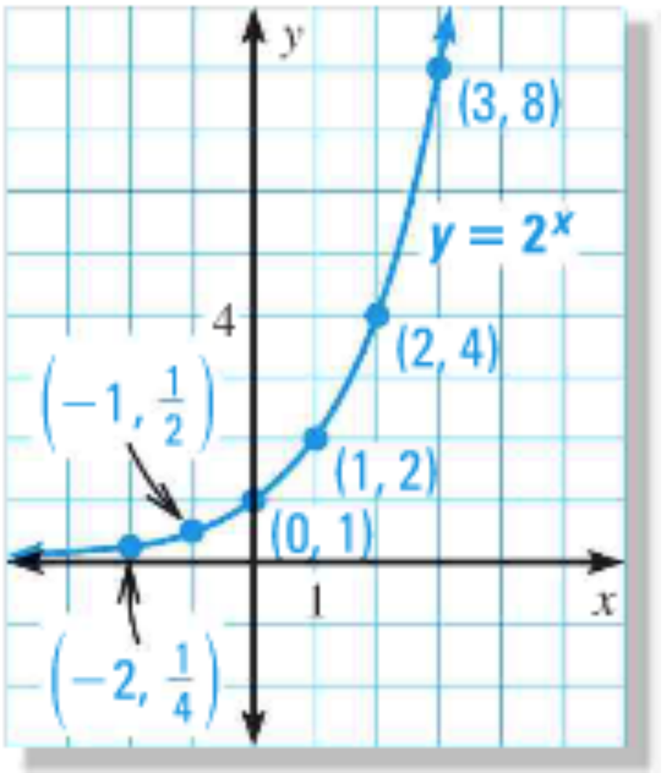
Graph $y = 2^x$.

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Step 1: Make a table of values.

Step 2: Plot the points from the table.

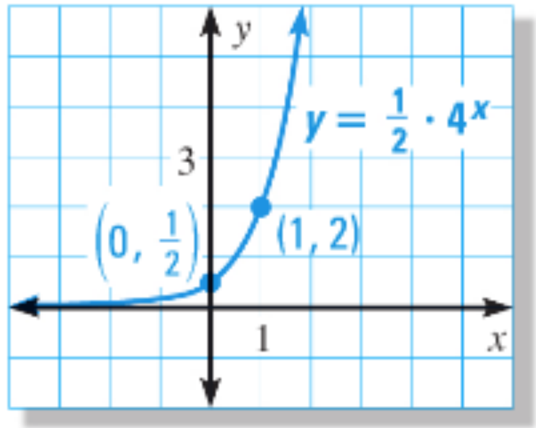
Step 3: Draw, from left to right, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the right.



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Asymptote: $y = 0$

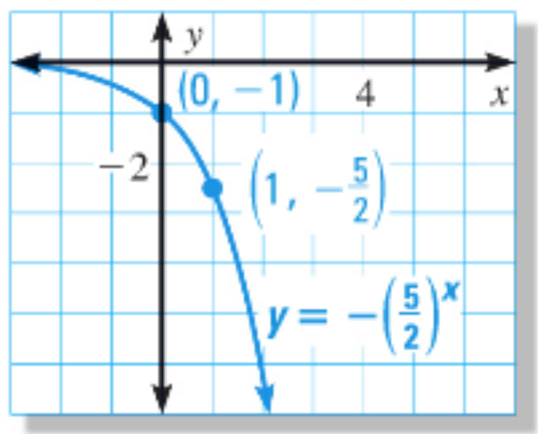
Graph the following functions and state domain, range, and asymptote.

2) $y = \frac{1}{2} \cdot 4^x$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Asymptote: $y = 0$

3) $y = -\left(\frac{5}{2}\right)^x$



Domain: $(-\infty, \infty)$
Range: $(-\infty, 0)$
Asymptote: $y = 0$

Exponential Functions

An exponential function has the form $\boldsymbol{y = ab^x}$ where $a \neq 0$ and the base b is a positive number other than 1.

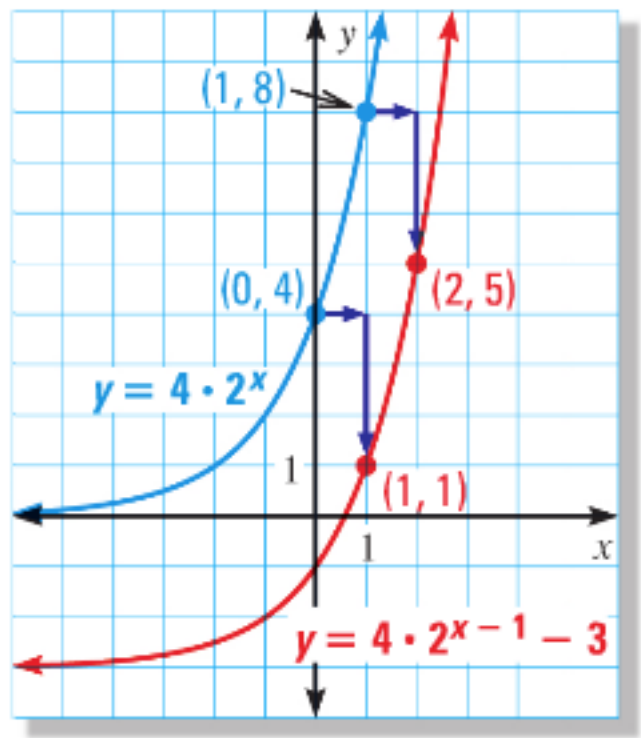
b is the growth or decay
a is the initial value (y-intercept)

Translations

To graph a function of the form $y = ab^{x-h} + k$, begin by sketching the graph of $y = ab^x$. Then translate the graph horizontally by h units and vertically by k units.

Graph $y = 4 \cdot 2^{x-1} - 3$. State the domain, range, and asymptote.

Begin by sketching the graph of $y = 4 \cdot 2^x$, which passes through (0, 4) and (1, 8). Then translate the graph right 1 unit and down 3 units to obtain the graph of $y = 4 \cdot 2^{x-1} - 3$.



The graph's asymptote is the line $y = -3$.
So domain: $(-\infty, \infty)$ and range $(-3, \infty)$

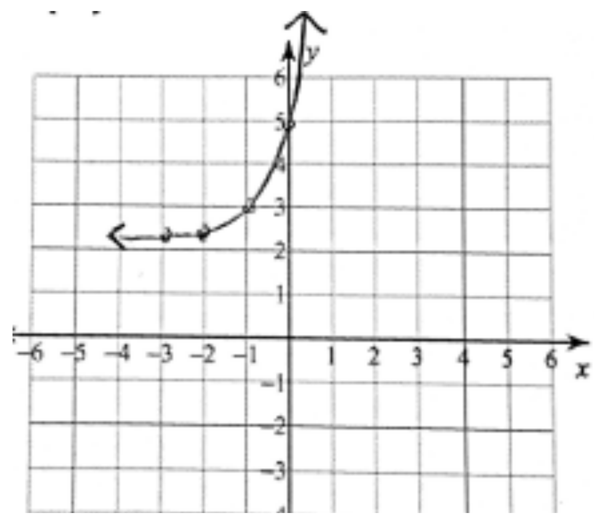
**asymptote is the line $y = k$

5) Graph $y = 3^{x+1} + 2$.

Then state the domain, range, and asymptote.

Make a table of values and sketch the graph.

x	-3	-2	-1	0	1
y	19/9	7/3	3	5	11



Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

Asymptote: $y = 2$

Homework

p. 482: 3, 4, 5, 7, 15, 17

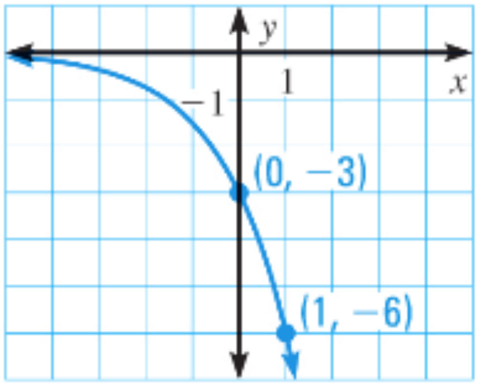
MATCHING GRAPHS Match the function with its graph.

3. $y = 3 \cdot 2^x$

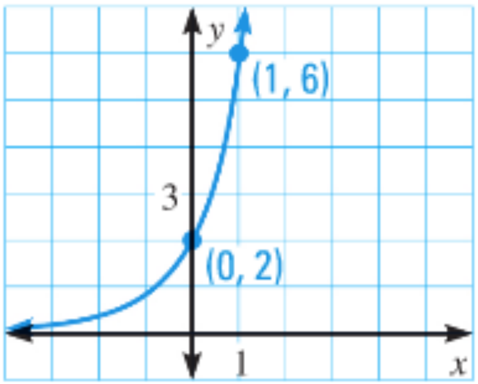
4. $y = -3 \cdot 2^x$

5. $y = 2 \cdot 3^x$

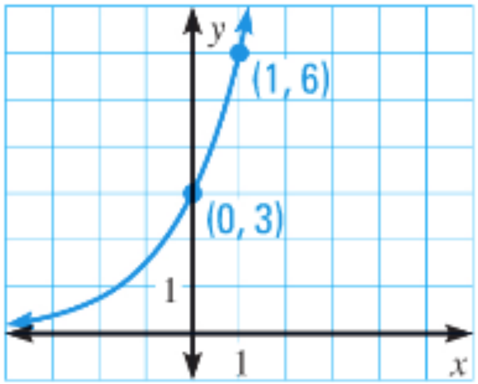
A.



B.



C.



GRAPHING FUNCTIONS Graph the function.

7. $y = -2^x$

TRANSLATING GRAPHS Graph the function. State the domain and range.

15. $y = -3 \cdot 2^{x+2}$

17. $y = 2^{x+1} + 3$ **Find the asymptote as well

Objective

Students will be able to understand exponential growth in real life problems and compound interest.

Exponential Functions (7.1-7.3) Quiz on Friday!

Exponential Growth Models

When a real-life quantity increases by a fixed percent each year (or other time period), the amount *y* of the quantity after *t* years can be modeled by the equation:

$$y = a\left(1 + r\right)^{t}$$

where *a* is the initial amount and *r* is the percent increase expressed as a decimal. Note that the quantity $1 + r$ is the growth factor.

In 1996, there were 2573 computer viruses and other computer security incidents. During the next 7 years, the number of incidents increased by about 92% each year.

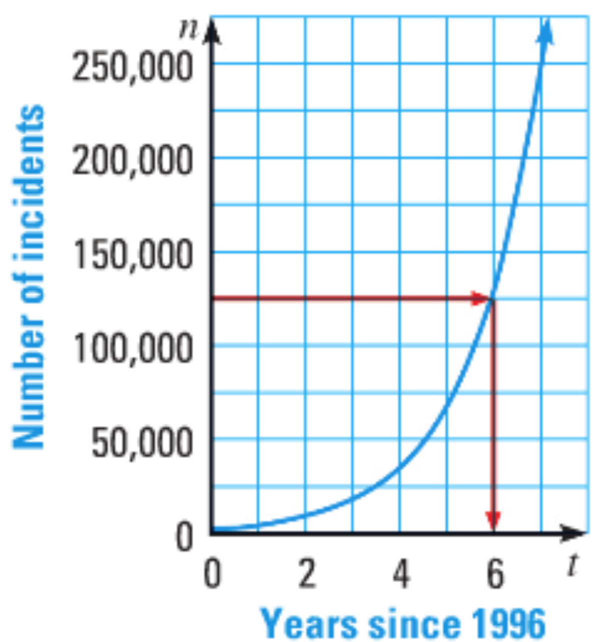
a) Write an exponential growth model giving the number *n* of incidents *t* years after 1996.

$n = a(1+r)^t$ a = 2573, r = 0.92 $n = 2573(1+0.92)^t$ $n = 2573(1.92)^t$

b) About how many incidents were there in 2003?

t = 7 $n = 2573(1.92)^7$ $n \approx 247{,}484$ computer security incidents

c) Graph the model.



d) Use the graph to estimate the year when there were about 125,000 computer security incidents.

(t ≈ 6), so 2002

In 1977, there were 41 breeding pairs of bald eagles in Maryland. Over the next 24 years, the number of breeding pairs increased by about 8.9% each year.

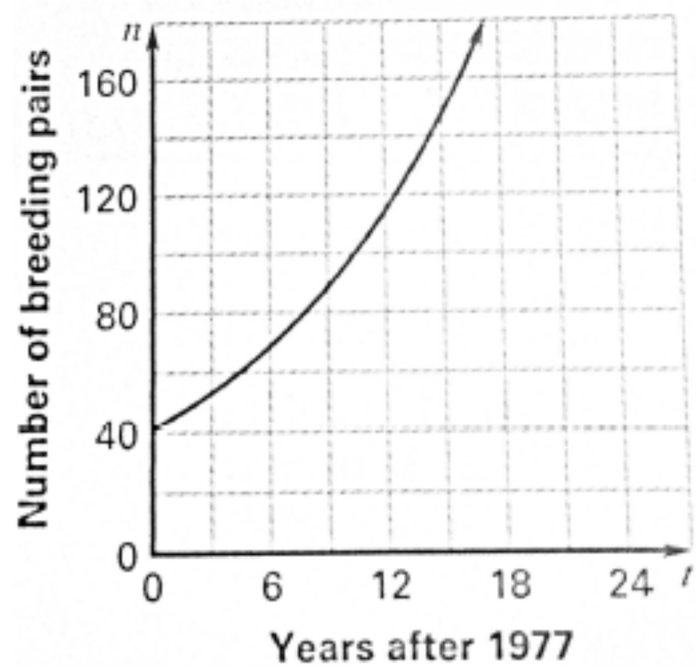
a) Write a model giving the number *n* of breeding pairs of bald eagles in Maryland *t* years after 1977.

$n = a(1+r)^t$ $a = 41$, $r = 0.089$ $n = 41(1+0.089)^t$ $n = 41(1.089)^t$

b) Make a table of values for the model.

t	0	6	12	18
n	41	68	114	190

c) Graph the model.



d) Use the graph to estimate how many breeding pairs of bald eagles were in Maryland in 1992.

(t = 15), so about 150 pairs of bald eagles

In the exponential growth model $y = 527(1.39)^x$, identify the initial amount, the growth factor, and the percent increase.

initial amount (a) = 527

growth factor ($1 + r$) = 1.39

percent increase (r%) = 39%

Compound Interest

Exponential growth functions are used in real-life situations involving *compound interest*. Compound interest is paid on the initial investment, called the *principal*, and on previously earned interest. Interest paid only on the principal is called *simple interest.*

Consider an initial principal *P* deposited in an account that pays interest at an annual rate *r* (expressed as a decimal), compounded *n* times per year. The amount *A* in the account after *t* years is given by this equation:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

You deposited \$4000 in an account that pays 2.92% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a) Quarterly

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$P = 4000$, $r = 0.0292$, $n = 4$, $t = 1$

$$A = 4000\left(1+\frac{0.0292}{4}\right)^{4\cdot 1}$$

$$A = 4000(1.0073)^{4}$$

$$A \approx 4118.09$$

The balance at the end of 1 year is \$4118.09

b) Daily

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$P = 4000$, $r = 0.0292$, $n = 365$, $t = 1$

$$A = 4000\left(1+\frac{0.0292}{365}\right)^{365\cdot 1}$$

$$A = 4000(1.00008)^{365}$$

$$A \approx 4118.52$$

The balance at the end of 1 year is \$4118.52

You deposited $2000 in an account that pays 4% annual interest. Find the balance after 3 year if the interest is compounded daily.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

P = 2000, *r* = 0.04, *n* = 365, *t* = 3

$$A = 2000\left(1+\frac{0.04}{365}\right)^{365\cdot 3}$$

$$A = 2000\left(1.0001\right)^{1095}$$

$$A \approx 2254.98$$

The balance at the end of 3 years is $2254.98

Homework p. 482: 25, 28, 29, 30, 35

25. ★ **MULTIPLE CHOICE** The student enrollment E of a high school was 1310 in 1998 and has increased by 10% per year since then. Which exponential growth model gives the school's student enrollment in terms of t, where t is the number of years since 1998?

Ⓐ $E = 0.1(1310)^t$ Ⓑ $E = 1310(0.1)^t$

Ⓒ $E = 1.1(1310)^t$ Ⓓ $E = 1310(1.1)^t$

WRITING MODELS In Exercises 28–30, write an exponential growth model that describes the situation.

28. In 1992, 1219 monk parakeets were observed in the United States. For the next 11 years, about 12% more parakeets were observed each year.

29. You deposit \$800 in an account that pays 2% annual interest compounded daily.

30. You purchase an antique table for \$450. The value of the table increases by 6% per year.

35. **DVD PLAYERS** From 1997 to 2002, the number n (in millions) of DVD players sold in the United States can be modeled by $n = 0.42(2.47)^t$ where t is the number of years since 1997.

   a. Identify the initial amount, the growth factor, and the annual percent increase.

   b. Graph the function. Estimate the number of DVD players sold in 2001.

Objective

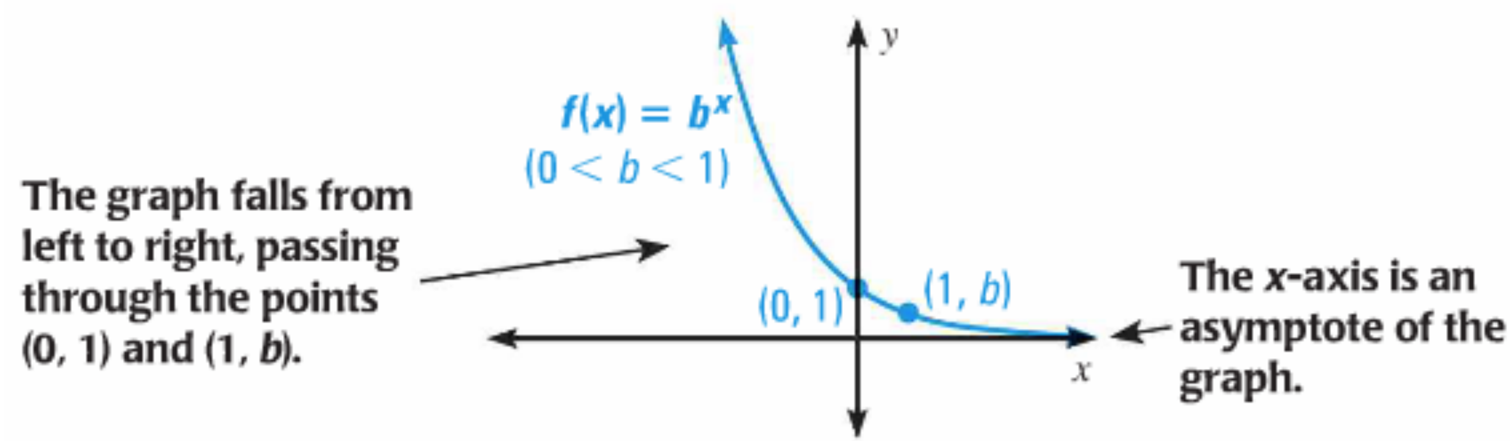
Students will be able to graph and model exponential decay functions.

Exponential Functions (7.1-7.3) Quiz on Friday!

If $a > 0$ and $0 < b < 1$, then the function $y = ab^x$ is an <u>exponential decay function</u>, and b is called the <u>decay factor</u>.

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b.



The domain of $f(x) = b^x$ is $(-\infty, \infty)$, the range is $(0, \infty)$, and the asymptote is the line $y = 0$.

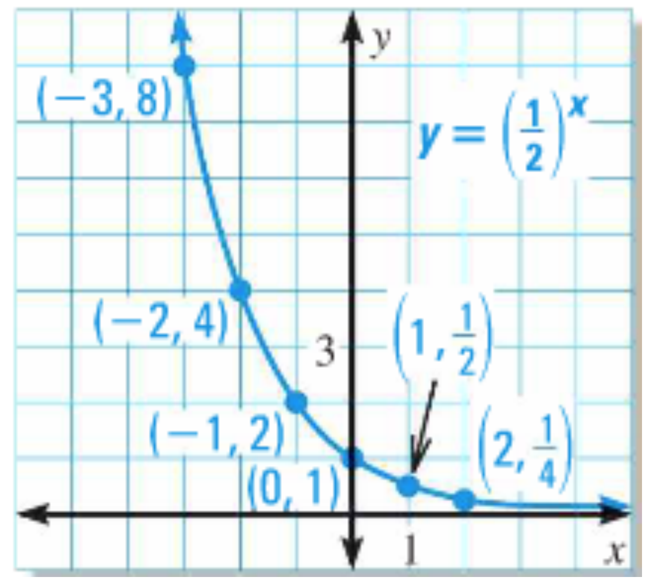
Graph $y = \left(\frac{1}{2}\right)^x$.

x	−3	−2	−1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

Step 1: Make a table of values.

Step 2: Plot the points from the table.

Step 3: Draw, from *right* to *left*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



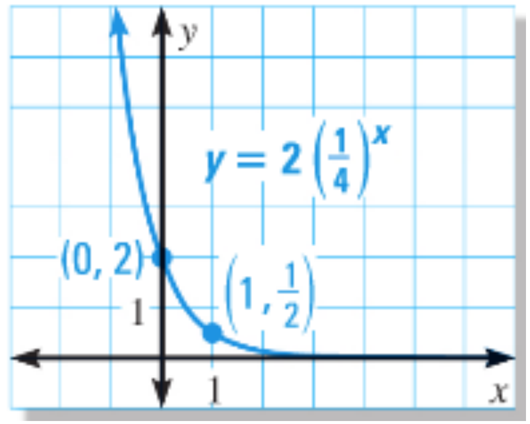
Domain: (-∞, ∞)

Range: (0, ∞)

Asymptote: $y = 0$

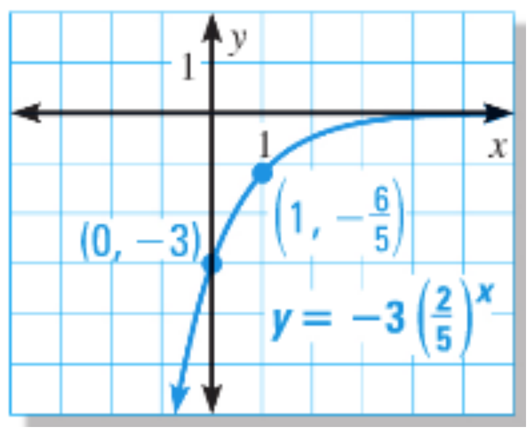
Graph the following functions and state domain, range, and asymptote.

2) $y = 2\left(\frac{1}{4}\right)^x$



Domain: (-∞, ∞)
Range: (0, ∞)
Asymptote: $y = 0$

3) $y = -3\left(\frac{2}{5}\right)^x$



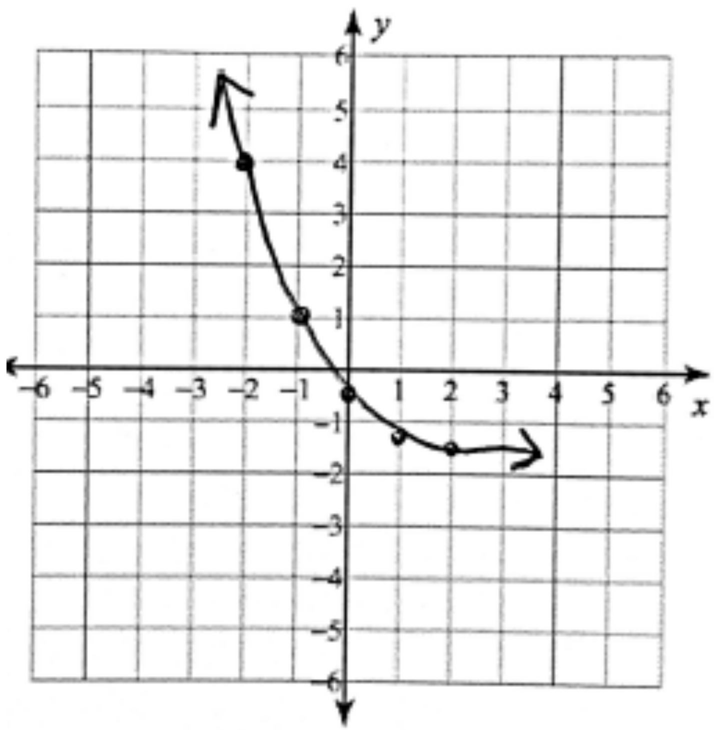
Domain: (-∞, ∞)
Range: (-∞, 0)
Asymptote: $y = 0$

Graph $y = 3\left(\frac{1}{2}\right)^{x+1} - 2$. State the domain, range, and asymptote.

**Translation

Make a table of values and sketch the graph.

x	-2	-1	0	1	2
y	4	1	-1/2	-5/4	-13/8



Domain: $(-\infty, \infty)$

Range: $(-2, \infty)$

Asymptote: $y = -2$

Exponential Decay Models

When a real-life quantity decreases by a fixed percent each year (or other time period), the amount *y* of the quantity after *t* years can be modeled by the equation:

$$y = a(1-r)^t$$

where *a* is the initial amount and *r* is the percent decrease expressed as a decimal. Note that the quantity $1 - r$ is the decay factor.

A new snowmobile costs \$4200. The value of the snowmobile decreases by 10% each year.

a) Write an exponential decay model giving the snowmobile's value *y* (in dollars) after *t* years.

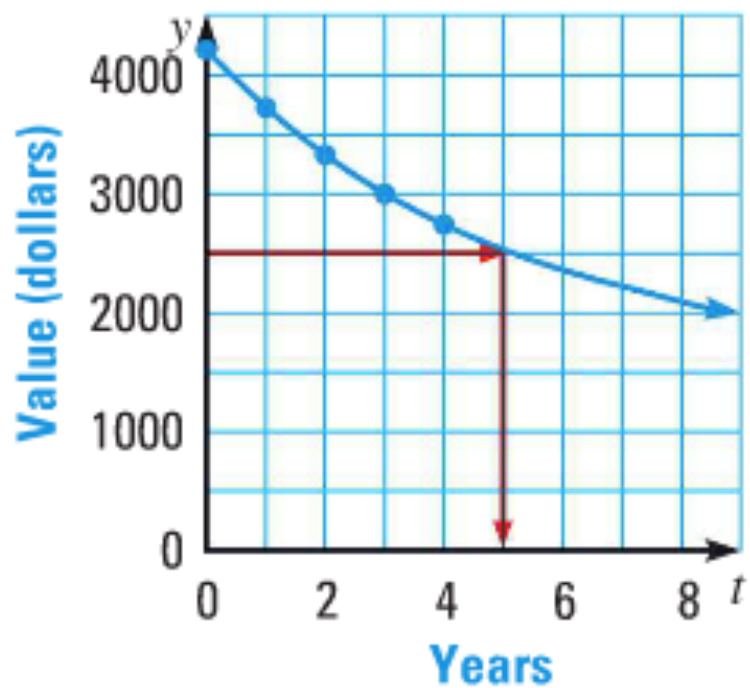
$y = a(1-r)^t$ $a = 4200$, $r = 0.10$ $y = 4200(1-0.10)^t$ $y = 4200(0.90)^t$

b) What is the value after 3 years?

$t = 3$ $y = 4200(0.90)^3$ $y \approx 3061.80$

\$3061.80

c) Graph the model.

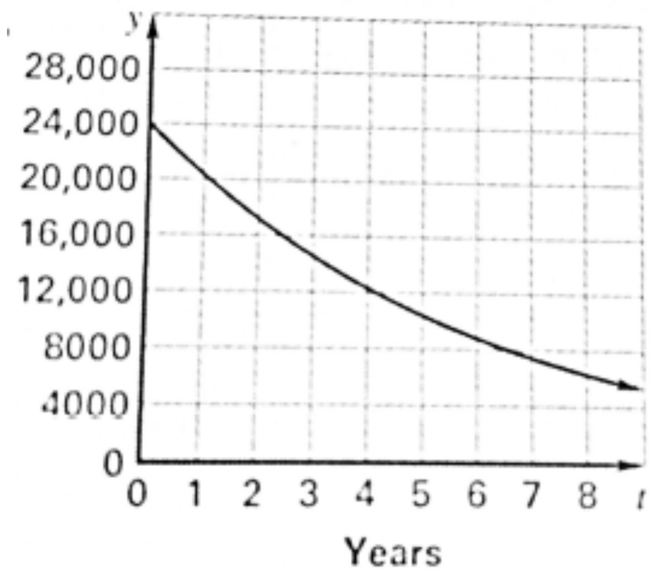


d) Use the graph to estimate when the value of the snowmobile will be \$2500.

t is between 4 and 5, so during the 4th year

The value of a car can be modeled by the equation $y = 24{,}000(0.845)^t$ where t is the number of years since the car was purchased.

a) Graph the model.



b) Estimate when the value of the car will be $10,000.

t is between 4 and 5, so during the 4th year

c) Use the model to predict the value of the car after 50 years. Is this a reasonable value? *Explain*.

$5.29 No; because a car does not normally last 50 years

Homework

p. 489: 3-6, 14, 19, 30, 31

CLASSIFYING FUNCTIONS Tell whether the function represents *exponential growth* or *exponential decay.*

3. $f(x) = 3\left(\frac{3}{4}\right)^x$

4. $f(x) = 4\left(\frac{5}{2}\right)^x$

5. $f(x) = \frac{2}{7} \cdot 4^x$

6. $f(x) = 25(0.25)^x$

GRAPHING FUNCTIONS Graph the function.

14. $h(x) = -3\left(\frac{3}{8}\right)^x$

TRANSLATING GRAPHS Graph the function. State the domain and range.

19. $y = \left(\frac{2}{3}\right)^{x-4} - 1$

**Find the asymptote as well

31. **BIKE COSTS** You buy a new mountain bike for $200. The value of the bike decreases by 25% each year.

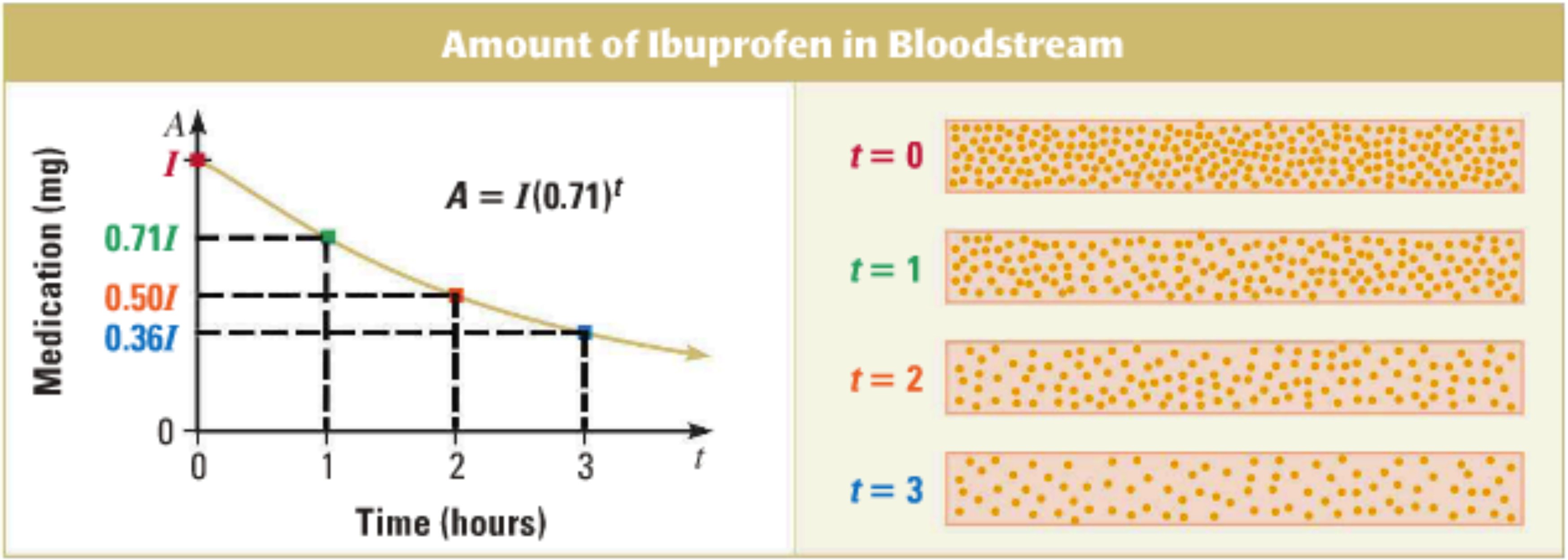
    a. Write a model giving the mountain bike's value y (in dollars) after t years. Use the model to estimate the value of the bike after 3 years.

    b. Graph the model.

    c. Estimate when the value of the bike will be $100.

Problem 30 is on next slide!

30. **MEDICINE** When a person takes a dosage of I milligrams of ibuprofen, the amount A (in milligrams) of medication remaining in the person's bloodstream after t hours can be modeled by the equation $A = I(0.71)^t$.



Find the amount of ibuprofen remaining in a person's bloodstream for the given dosage and elapsed time since the medication was taken.

a. Dosage: 200 mg
Time: 1.5 hours

b. Dosage: 325 mg
Time: 3.5 hours

c. Dosage: 400 mg
Time: 5 hours

Warm Up

You invest \$500 in the stock of a company. The value of the stock decreases by 2% each year. *Describe* and correct the error in writing a model for the value of the stock after *t* years.

$$y = \begin{pmatrix}\text{Initial}\\ \text{amount}\end{pmatrix}\begin{pmatrix}\text{Decay}\\ \text{factor}\end{pmatrix}^{t}$$

$$y = 500(0.02)^t$$

The decay factor 1 – *r*, not r

$y = 500(0.98)^t$

Homework Questions?

p. 489: 3-6, 14, 19, 30, 31

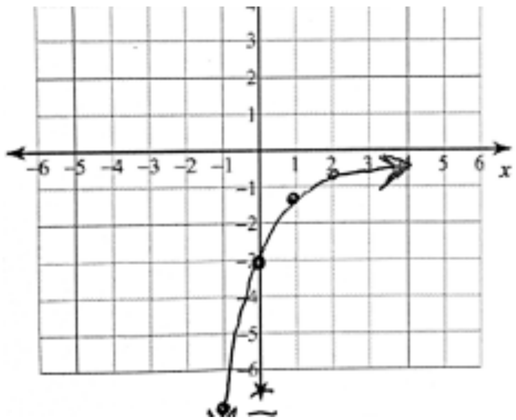
3) exponential decay

4) exponential growth

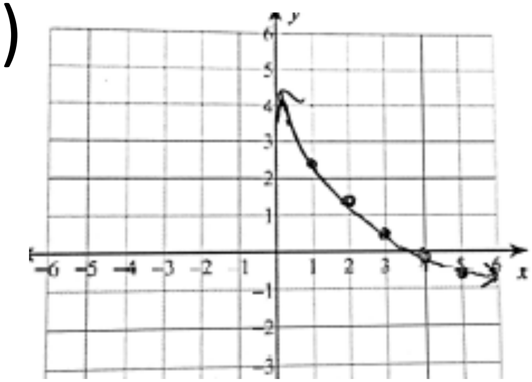
5) exponential growth

6) exponential decay

14)



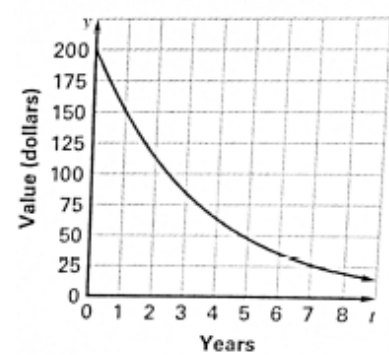
19)



Domain: $(-\infty, \infty)$
Range: $(-1, \infty)$
Asymptote: $y = -1$

30) a) about 119.65 mg
b) about 98.01 mg
c) about 72.17 mg

31) a) $y = 200(0.75)^t$; about \$84.38
b)



c) after about 2.5 years

Objective

Students will be able to understand continuously compounded interest.

Students will be able to correct their mistakes from their rational functions test.

Exponential Functions (7.1-7.3) Quiz on Friday!

Continuously Compounded Interest

When interest is compounded *continuously*, the amount *A* in an account after *t* years is given by the formula:

$$A = Pe^{rt}$$

where *P* is the principal and *r* is the annual interest rate expressed as a decimal.

You deposit \$4000 in an account that pays 6% annual interest compounded continuously. What is the balance after 1 year?

$A = Pe^{rt}$	Write formula
$= 4000e^{0.06(1)}$	Substitute 4000 for *P*, 0.06 for *r*, and 1 for *t*
≈ 4247.35	Use a calculator

The balance at the end of 1 year is \$4247.35.

You deposit $2500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time.

a) 2 years

$2762.93

b) 5 years

$3210.06

c) 7.5 years

$3637.48

Homework

p. 497: 57, 58

57. **FINANCE** You deposit \$2000 in an account that pays 4% annual interest compounded continuously. What is the balance after 5 years?

58. **FINANCE** You deposit \$800 in an account that pays 2.65% annual interest compounded continuously. What is the balance after 12.5 years?

Objective

Students will be able to understand the natural base *e* functions.

Natural Base *e*

The natural base *e* is irrational. It is defined as follows:

As *n* approaches $+\infty$, $\left(1+\frac{1}{n}\right)^{n}$

approaches $e \approx 2.718281828$

Simplify the following expressions.

1) $e^2 \cdot e^5 = e^{2+5} = e^7$

2) $\dfrac{12e^4}{3e^3} = 4e^{4-3} = 4e$

3) $\left(5e^{-3x}\right)^2 = 5^2 e^{-6x} = \dfrac{25}{e^{6x}}$

Simplify the following expressions.

4) $e^7 \cdot e^4 \quad = e^{7+4} \quad = e^{11}$

5) $2e^{-3} \cdot 6e^5 \quad = 12e^{-3+5} \quad = 12e^2$

6) $\dfrac{24e^8}{4e^5} \quad = 6e^{8-5} \quad = 6e^3$

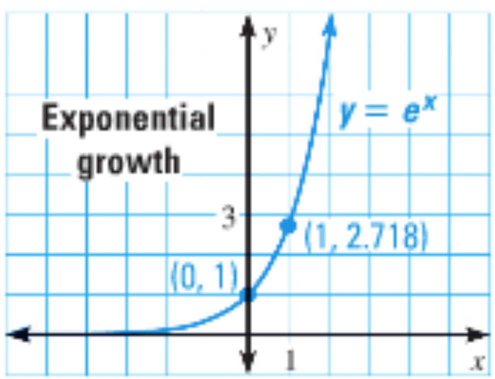
7) $\left(10e^{-4x}\right)^3 = 10^3 e^{-12x} = \dfrac{1000}{e^{12x}}$

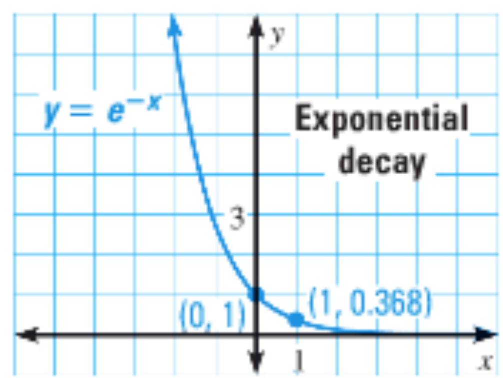
Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function.*

- If $a > 0$ and $r > 0$, the function is an exponential growth function
- If $a > 0$ and $r < 0$, the function is an exponential decay function

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown below.





The length *l* (in centimeters) of a tiger shark can be modeled by the function $l = 337 - 276e^{-0.178t}$ where *t* is the shark's age (in years).

a) What is the length of a tiger shark that is 3 years old?

about 175 centimeters

b) What is the length of a tiger shark that is 5 years old?

about 224 centimeters

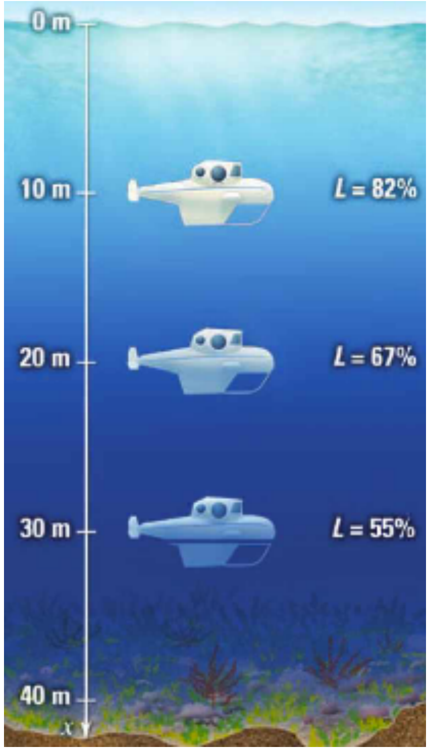
c) How old is a tiger shark that is 281 centimeters long?

about 9 years old

The percent *L* of surface light that filters down through bodies of water can be modeled by the exponential function $L(x) = 100e^{kx}$ where *k* is a measure of murkiness of water and *x* is the depth below the surface (in meters).

a) A recreational submarine is traveling in clear water with a *k*-value of about -0.02. Write an equation giving the percent of surface light that filters down through clear water as a function of depth.

$$L(x) = 100e^{-0.02x}$$



b) What is the percent of surface light available at a depth of 40 meters? about 45%

c) How deep can the submarine descend in clear water before only 50% of surface light is available? about 35 meters

Homework p. 495: 5, 12, 13, 14, 55, 56

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

5. $(2e^{3x})^3$

12. $\frac{4e^x}{e^{4x}}$

13. $\sqrt[3]{8e^{9x}}$

14. $\frac{6e^{4x}}{8e}$

55. CAMERA PHONES The number of camera phones shipped globally can be modeled by the function $y = 1.28e^{1.31x}$ where x is the number of years since 1997 and y is the number of camera phones shipped (in millions). How many camera phones were shipped in 2002?

56. BIOLOGY Scientists used traps to study the Formosan subterranean termite population in New Orleans. The mean number y of termites collected annually can be modeled by $y = 738e^{0.345t}$ where t is the number of years since 1989. What was the mean number of termites collected in 1999?

Objective

Students will be able to evaluate logarithms.

Solve for *x* in the following problems.

1) $2^x = 8$

x = 3

2) $3^x = 81$

x = 4

3) $4^x = \frac{1}{16}$

x = -2

Logarithm with Base *b*

Let *b* and *y* be positive numbers with $b \neq 1$. The logarithm of *y* with base *b* is denoted by $\log_b y = x$ and is defined as follows:

$$\log_b y = x \text{ if and only if } b^x = y$$

The expression $\log_b y = x$ is read as "log base *b* of *y*."

Logarithm with Base *b*

$$b^x = y$$

is in *exponential form* where *b* is the base, *x* is the exponent, and *y* is the solution

$$\log_b y = x$$

is in *logarithmic form* where *b* is the base, *x* is the solution, and *y* is the number you are taking the log of

Convert from exponential to logarithmic form:

$$b^x = y \rightarrow \log_b y = x$$

1) $3^2 = 9$ b = 3 x = 2 y = 9 $\log_3 9 = 2$

2) $2^4 = 16$ b = 2 x = 4 y = 16 $\log_2 16 = 4$

3) $\left(\frac{1}{4}\right)^{-3} = 64$ b = ¼ x = -3 y = 64 $\log_{1/4} 64 = -3$

Convert from logarithmic form to exponential form:

$$\log_b y = x \rightarrow b^x = y$$

Swirl method: $\log_b x = y \longrightarrow b^y = x$

1) $\log_9 81 = 2$ b = 9 x = 2 y = 81 $9^2 = 81$

2) $\log_{1/2} 32 = -5$ $\left(\frac{1}{2}\right)^{-5} = 32$

3) $\log_{12} 12 = 1$ $12^1 = 12$

4) $\log_4 1 = 0$ $4^0 = 1$

Let b be a positive real number such that $b \neq 1$.

Logarithm of 1:

$$\log_b 1 = 0 \quad \text{because} \quad b^0 = 1$$

**Number 4 from previous slide

Logarithm of *b* with Base *b*:

$$\log_b b = 1 \quad \text{because} \quad b^1 = b$$

**Number 3 from previous slide

Evaluate the following logarithms.

**Ask yourself what power of *b* gives you *y*?

1) $\log_4 64$

4 to what power gives 64? $4^x = 64$, what is x?

$4^3 = 64$, so $\log_4 64 = 3$

2) $\log_5\left(\frac{1}{5}\right)$

5 to what power gives 1/5? $5^x = 1/5$, what is x?

$5^{-1} = 1/5$, so $\log_5(1/5) = -1$

3) $\log_{1/5} 125$

$1/5^x = 125$, what is x? -3

4) $\log_{36} 6$

1/2

Special Logarithms

A common logarithm is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A natural logarithm is a logarithm with base *e*. It can be denoted by $\log_e$, but is more often denoted by ln.

Common Logarithm:

$$\log_{10} x = \log x$$

Natural Logarithm:

$$\log_e x = \ln x$$

Use a calculator to evaluate the logarithm:

1) log 8 LOG 8) ENTER 0.903089987 $10^{0.903} \approx 8$

2) ln 0.3 LN .3) ENTER -1.203972804 $e^{-1.204} \approx 0.3$

Homework

p. 503: 3-15 (odds), 21, 23

EXPONENTIAL FORM **Rewrite the equation in exponential form.**

3. $\log_4 16 = 2$

5. $\log_6 \frac{1}{36} = -2$

7. ERROR ANALYSIS *Describe* and correct the error in rewriting the equation $2^{-3} = \frac{1}{8}$ in logarithmic form.

$\log_2 -3 = \frac{1}{8}$ ✗

EVALUATING LOGARITHMS **Evaluate the logarithm without using a calculator.**

9. $\log_7 49$

11. $\log_2 64$

13. $\log_{1/2} 8$

15. $\log_{16} \frac{1}{4}$

CALCULATING LOGARITHMS **Use a calculator to evaluate the logarithm.**

21. $\ln 6$

23. $\log 6.213$

Objective

Students will be able to find inverse functions.

Homework Quizzes; grades will be entered into your homework category; they are short (5 minute) quizzes; you will be warned ahead of time when you have one

Inverse Functions

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b x$ is the inverse of the exponential function $f(x) = b^x$.

This means that

$$g(f(x)) = \log_b b^x = x \qquad ✔$$

and

$$f(g(x)) = b^{\log_b x} = x \qquad ✔$$

Use inverse properties to simplify the following expressions:

1) $10^{\log 4} = 4$ $\quad 10^{\log_{10} 4}$ $\quad b^{\log_b x} = x$

2) $\log_3 3^x = x$ $\quad \log_b b^x = x$

3) $\log_5 25^x$

$= \log_5 (5^2)^x$ Express 25 as a power with base 5

$= \log_5 5^{2x}$ Power of a power property

$= 2x$ $\quad \log_b b^x = x$

Find the inverse of the following functions:

1) $y = 6^x$

$x = 6^y$ Switch x and y

$y = \log_6 x$ Write in logarithmic form (solve for y)

The inverse of $y = 6^x$ is $y = \log_6 x$.

2) $y = \log_5 x$

$x = \log_5 y$ Switch x and y

$y = 5^x$ Write in exponential form (solve for y)

The inverse of $y = \log_5 x$ is $y = 5^x$.

Inverse Recall:

When taking an inverse, you first flip x and y.

**Switch your domain and range

If you start with a function in exponential form, what form should the inverse be in?

logarithmic form

If you start with a function in logarithmic form, what form should the inverse be in?

exponential form

logarithms and exponents are inverses of one another; they undo each other

Simplify the expression.

1) $8^{\log_8 x}$

x

2) $\log_7 7^{-3x}$

$-3x$

3) $\log_2 64^x$

$6x$

4) $e^{\ln 20}$

20

5) Find the inverse of $y = 4^x$.

$y = \log_4 x$

6) Find the inverse of $y = \ln x$.

$y = e^x$

Homework

p. 504: 29-35 (odds), 37 – 39

USING INVERSE PROPERTIES **Simplify the expression.**

29. $\log_5 5^x$ **31.** $10^{\log 8}$ **33.** $\log_3 81^x$ **35.** $\log_2 32^x$

FINDING INVERSES **Find the inverse of the function.**

37. $y = \log_8 x$ **38.** $y = 7^x$ **39.** $y = (0.4)^x$

Objective

Students will be able to find inverse functions.

Homework Quizzes; grades will be entered into your homework category; they are short (5 minute) quizzes; you will be warned ahead of time when you have one

We will have our first homework quiz on Monday!

What is the date on Tuesday?

Find the inverse of the following functions:

1) $y = 6^{x-2}$

$x = 6^{y-2}$ Switch x and y

$y - 2 = \log_6(x)$ Write in logarithmic form

$y = \log_6(x) + 2$ Solve for y

The inverse of $y = 6^{x-2}$ is $y = \log_6(x) + 2$

2) $y = \ln(x+3)$

$x = \ln(y+3)$ Switch x and y

$x = \log_e(y+3)$ Change *ln* to $\log_e$

$e^x = y + 3$ Write in exponential form

$e^x - 3 = y$ Solve for y

The inverse of $y = \ln(x + 3)$ is $y = e^x - 3$

Find the inverse of the following functions:

1) $y = \log(x - 5)$ $y = 10^x + 5$

2) $y = e^x + 1$ $y = \ln(x - 1)$

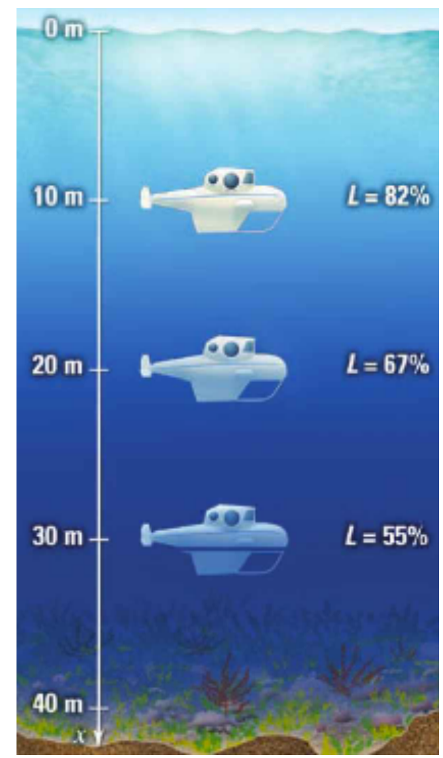
3) $y = \log_6(x + 4)$ $y = 6^x - 4$

4) $y = \log_{1/2}(2x) + 3$ $y = \frac{1}{2}\left(\frac{1}{2}\right)^{x-3}$

From Monday….

The percent *L* of surface light that filters down through bodies of water can be modeled by the exponential function $L(x) = 100e^{-0.02x}$ where *k* is a measure of murkiness of water and *x* is the depth below the surface (in meters).

c) Exactly (rounded to the hundredths place) how deep can the submarine descend in clear water before only 50% of surface light is available?



$$50 = 100e^{-0.02x} \qquad \log_e(0.5) = -0.02x$$

$$0.5 = e^{-0.02x} \qquad \ln(0.5) = -0.02x$$

$$\frac{\ln(0.5)}{-0.02} = x$$

34.65 meters

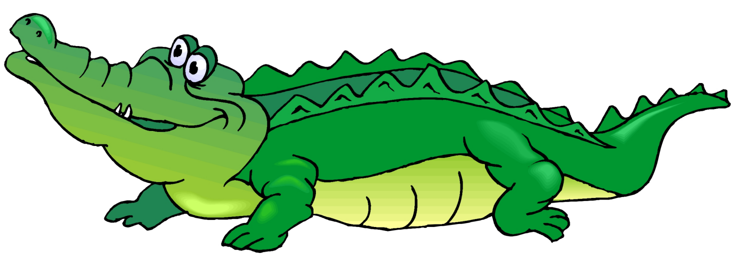
Biologists have found that an alligator's length *l* (in inches) and weight *w* (in pounds) are related by the function

$$L = 27.1\ln(w) - 32.8$$

Find the <u>exact</u> weight of an alligator that is 10 feet long. **Be careful with the units

$$120 = 27.1\ln(w) - 32.8$$

$$152.8 = 27.1\ln(w)$$



$$5.638 = \ln(w)$$

$$5.638 = \log_e(w)$$

$$e^{5.638} = w$$

281 pounds

Homework

p. 504: 41 – 44, 58

FINDING INVERSES **Find the inverse of the function.**

41. $y = e^{x+2}$

42. $y = 2^x - 3$

43. $y = \ln(x + 1)$

44. $y = 6 + \log x$

58. ALTIMETER Skydivers use an instrument called an altimeter to track their altitude as they fall. The altimeter determines altitude by measuring air pressure. The altitude h (in meters) above sea level is related to the air pressure P (in pascals) by the function in the diagram below.

$$h = -8005 \ln \frac{P}{101{,}300}$$

What is the altitude above sea level when the air pressure is 57,000 pascals?

Also, what is the air pressure when the altitude above sea level is 6000 meters?

Objective

Students will be able to graph logarithmic functions.

Logarithmic Functions (7.4-7.5) Quiz on Friday!!!

Do not forget about pi day tomorrow!!!

Graphing Logarithmic Functions

What determines whether an exponential function is a growth or decay function?

b is the growth or decay

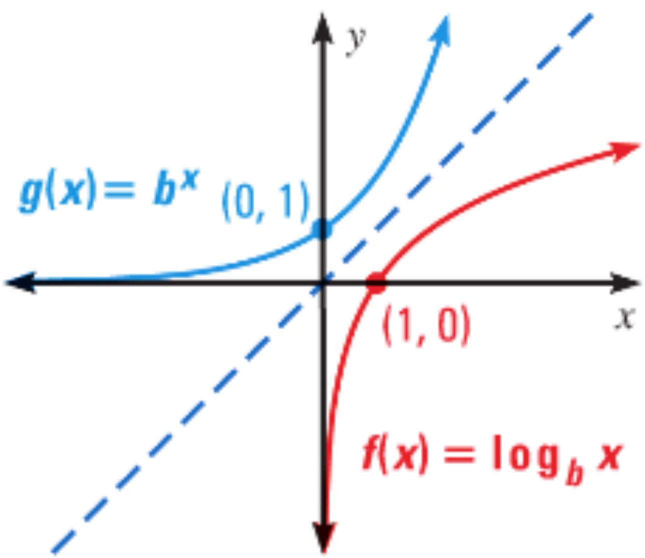
If $b > 1$, then it is an exponential growth function
If $0 < b < 1$, then it is an exponential decay function

You can use the inverse relationship between exponential and logarithmic functions to graph logarithmic functions.

Parent Graphs for Logarithmic Functions

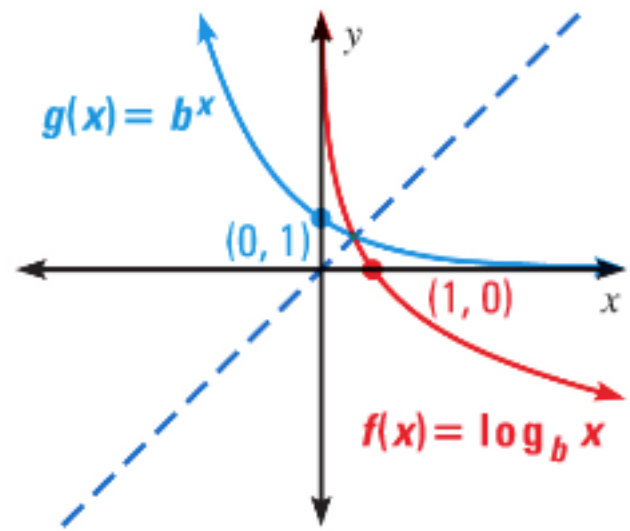
The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and $0 < b < 1$. Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$



**y-axis is the vertical asymptote; x = 0

Graph of $f(x) = \log_b x$ for $0 < b < 1$



Domain: $(0, \infty)$ Range: $(-\infty, \infty)$ Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

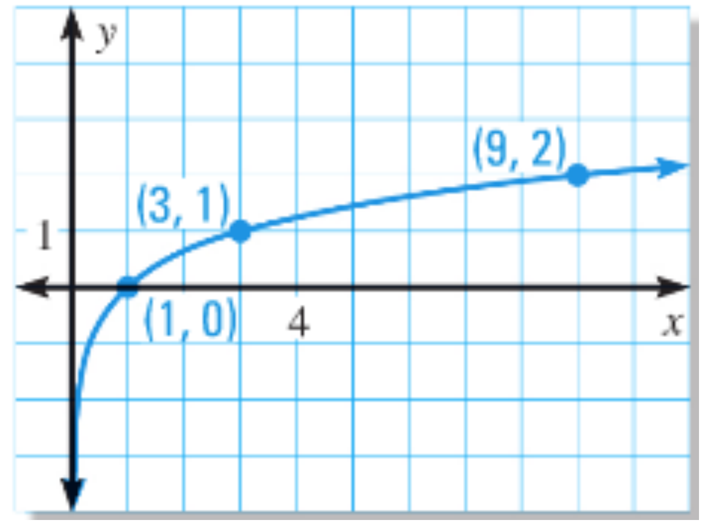
Graph $y = \log_3 x$. ** $3^y = x$

x	y
1	0
3	1
9	2

Step 1: Make a table of values.

**Have them be convenient points

Step 2: Plot the points from the table.



Step 3: From left to right, draw a curve that starts to the right of the *y*-axis and moves up through the plotted points.

Domain: $(0, \infty)$

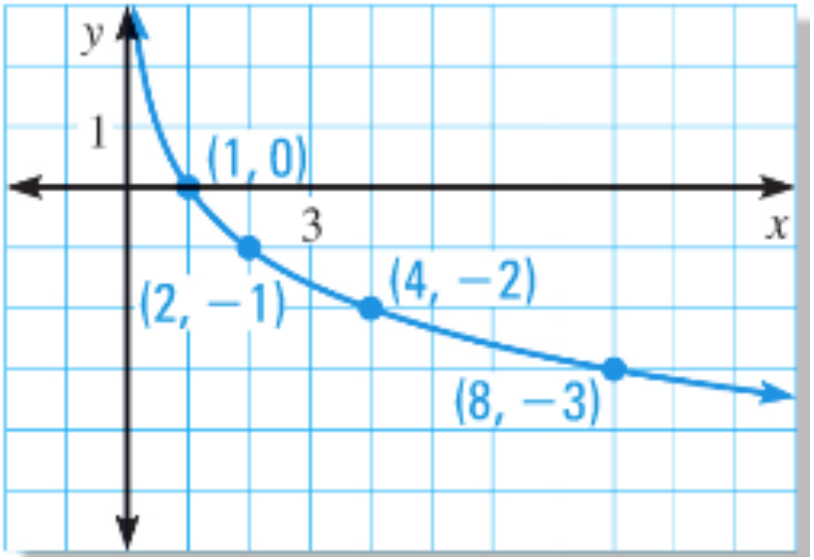
Range: $(-\infty, \infty)$

Asymptote: $x = 0$

Graph $y = \log_{1/2}x$.

** $(1/2)^y = x$

x	1	2	4	8
y	0	-1	-2	-3



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 0$

Translations

You can graph a logarithmic function of the form $y = \log_b(x-h)+k$ by translating the graph of the parent function $y = \log_b x$.

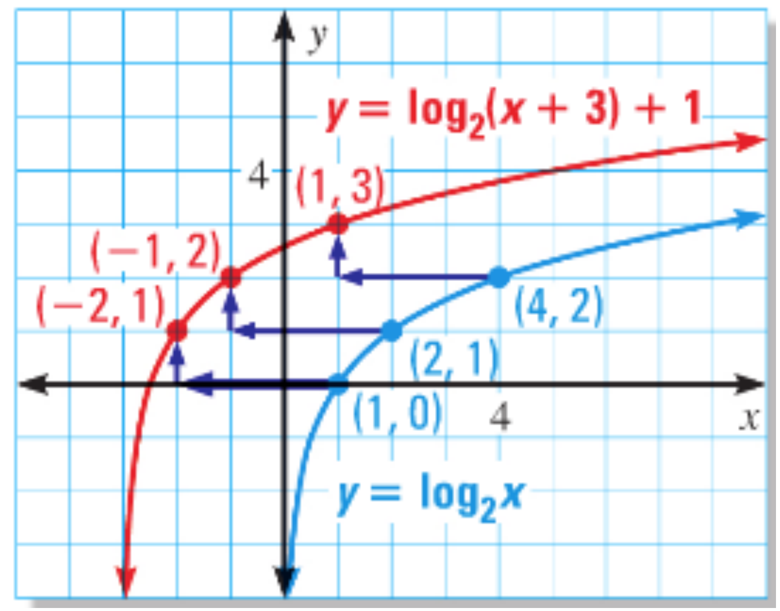
**Remember: you are translating the function opposite of h (to the right or left) and k (up or down)

Vertical asymptote is $x = h$

Graph $y = \log_2(x+3)+1$.

State the domain, range, and asymptote.

Step 1: Sketch the graph of the parent function $y = \log_2 x$, which passes through (1, 0), (2, 1), and (4, 2).



Step 2: Translate the parent graph left 3 units and up 1 unit. The translated graph passes through (-2, 1), (-1, 2), and (1, 3).

** $y - 1 = \log_2(x + 3)$

$(2)^{y-1} = x + 3$

$x = (2)^{y-1} - 3$

Domain: (-3, ∞)

Range: (-∞, ∞)

Asymptote: $x = -3$

Graph $y = \log_4(x+1) - 2$.

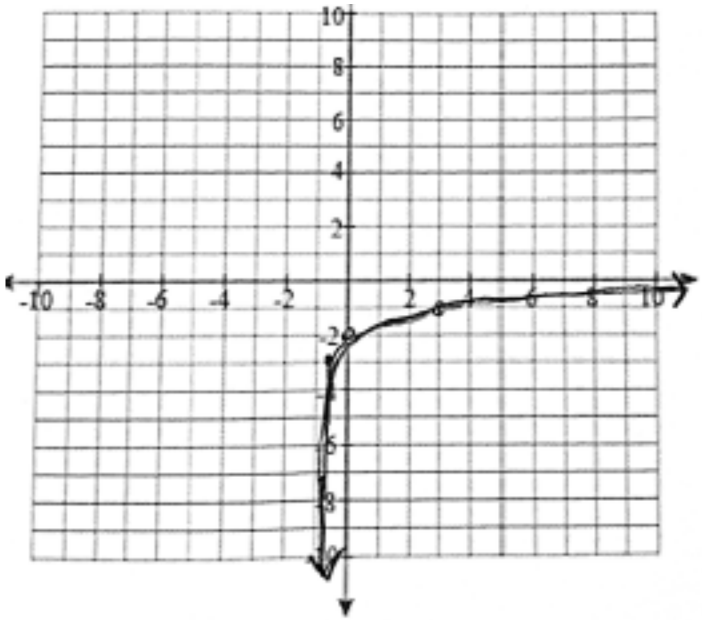
State the domain, range, and asymptote.

Parent function: $y = \log_4 x$

** $4^y = x$

x	y
1/4	-1
1	0
4	1
16	2

Each point gets moved to the left one unit and down 2 units



** $y + 2 = \log_4(x + 1)$

$(4)^{y+2} = x + 1$

$x = (4)^{y+2} - 1$

x	y
-3/4	-3
0	-2
3	-1
15	0

Domain: $(-1, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -1$

Homework

p. 504: 46, 47, 49, 52, 53

GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

**Find the asymptote as well

46. $y = \log_6 x$

47. $y = \log_{1/3} x$

49. $y = \log_2 (x - 3)$

52. $g(x) = \log_6 (x - 4) + 2$

53. $h(x) = \log_5 (x + 1) - 3$

Objective

Students will be able to apply properties of logarithms.

Logarithmic Functions (7.4-7.5) Quiz on Friday!!!

Properties of Logarithms

Let *b*, *m*, and *n* be positive number such that b ≠ 1.

Product Property:

$$\log_b mn = \log_b m + \log_b n$$

Quotient Property:

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Power Property:

$$\log_b m^n = n \log_b m$$

Use $\log_4 3 \approx 0.792$ and $\log_4 7 \approx 1.404$ to evaluate the following logarithms:

1) $\log_4 \frac{3}{7}$

$= \log_4 3 - \log_4 7$ quotient property

$\approx 0.792 - 1.404$ use given values of $\log_4 3$ and $\log_4 7$

$= -0.612$ simplify

2) $\log_4 21 = \log_4 (3 \cdot 7)$ write 21 as 3•7

$= \log_4 3 + \log_4 7$ product property

$\approx 0.792 + 1.404$ use given values of $\log_4 3$ and $\log_4 7$

$= 2.196$ simplify

3) $\log_4 49 = \log_4 7^2$ write 49 as 7^2

$= 2\log_4 7$ power property

$\approx 2(1.404)$ use given value of $\log_4 7$

$= 2.808$ simplify

You can use the properties of logarithms to expand and condense logarithmic expressions.

1) Expand $\log_6 \frac{5x^3}{y}$ $= \log_6 5x^3 - \log_6 y$

quotient property

$= \log_6 5 + \log_6 x^3 - \log_6 y$ $= \log_6 5 + 3\log_6 x - \log_6 y$

product property power property

2) Condense $\ln 4 + 3\ln 3 - \ln 12$

$= \ln 4 + \ln 3^3 - \ln 12$ $= \ln 4 + \ln 27 - \ln 12$

$= \ln 4(27) - \ln 12$ $= \ln 108 - \ln 12$ $= \ln \frac{108}{12}$

$= \ln 9$

Change-of-Base Formula

Logarithms with any base other than 10 or *e* can be written in terms of common or natural logarithms using the *change-of-base formula*. This allows you to evaluate any logarithm using a calculator.

If *a*, *b*, and *c* are positive numbers with b ≠ 1 and c ≠ 1, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Use the change-of-base formula to evaluate the following logarithms:

1) $\log_3 8 = \dfrac{\log 8}{\log 3} \approx \dfrac{0.9031}{0.4771} \approx 1.893$

OR $= \dfrac{\ln 8}{\ln 3} \approx \dfrac{2.0794}{1.0986} \approx 1.893$

2) $\log_{26} 9 = \dfrac{\log 9}{\log 26} \approx \dfrac{0.9542}{1.4150} \approx 0.674$

Homework p. 510: 3-9, 15-21 (odds), 28, 33-39 (odds), 45, 47

MATCHING EXPRESSIONS Match the expression with the logarithm that has the same value.

3. $\ln 6 - \ln 2$ **4.** $2 \ln 6$ **5.** $6 \ln 2$ **6.** $\ln 6 + \ln 2$

A. $\ln 64$ **B.** $\ln 3$ **C.** $\ln 12$ **D.** $\ln 36$

APPROXIMATING EXPRESSIONS Use $\log 4 \approx 0.602$ and $\log 12 \approx 1.079$ to evaluate the logarithm.

7. $\log 3$ **8.** $\log 48$ **9.** $\log 16$

EXPANDING EXPRESSIONS Expand the expression.

15. $\log_3 4x$ **17.** $\log 3x^4$ **19.** $\log_2 \frac{2}{5}$ **21.** $\log_4 \frac{x}{3y}$ **28.** $\ln \frac{6x^2}{y^4}$

CONDENSING EXPRESSIONS Condense the expression.

33. $\log_4 7 - \log_4 10$ **35.** $2 \log x + \log 11$ **37.** $5 \log x - 4 \log y$

39. $\ln 40 + 2 \ln \frac{1}{2} + \ln x$

CHANGE-OF-BASE FORMULA Use the change-of-base formula to evaluate the logarithm. **45.** $\log_4 7$ **47.** $\log_3 15$