

# Objective

Students will be able to graph rational functions and identify appropriate values for domain, range, and asymptotes.

Midterm is on Thursday!

# Rational functions

A rational function has the form

$$f(x) = \frac{p(x)}{q(x)}$$

where  $p(x)$  and  $q(x)$  are polynomials and  $q(x) \neq 0$ .

The inverse variation function  $f(x) = \frac{a}{x}$  is a rational function. The graph of this function when  $a = 1$  is shown below.

## Parent Function for Simple Rational Functions

The graph of the parent function  $f(x) = \frac{1}{x}$  is a *hyperbola*, which consists of two symmetrical parts called *branches*.

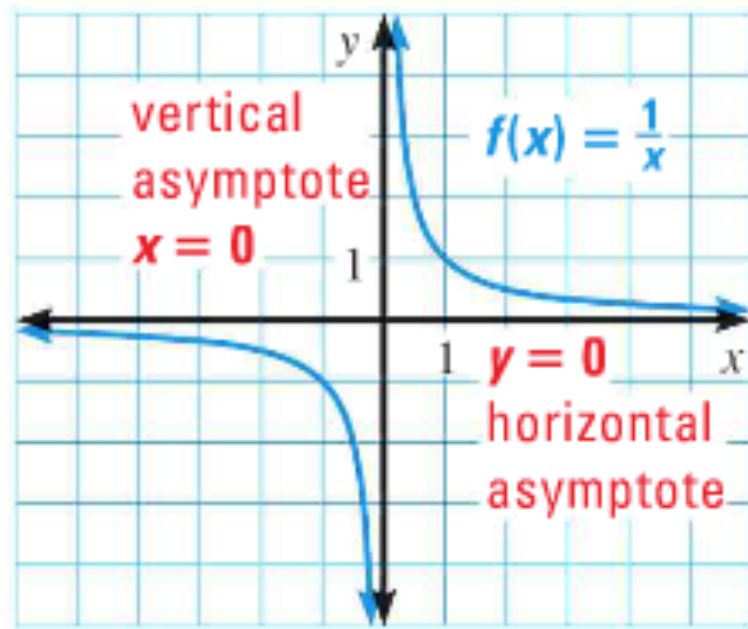
The domain and range are all nonzero real numbers.

Domain:  $(-\infty, 0) \cup (0, \infty)$

Range:  $(-\infty, 0) \cup (0, \infty)$

Any function of the form  $g(x) = \frac{a}{x}$  ( $a \neq 0$ ) has the same asymptotes, domain, and range as the function

$$f(x) = \frac{1}{x}$$



Graph the function  $y = \frac{6}{x}$ .

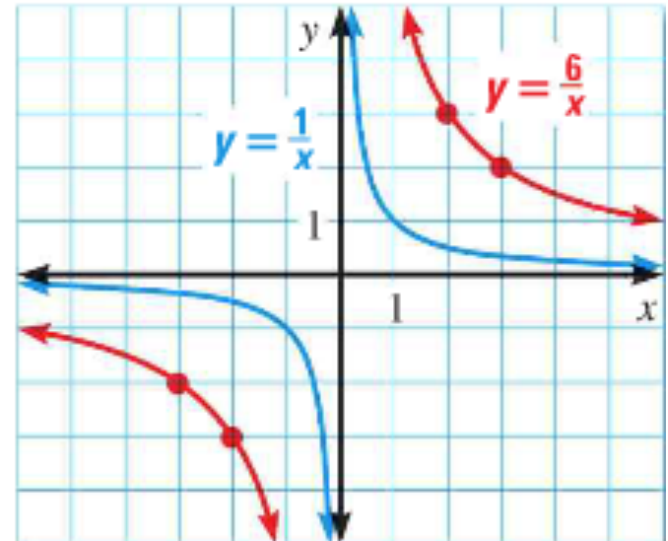
**\*\*When creating your table of values, find two points to the right and left of your vertical asymptote**

Compare the graph with the graph of  $y = \frac{1}{x}$ .

**Step 1:** Draw the asymptotes of  $x = 0$  and  $y = 0$ .

**Step 2:** Create a table of values. Plot points to the left and to the right of the vertical asymptote, such as  $(-3, -2)$ ,  $(-2, -3)$ ,  $(2, 3)$ , and  $(3, 2)$ .

**Step 3:** Draw the branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



The graph  $y = 6 / x$  lies further from the axes than the graph of  $y = 1 / x$ . Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

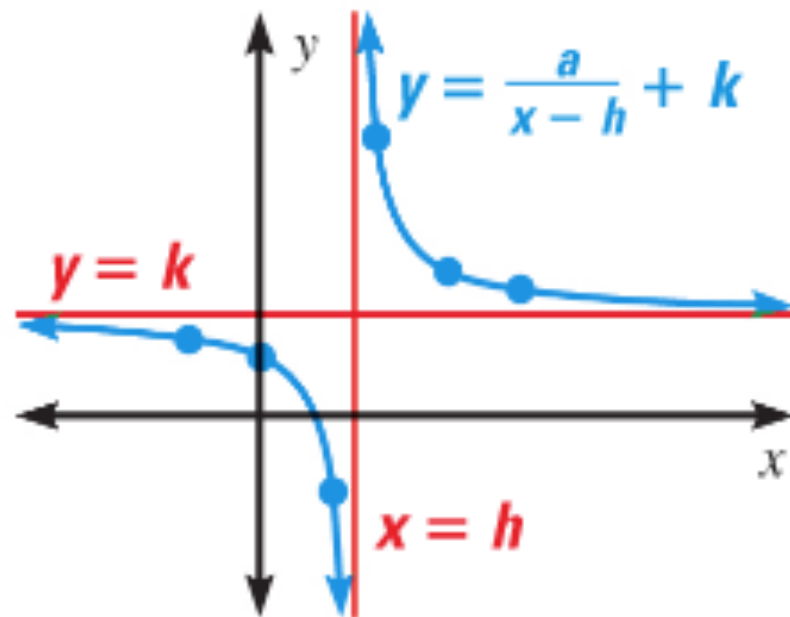
# Graphing Translations of Simple Rational Functions

To graph a rational function of the form  $y = \frac{a}{x-h} + k$ , follow these steps:

**Step 1:** Draw the asymptotes of  $x = h$  and  $y = k$ .

**Step 2:** Create a table of values. Plot points to the left and to the right of the vertical asymptote.

**Step 3:** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



**\*\*asymptotes should be drawn in with dotted lines!**

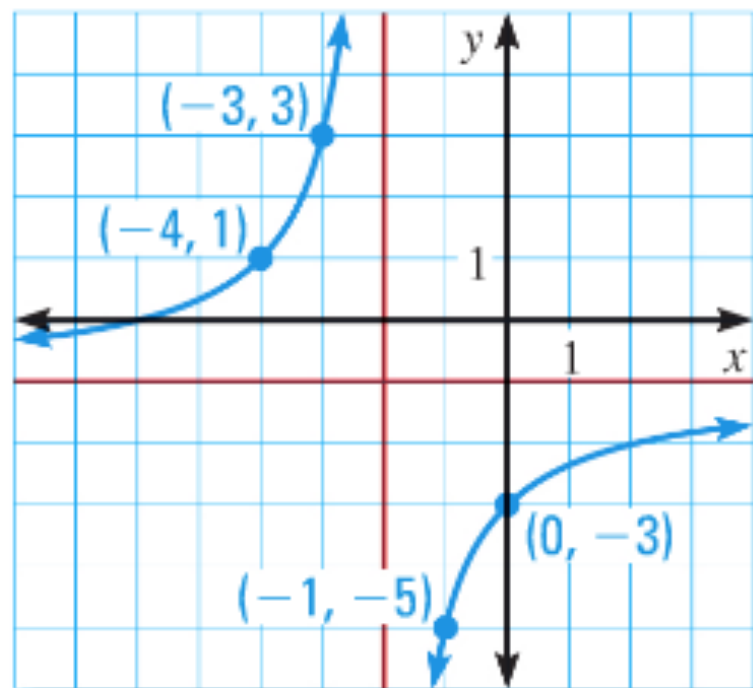
1) Graph the function  $y = \frac{-4}{x+2} - 1$ .

Find the asymptotes. State the domain and range.

**Step 1:** Draw the asymptotes of  $x = -2$  and  $y = -1$ .

**Step 2:** Create a table of values. Plot points to the left of the vertical asymptote, such as  $(-3, 3)$  and  $(-4, 1)$ , and to the right, such as  $(-1, -5)$ , and  $(0, -3)$ .

**Step 3:** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



**\*\*asymptotes should be drawn in with dotted lines!**

Domain:  $(-\infty, -2) \cup (-2, \infty)$

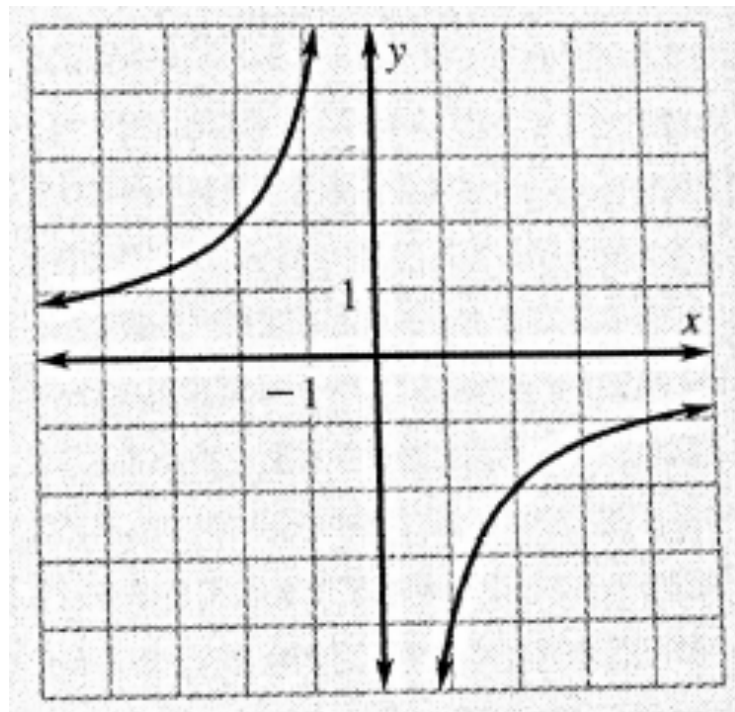
Range:  $(-\infty, -1) \cup (-1, \infty)$

2) Graph the function  $f(x) = \frac{-4}{x}$ .

Find the asymptotes. State the domain and range.

Asymptotes:  $x = 0$   
 $y = 0$

<b>x</b>	<b>y</b>
-2	2
-1	4
1	-4
2	-2



Domain:  $(-\infty, 0) \cup (0, \infty)$

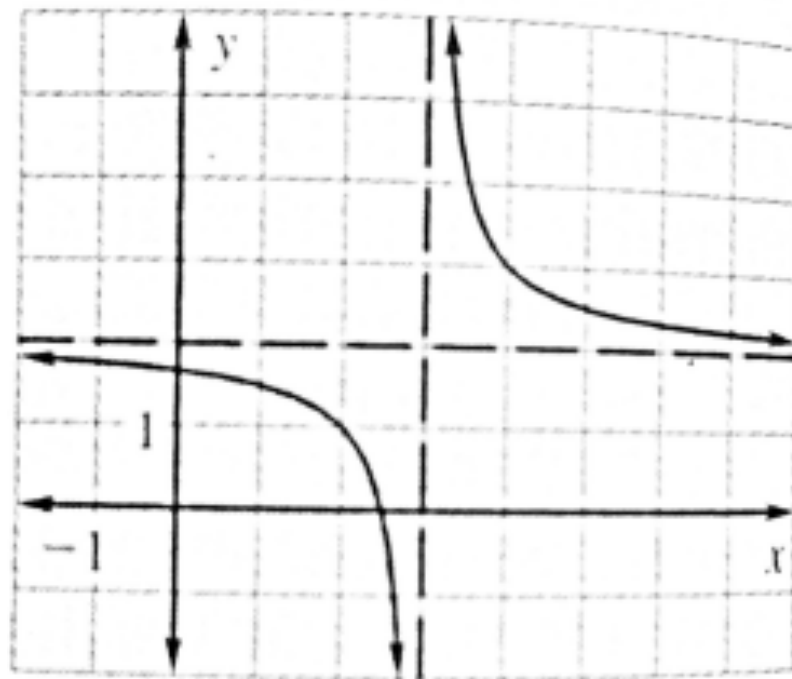
Range:  $(-\infty, 0) \cup (0, \infty)$

3) Graph the function  $y = \frac{1}{x-3} + 2$ .

Find the asymptotes. State the domain and range.

Asymptotes:  $x = 3$   
 $y = 2$

<b>x</b>	<b>y</b>
0	$5/3$
2	1
4	3
7	$9/4$



Domain:  $(-\infty, 3) \cup (3, \infty)$

Range:  $(-\infty, 2) \cup (2, \infty)$



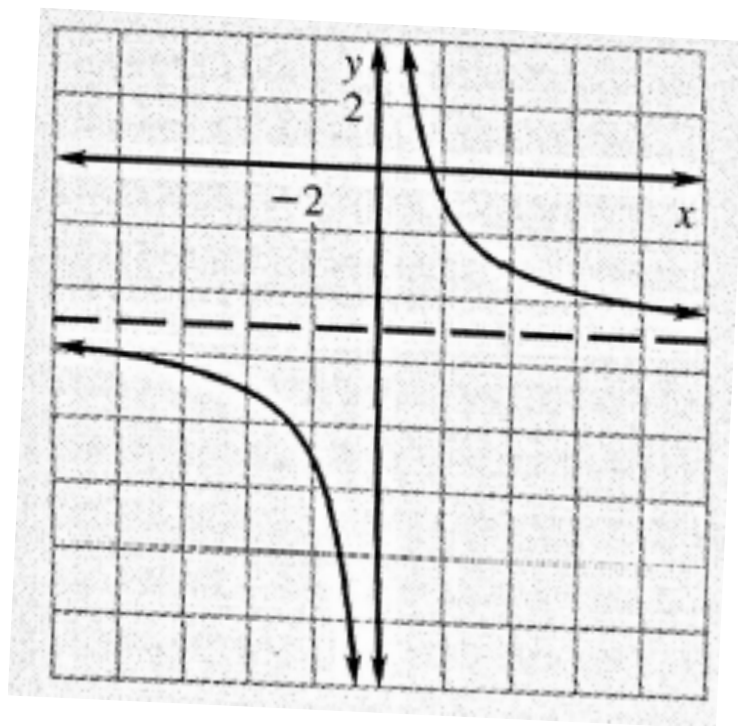
4) Graph the function  $y = \frac{8}{x} - 5$ .

Find the asymptotes. State the domain and range.

Asymptotes:  $x = 0$

$y = -5$

<b>x</b>	<b>y</b>
-4	-7
-2	-9
1	3
2	-1



Domain:  $(-\infty, 0) \cup (0, \infty)$

Range:  $(-\infty, -5) \cup (-5, \infty)$

# Homework p.561: 5, 11, 13, 15, 19, 23

**GRAPHING FUNCTIONS** Graph the function. Compare the graph with the graph of  $y = \frac{1}{x}$ .

5.  $y = \frac{-5}{x}$

**GRAPHING FUNCTIONS** Graph the function. State the domain and range.

**\*\*Find the asymptote as well**

11.  $y = \frac{4}{x} + 3$     13.  $y = \frac{6}{x-1}$     15.  $y = \frac{-5}{x} - 7$     19.  $y = \frac{-4}{x+4} + 3$

23. **★ MULTIPLE CHOICE** What are the asymptotes of the graph of  $y = \frac{3}{x+8} - 3$ ?

- (A)  $x = 8, y = 3$     (B)  $x = 8, y = -3$     (C)  $x = -8, y = 3$     (D)  $x = -8, y = -3$

# Objective

Students will be able to graph rational functions and identify appropriate values for domain, range, and asymptotes.

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\*\*Remember, I am in the math lab (341) after school today!

# Other Rational Functions

All rational functions have the form  $y = \frac{ax + b}{cx + d}$   
also have graphs that are hyperbolas.

- The vertical asymptote of the graph is the line  $x = -\frac{d}{c}$ , because the function is undefined when the denominator  $cx + d$  is zero    **\*\*set denominator equal to 0 and solve for x**

**\*\*\*same as rational functions on Friday!**

- The horizontal asymptote is the line  $y = \frac{a}{c}$   
**\*\*dividing leading coefficients**

**\*\*\*different from rational functions on Friday!**

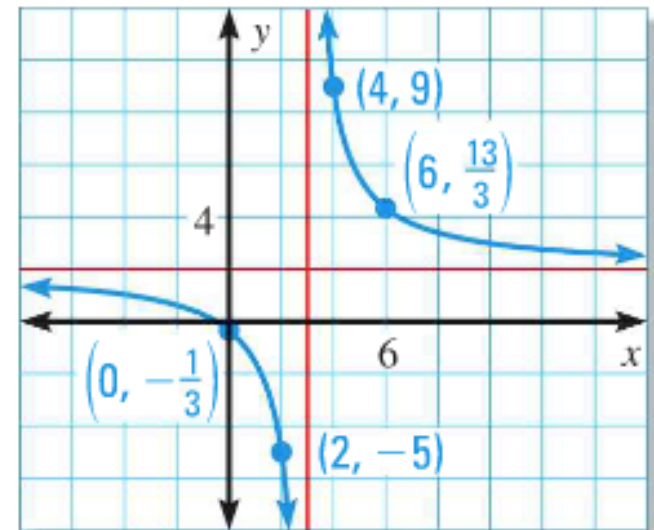
Graph the function  $y = \frac{2x+1}{x-3}$ . \*\*asymptotes should be drawn in with dotted lines!

Find asymptotes. State the domain and range.

**Step 1:** Find and draw the asymptotes. Solve  $x - 3 = 0$  for  $x$  to find the vertical asymptote  $x = 3$ . The horizontal asymptote is the line  $y = \frac{a}{c} = \frac{2}{1} = 2$

**Step 2:** Create your table of values. Plot points to the left of the vertical asymptote, such as  $(2, -5)$  and  $(0, -1/3)$ , and to the right, such as  $(4, 9)$ , and  $(6, 13/3)$ .

**Step 3:** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



Domain:  $(-\infty, 3) \cup (3, \infty)$

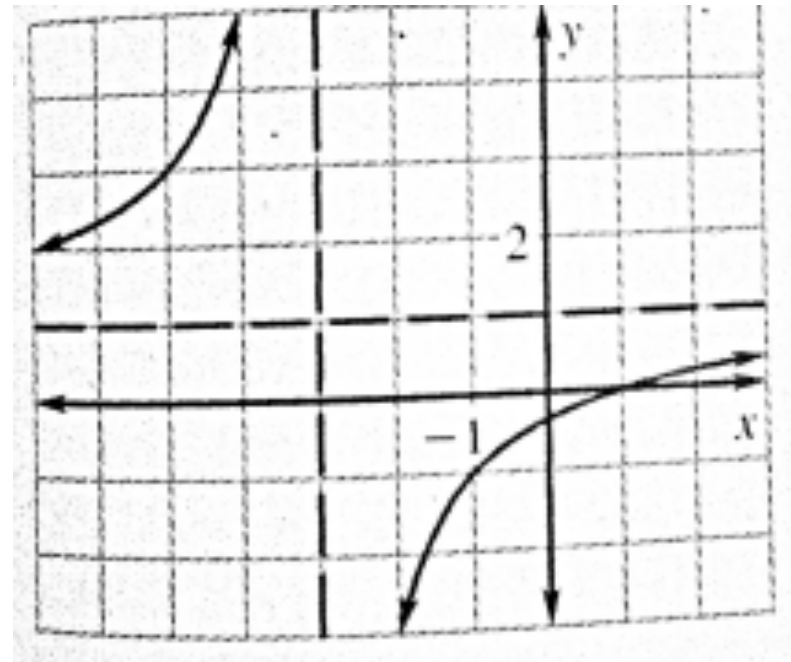
Range:  $(-\infty, 2) \cup (2, \infty)$

Graph the function  $y = \frac{x-1}{x+3}$ . \*\*When creating your table of values, find two points to the right and left of your vertical asymptote

Find asymptotes. State the domain and range.

Asymptotes:  $x = -3$   
 $y = 1$

<b>x</b>	<b>y</b>
-6	$7/3$
-4	5
-2	-3
0	$-1/3$



Domain:  $(-\infty, -3) \cup (-3, \infty)$

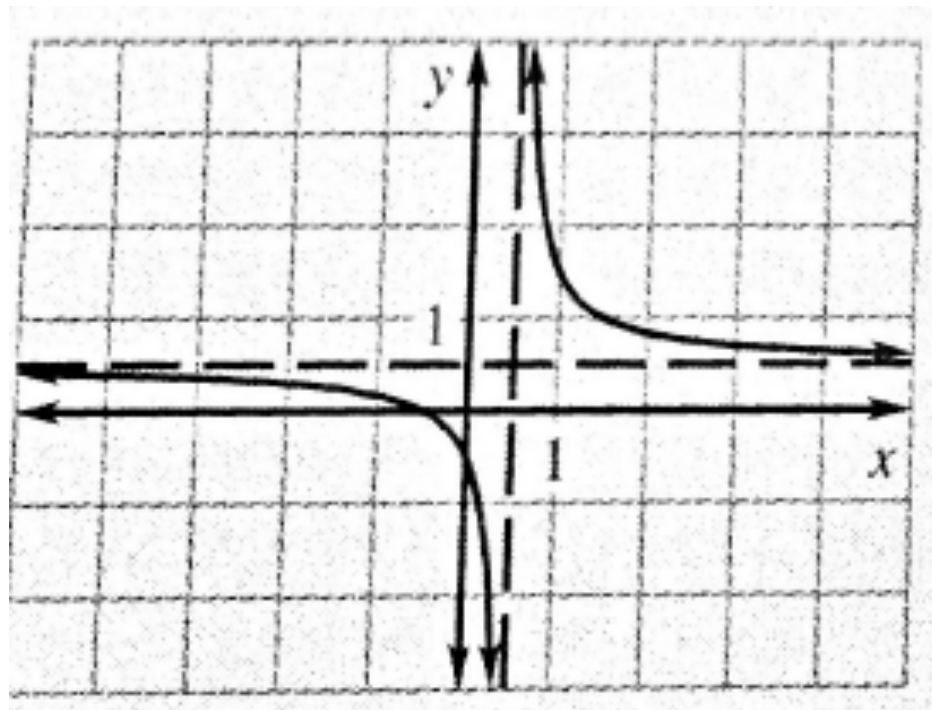
Range:  $(-\infty, 1) \cup (1, \infty)$

Graph the function  $y = \frac{2x+1}{4x-2}$ .

Find asymptotes. State the domain and range.

Asymptotes:  $x = 1/2$   
 $y = 1/2$

x	y
-1	1/6
0	-1/2
1	3/2
2	5/6



Domain:  $(-\infty, 1/2) \cup (1/2, \infty)$

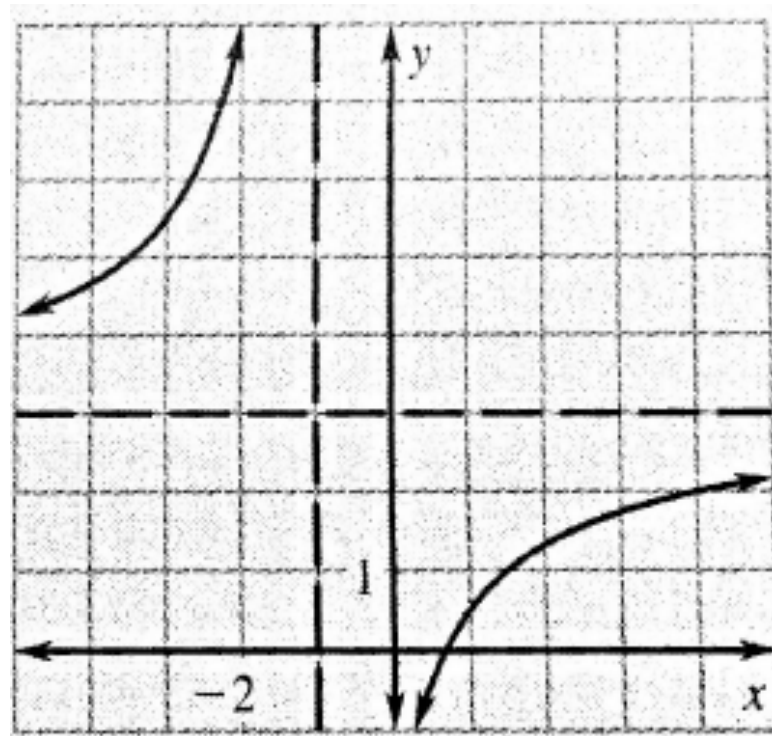
Range:  $(-\infty, 1/2) \cup (1/2, \infty)$

Graph the function  $y = \frac{-3x + 2}{-x - 1}$ .

Find asymptotes. State the domain and range.

Asymptotes:  $x = -1$   
 $y = 3$

<b>x</b>	<b>y</b>
-4	$14/3$
-2	8
0	-2
2	$4/3$



Domain:  $(-\infty, -1) \cup (-1, \infty)$

Range:  $(-\infty, 3) \cup (3, \infty)$

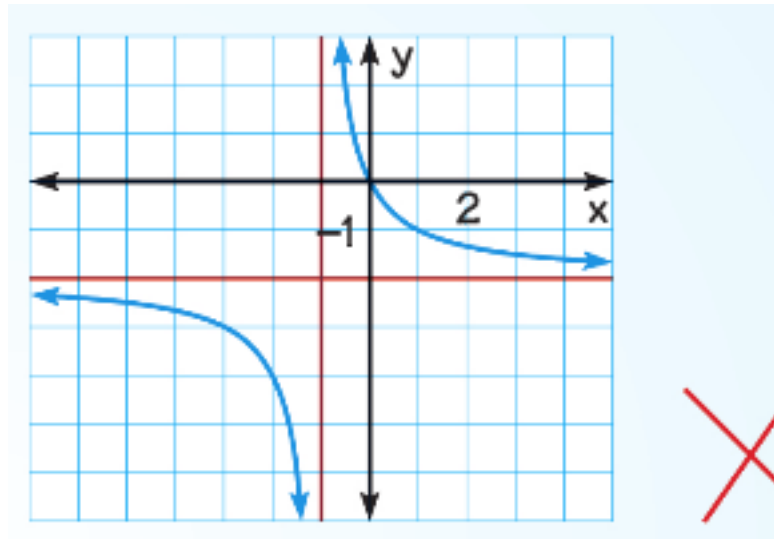


# Homework

p. 562: 26, 27, 28, 30, 34

**ERROR ANALYSIS** Describe and correct the error in the graph.

26.  $y = \frac{2}{x-1} - 2$



**GRAPHING FUNCTIONS** Graph the function. State the domain and range.

27.  $y = \frac{x+4}{x-3}$

28.  $y = \frac{x-1}{x+5}$

\*\*Find the asymptote as well

30.  $y = \frac{8x+3}{2x-6}$

34.  $h(x) = \frac{5x+3}{-x+10}$

# Objective

Students will be able to simplify and multiply rational expressions.

Midterm is on Thursday!

# Refresher:

Factor the following problems:

1)  $x^2 - 4x - 12$

$$(x - 6)(x + 2)$$

2)  $7z^2 - 20z - 3$

$$(7z + 1)(z - 3)$$

3)  $9y^2 - 64$

$$(3y + 8)(3y - 8)$$

4)  $8x^3 - 125$

\*\*Difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(2x - 5)(4x^2 + 10x + 25)$$

# Simplifying Rational Expressions

$$f(x) = \frac{\cancel{x} + 4}{\cancel{x} + 1}$$
$$= \frac{4}{1} = 4?$$

**NO!!!**



Simplify:  $\frac{x^2 - 2x - 15}{x^2 - 9}$

$$= \frac{(x+3)(x-5)}{(x+3)(x-3)}$$

Factor numerator and denominator.

$$= \frac{\cancel{(x+3)}(x-5)}{\cancel{(x+3)}(x-3)}$$

Divide out common factor.

$$= \frac{x-5}{x-3}$$

Simplified form

Simplify the following expressions,  
if possible.

**\*\*The most simplified form is when everything is multiplied out; there should not be any parentheses left**

$$1) \frac{2(x+1)}{(x+1)(x+3)}$$

$$\frac{2}{x+3}$$

$$2) \frac{40x+20}{10x+30}$$

$$\frac{4x+2}{x+3}$$

$$3) \frac{4}{x(x+2)}$$

$$\frac{4}{x^2+2x}$$

Simplify the following expressions,  
if possible.

$$4) \quad \frac{x+4}{x^2-16} \qquad \frac{1}{x-4}$$

$$5) \quad \frac{x^2-2x-3}{x^2-x-6} \qquad \frac{x+1}{x+2}$$

$$6) \quad \frac{2x^2+10x}{3x^2+16x+5} \qquad \frac{2x}{3x+1}$$

Multiply:  $\frac{x+2}{x^3-27} \cdot (x^2+3x+9)$

$$= \frac{x+2}{x^3-27} \cdot \frac{x^2+3x+9}{1}$$

Write polynomial as a rational expression.

$$= \frac{(x+2)(x^2+3x+9)}{(x-3)(x^2+3x+9)}$$

Factor denominator.

$$= \frac{(x+2)(\cancel{x^2+3x+9})}{(x-3)(\cancel{x^2+3x+9})}$$

Divide out common factors.

$$= \frac{x+2}{x-3}$$

Simplified form



Multiply the following expressions.  
Simplify the result.

$$1) \frac{3x^5y^2}{8xy} \cdot \frac{6xy^2}{9x^3y}$$

$$2) \frac{2x^2 - 10}{x^2 - 25} \cdot \frac{x + 3}{2x^2}$$

$$\frac{x^2y^2}{4}$$

$$\frac{x + 3}{x^2 + 5x}$$

$$3) \frac{x + 5}{x^3 - 1} \cdot (x^2 + x + 1)$$

$$\frac{x + 5}{x - 1}$$

# Homework p.577: 13, 14, 15, 17, 25, 29-33 (odds)

**SIMPLIFYING** Simplify the rational expression, if possible.

$$13. \frac{x^2 - 36}{x^2 + 12x + 36}$$

$$14. \frac{3x^3 + 6x^2 + 12x}{x^3 - 8}$$

$$15. \frac{8x^2 + 10x - 3}{6x^2 + 13x + 6}$$

$$17. \frac{x^3 - 5x^2 - 3x + 15}{x^2 - 8x + 15}$$

**MULTIPLYING** Multiply the expressions. Simplify the result.

$$25. \frac{48x^5y^3}{y^4} \cdot \frac{x^2y}{6x^3y^2}$$

$$29. \frac{x + 5}{4x - 16} \cdot \frac{2x^2 - 32}{x^2 - 25}$$

$$31. \frac{x^2 - 3x - 10}{x^2 - 2x - 15} \cdot (x^2 + 10x + 21)$$

$$33. \frac{4x^2 + 20x}{x^3 + 4x^2} \cdot (x^2 + 8x + 16)$$

# Objective

Students will be able to divide polynomial expressions.

**Graphing, Multiplying, and Dividing Rational Expressions Quiz (8.2, 8.4) on Wednesday!**

**\*\*NO graphing calculator**

**New hall passes**

# Dividing Rational Expressions

What can you do when you have a fraction and you divide it by another fraction?

To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression.

Let  $a$ ,  $b$ ,  $c$ , and  $d$  be expressions with  $b \neq 0$ ,  $c \neq 0$ , and  $d \neq 0$ .

<b>Property</b>	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$	<b>Simplify <math>\frac{ad}{bc}</math> if possible.</b>
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<b>Examples</b>	$\frac{2}{5} \div \frac{7}{3} = \frac{2}{5} \cdot \frac{3}{7} = \frac{6}{35}$
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$$\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)}$$

Divide:  $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$

$= \frac{7x}{2x-10} \cdot \frac{x^2-11x+30}{x^2-6x}$  Multiply by reciprocal.

$= \frac{7x}{2(x-5)} \cdot \frac{(x-5)(x-6)}{x(x-6)}$  Factor.

$= \frac{7\cancel{x}(x-5)\cancel{(x-6)}}{2(\cancel{x-5})\cancel{(x)}(x-6)}$  Divide out common factors.

$= \frac{7}{2}$  Simplified form

Divide:  $\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$

$$= \frac{6x^2 + x - 15}{4x^2} \cdot \frac{1}{3x^2 + 5x}$$

Multiply by reciprocal.

$$= \frac{(3x + 5)(2x - 3)}{4x^2} \cdot \frac{1}{x(3x + 5)}$$

Factor.

$$= \frac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}}$$

Divide out common factors.

$$= \frac{2x - 3}{4x^3}$$

Simplified form

Divide the following expressions.  
Simplify the result.

$$1) \quad \frac{4x}{5x-20} \div \frac{x^2-2x}{x^2-6x+8} \qquad \frac{4}{5}$$

$$2) \quad \frac{2x^2+3x-5}{6x} \cdot (2x^2+5x) \qquad \frac{x-1}{6x^2}$$

# Homework

p. 578: 35-43 (odds)

**DIVIDING** Divide the expressions. Simplify the result.

$$35. \frac{8x^2y^2z}{xz^3} \div \frac{10xy}{x^4z}$$

$$37. \frac{8x^2}{x+4} \div \frac{x}{2(x-4)}$$

$$39. \frac{x^2 - 4x - 5}{x+5} \div (x^2 + 6x + 5)$$

$$41. \frac{x^2 - x - 2}{x^2 + 4x - 5} \div \frac{x - 2}{5x + 25}$$

$$43. \frac{x^2 + 12x + 32}{6x + 42} \div \frac{x^2 + 4x}{x^2 - 49}$$



# Objective

Students will be able to use their knowledge on graphing, multiplying, and dividing rational expressions to study for their quiz tomorrow.

**Graphing, Multiplying, and Dividing Rational Expressions Quiz (8.2, 8.4) tomorrow!**

**\*\*NO graphing calculator**

# Graphing Rational Expressions Refresher

Two Forms:

$$y = \frac{a}{x - h} + k$$

$$y = \frac{ax + b}{cx + d}$$

V.A:  $x = h$  ( $x - h = 0$ )

V.A:  $x = -d/c$  ( $cx + d = 0$ )

H.A:  $y = k$

H.A:  $y = a/c$  (L.Cs)

Domain: all real numbers except V.A;  $(-\infty, \text{V.A.}) \cup (\text{V.A.}, \infty)$

Range: all real numbers except H. A;  $(-\infty, \text{H.A.}) \cup (\text{H.A.}, \infty)$

To graph: Sketch and label asymptotes, create a table of values (you need to have at least two points on each side of the vertical asymptote), plot points and draw branches so they approach the asymptotes

# Objective

Students will be able to add and subtract rational expressions.

**There is a free PreCalculus class this summer at Wright College, if you earn a C or higher, you would be able to take AP Calculus next year. If you are interested, I can email you more information.**

# Adding or Subtracting with Like Denominators

To add (or subtract) rational expressions with *like* denominators, simply add (or subtract) their numerators. Then place the result over the common denominator.

Let  $a$ ,  $b$ , and  $c$  be expressions with  $c \neq 0$ .

	Addition	Subtraction
Properties	$\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$	$\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$
Examples	$\frac{3x}{5x^2} + \frac{7}{5x^2} = \frac{3x + 7}{5x^2}$	$\frac{9x^3}{x + 1} - \frac{x^2}{x + 1} = \frac{9x^3 - x^2}{x + 1}$

Perform the indicated operation and simplify.

$$1) \quad \frac{7}{4x} + \frac{3}{4x} = \frac{7+3}{4x} = \frac{10}{4x} = \frac{5}{2x}$$

$$2) \quad \frac{2x}{x+6} - \frac{5}{x+6} = \frac{2x-5}{x+6}$$

Perform the indicated operation and simplify.

$$3) \frac{7}{12x} - \frac{5}{12x} = \frac{2}{12x} = \frac{1}{6x}$$

$$4) \frac{2}{3x^2} + \frac{1}{3x^2} = \frac{3}{3x^2} = \frac{1}{x^2}$$

Perform the indicated operation and simplify.

$$5) \quad \frac{4x}{x-2} - \frac{x}{x-2} = \frac{3x}{x-2}$$

$$6) \quad \frac{2x^2}{x^2+1} + \frac{2}{x^2+1} = \frac{2x^2+2}{x^2+1} = \frac{2(x^2+1)}{x^2+1} = 2$$

What if we have unlike  
denominators?

Need to a common denominator;  
find the least common multiple (LCM)!



Find the least common multiple of  $4x^2 - 16$  and  $6x^2 - 24x + 24$ .

Step 1: **Factor** each polynomial. Write the numerical factors as a products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = 4(x - 2)(x + 2) = (2^2)(x - 2)(x + 2)$$

$$\begin{aligned} 6x^2 - 24x + 24 &= 6(x^2 - 4x + 4) = 6(x - 2)(x - 2) \\ &= (2)(3)(x - 2)^2 \end{aligned}$$

Step 2: **Form** the LCM by writing each factor to the **highest** power it occurs in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

Find the least common multiple of the polynomials.

1)  $5x^3$  and  $10x^2 - 15x$

$$5x^3(2x - 3)$$

2)  $8x - 16$  and  $12x^2 + 12x - 72$

$$24(x - 2)(x + 3)$$

# Homework

p. 586: 3-15 (odds)

**LIKE DENOMINATORS** Perform the indicated operation and simplify.

3.  $\frac{15}{4x} + \frac{5}{4x}$     5.  $\frac{9}{x+1} - \frac{2x}{x+1}$     7.  $\frac{5x}{x+3} + \frac{15}{x+3}$

**FINDING LCMS** Find the least common multiple of the polynomials.

9.  $3x$  and  $3(x - 2)$     11.  $2x$  and  $2x(x - 5)$

13.  $x^2 - 25$ ,  $x$ , and  $x - 5$

15. **★ MULTIPLE CHOICE** What is the least common multiple of the polynomials  $3x^2 - 9x$  and  $6x^2$ ?

(A)  $3x(x - 3)$

(B)  $6x^2$

(C)  $6x(x - 3)$

(D)  $6x^2(x - 3)$

# Objective

Students will be able to add and subtract rational expressions.

**Add and Subtract Rational Expressions**

**(8.5) Quiz on Friday!!!!**

**Homework Quiz tomorrow!**

There is a free PreCalculus class this summer at Wright College, if you earn a C or higher, you would be able to take AP Calculus next year. If you are interested, I can email you more information.

# Adding or Subtracting with Unlike Denominators

To add (or subtract) rational expressions with unlike denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add (or subtract).

Let  $a$ ,  $b$ ,  $c$ , and  $d$  be expressions with  $c \neq 0$  and  $d \neq 0$ .

**Addition**

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

**Subtraction**

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

You can always find a common denominator of two rational expressions by **multiplying the denominators**, as shown above. However, if you use the **LCM (LCD)**, you may have to do less simplifying.

Add:  $\frac{7}{9x^2} + \frac{x}{3x^2 + 3x}$

$$9x^2 = 3^2x^2$$

Find the **LCM (LCD)** of denominators:  $3x^2 + 3x = 3x(x + 1)$

$$\text{LCD: } 3^2x^2(x + 1) = 9x^2(x + 1)$$

$$= \frac{7}{9x^2} + \frac{x}{3x(x+1)}$$

$$= \frac{7}{9x^2} \cdot \frac{x+1}{x+1} + \frac{x}{3x(x+1)} \cdot \frac{3x}{3x}$$

$$= \frac{7x+7}{9x^2(x+1)} + \frac{3x^2}{9x^2(x+1)}$$

$$= \frac{3x^2 + 7x + 7}{9x^2(x+1)} = \frac{3x^2 + 7x + 7}{9x^3 + 9x^2}$$

Factor denominators

LCD is  $9x^2(x + 1)$ ; figure out what to multiply each part by so that it becomes the LCD

Multiply

Add numerators

Factor numerator out to see if anything cancels out

Subtract:  $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$

$$= \frac{x+2}{2(x-1)} - \frac{-2x-1}{(x-1)(x-3)}$$

Factor denominators

$$= \frac{x+2}{2(x-1)} \cdot \frac{x-3}{x-3} - \frac{-2x-1}{(x-1)(x-3)} \cdot \frac{2}{2}$$

LCD is  $2(x-1)(x-3)$

$$= \frac{x^2-x-6}{2(x-1)(x-3)} - \frac{-4x-2}{2(x-1)(x-3)}$$

Multiply

$$= \frac{x^2-x-6-(-4x-2)}{2(x-1)(x-3)}$$

Subtract numerators

$$= \frac{x^2+3x-4}{2(x-1)(x-3)}$$

Simplify numerators

$$= \frac{\cancel{(x-1)}(x+4)}{2\cancel{(x-1)}(x-3)} = \frac{x+4}{2(x-3)} = \frac{x+4}{2x-6}$$

Factor numerator

Simplify

# Homework

p. 586: 17-27 (odds)

**UNLIKE DENOMINATORS** Perform the indicated operation and simplify.

17.  $\frac{8}{3x^2} - \frac{5}{4x}$

19.  $\frac{12}{x^2 + 5x - 24} + \frac{3}{x - 3}$

21.  $\frac{9}{x - 3} + \frac{2x}{x + 1}$

23.  $\frac{-15x}{x^2 - 8x + 16} + \frac{12}{x - 4}$

25. **ERROR ANALYSIS** Describe and correct the error in adding the rational expressions.

$$\frac{x}{x+2} + \frac{4}{x-5} = \frac{x+4}{(x+2)(x-5)}$$



**UNLIKE DENOMINATORS** Perform the indicated operation(s) and simplify.

27.  $\frac{x}{x^2 - 9} + \frac{x + 1}{x^2 + 6x + 9}$



# Objective

Students will be able to add and subtract rational expressions.

**Add and Subtract Rational Expressions  
(8.5) Quiz tomorrow!!!!**

**Rational Functions (Chapter 8) Test on  
Thursday, May 4<sup>th</sup>**

I send each of you an email about your AP exams.  
Make sure you fill it out! It will take 2 minutes!

# Simplifying Complex Fractions

A complex fraction is a fraction that contains a fraction in its numerator or denominator.

$$\text{Example: } \frac{\frac{4}{9}}{\frac{9}{25} + \frac{5}{9}} = \frac{\frac{4}{9} \cdot \frac{25}{25}}{\frac{9}{25} \cdot \frac{9}{9} + \frac{5}{9} \cdot \frac{25}{25}}$$

$$= \frac{\frac{100}{225}}{\frac{81}{225} + \frac{125}{225}} = \frac{\frac{100}{225}}{\frac{206}{225}} = \frac{100}{225} \cdot \frac{225}{206} = \frac{100}{206} = \frac{50}{103}$$

1) Simplify:

$$\frac{\frac{4}{x} - 3}{1 + \frac{x}{2}}$$

$$= \frac{\frac{4}{x} \left( \frac{2}{2} \right) - 3 \left( \frac{2x}{2x} \right)}{1 \left( \frac{2x}{2x} \right) + \frac{x}{2} \left( \frac{x}{x} \right)} = \frac{\frac{8}{2x} - \frac{6x}{2x}}{\frac{2x}{2x} + \frac{x^2}{2x}}$$

$$= \frac{\frac{8-6x}{2x}}{\frac{2x+x^2}{2x}} = \frac{8-6x}{\cancel{2x}} \cdot \frac{\cancel{2x}}{2x+x^2} = \frac{8-6x}{2x+x^2}$$

Find the **LCD** of all fractions in numerator and denominator

**LCD** is **2x**; figure out what to multiply each part by so that it becomes the LCD

Multiply

Write numerator and denominator as a single fraction

Divide numerator by denominator (multiply by reciprocal of denominator)

Simply

2) Simplify: 
$$\frac{\frac{5}{x+4}}{\frac{1}{x+4} + \frac{2}{x}}$$

The **LCD** of all the fractions in the numerator and denominator is  $x(x+4)$ .

Multiply numerator and denominator by the **LCD**

$$= \frac{\frac{5}{x+4} \left( \frac{x}{x} \right)}{\frac{1}{x+4} \left( \frac{x}{x} \right) + \frac{2}{x} \left( \frac{x+4}{x+4} \right)} = \frac{\frac{5x}{x(x+4)}}{\frac{x}{x(x+4)} + \frac{2x+8}{x(x+4)}} = \frac{\frac{5x}{x(x+4)}}{\frac{3x+8}{x(x+4)}}$$

$$= \frac{5x}{\cancel{x(x+4)}} \cdot \frac{\cancel{x(x+4)}}{3x+8} = \frac{5x}{3x+8}$$

# Homework

p. 587: 31-33

**SIMPLIFYING COMPLEX FRACTIONS** Simplify the complex fraction.

31. 
$$\frac{\frac{x}{3} - 6}{10 + \frac{4}{x}}$$

32. 
$$\frac{15 - \frac{2}{x}}{\frac{x}{5} + 4}$$

33. 
$$\frac{\frac{16}{x-2}}{\frac{4}{x+1} + \frac{6}{x}}$$

**STUDY FOR QUIZ TOMORROW!!!**