Objective

Students will be able to graph rational functions and identify appropriate values for domain, range, and asymptotes.

Midterm is on Thursday!

Rational functions

A rational function has the form

$$f(x) = \frac{p(x)}{q(x)}$$

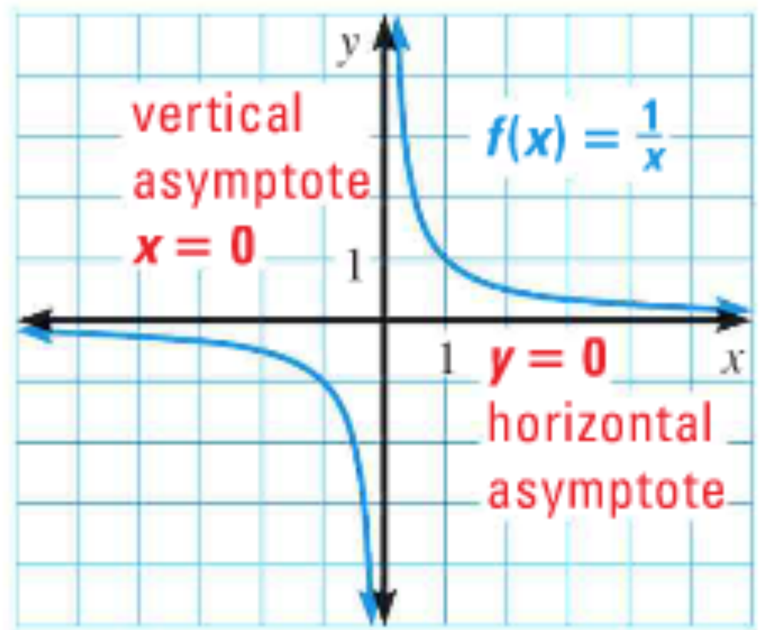
where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

The inverse variation function $f(x) = \frac{a}{x}$ is a rational function. The graph of this function when *a* = 1 is shown below.

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called *branches*. The domain and range are all nonzero real numbers.

Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$



Any function of the form $g(x) = \frac{a}{x}$ ($a \neq 0$) has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$

Graph the function $y = \dfrac{6}{x}$.

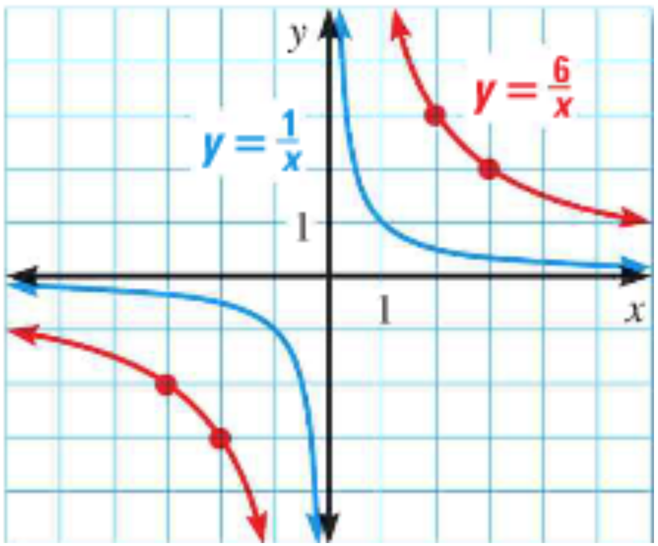
**When creating your table of values, find two points to the right and left of your vertical asymptote

Compare the graph with the graph of $y = \dfrac{1}{x}$.

Step 1: Draw the asymptotes of $x = 0$ and $y = 0$.

Step 2: Create a table of values. Plot points to the left and to the right of the vertical asymptote, such as (-3, -2), (-2, -3), (2, 3), and (3, 2).

Step 3: Draw the branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



The graph $y = 6 / x$ lies further from the axes than the graph of $y = 1 / x$. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

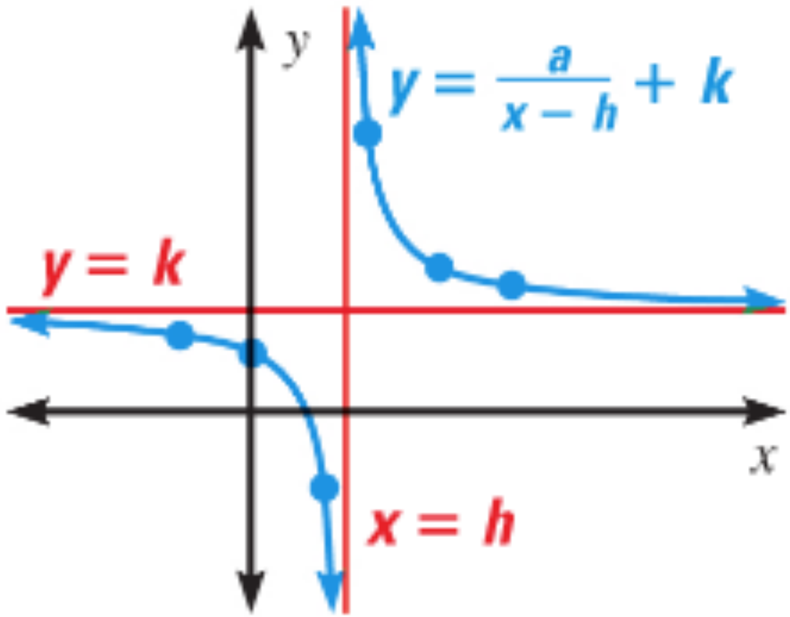
Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \dfrac{a}{x-h} + k$, follow these steps:

Step 1: Draw the asymptotes of $x = h$ and $y = k$.

Step 2: Create a table of values. Plot points to the left and to the right of the vertical asymptote.

Step 3: Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



**asymptotes should be drawn in with dotted lines!

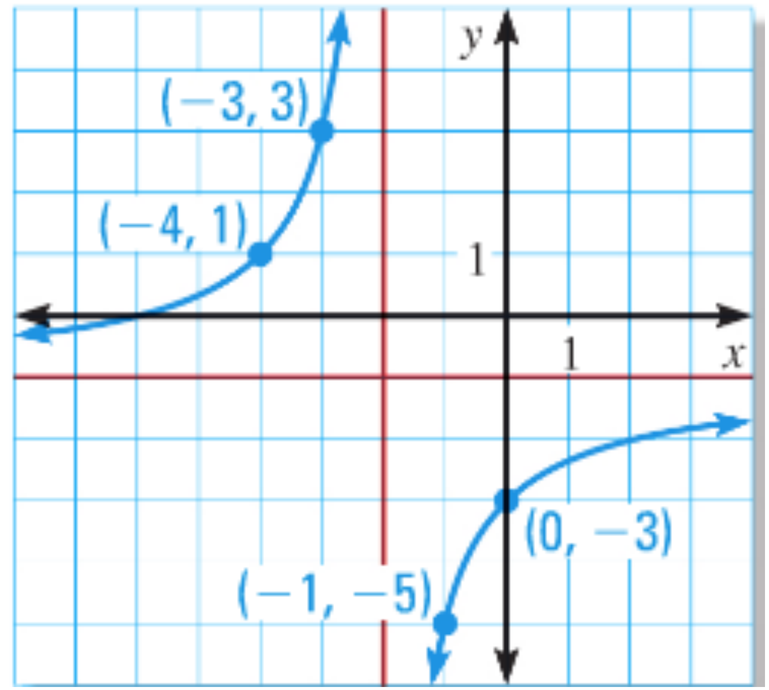
1) Graph the function $y = \dfrac{-4}{x+2} - 1$.

Find the asymptotes. State the domain and range.

Step 1: Draw the asymptotes of $x = -2$ and $y = -1$.

Step 2: Create a table of values. Plot points to the left of the vertical asymptote, such as (-3, 3) and (-4, 1), and to the right, such as (-1, -5), and (0, -3).

Step 3: Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



**asymptotes should be drawn in with dotted lines!

Domain: (-∞, -2)U(-2, ∞)

Range: (-∞, -1)U(-1, ∞)

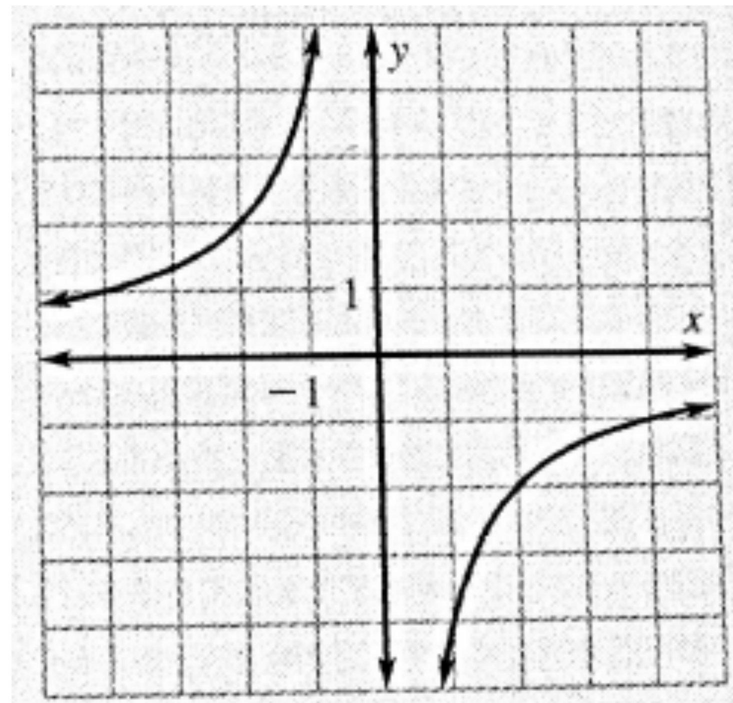
2) Graph the function $f(x) = \dfrac{-4}{x}$.

Find the asymptotes. State the domain and range.

Asymptotes: x = 0

y = 0

x	y
-2	2
-1	4
1	-4
2	-2



Domain: (-∞, 0)U(0, ∞)

Range: (-∞, 0)U(0, ∞)

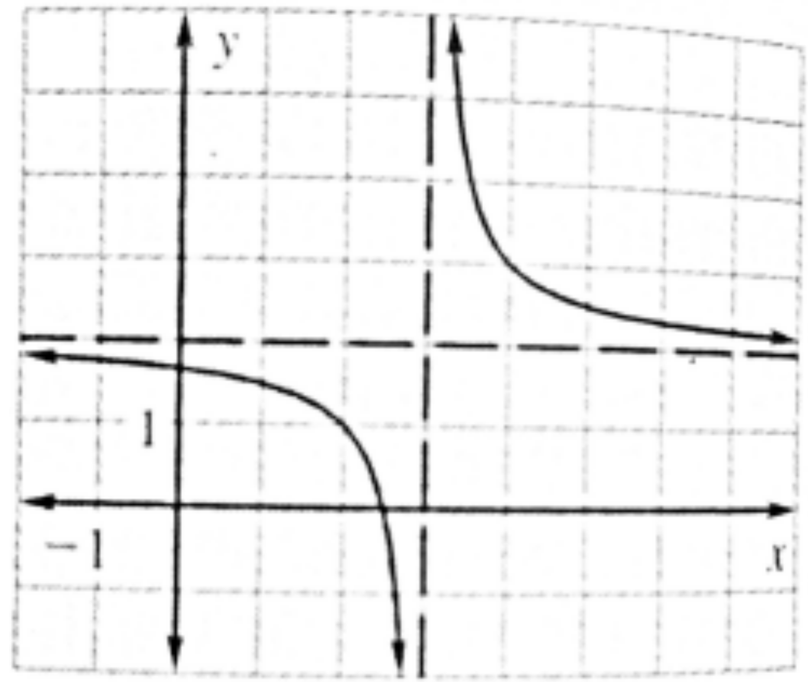
3) Graph the function $y = \dfrac{1}{x-3} + 2$.

Find the asymptotes. State the domain and range.

Asymptotes: x = 3

y = 2

x	y
0	5/3
2	1
4	3
7	9/4



Domain: (-∞, 3)U(3, ∞)

Range: (-∞, 2)U(2, ∞)

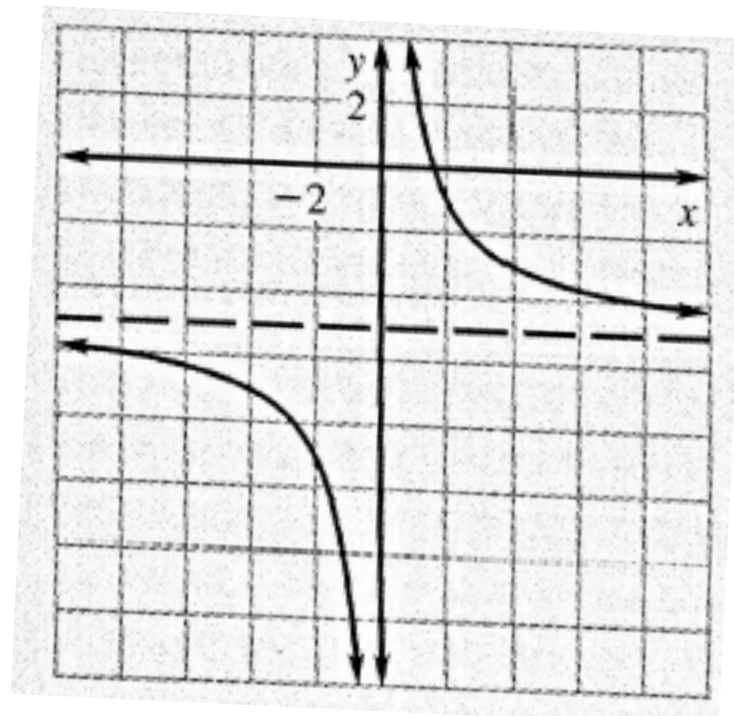
4) Graph the function $y = \frac{8}{x} - 5$.

Find the asymptotes. State the domain and range.

Asymptotes: x = 0

y = -5

x	y
-4	-7
-2	-9
1	3
2	-1



Domain: (-∞, 0)U(0, ∞)

Range: (-∞, -5)U(-5, ∞)

Homework p.561: 5, 11, 13, 15, 19, 23

GRAPHING FUNCTIONS **Graph the function. Compare the graph with the graph of $y = \frac{1}{x}$.**

5. $y = \frac{-5}{x}$

GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

**Find the asymptote as well

11. $y = \frac{4}{x} + 3$ **13.** $y = \frac{6}{x - 1}$ **15.** $y = \frac{-5}{x} - 7$ **19.** $y = \frac{-4}{x + 4} + 3$

23. ★ **MULTIPLE CHOICE** What are the asymptotes of the graph of $y = \frac{3}{x + 8} - 3$?

(A) $x = 8, y = 3$ (B) $x = 8, y = -3$ (C) $x = -8, y = 3$ (D) $x = -8, y = -3$

Objective

Students will be able to graph rational functions and identify appropriate values for domain, range, and asymptotes.

Midterm is on Thursday!

**Remember, I am in the math lab (341) after school today!

Other Rational Functions

All rational functions have the form $y = \dfrac{ax+b}{cx+d}$ also have graphs that are hyperbolas.

- The vertical asymptote of the graph is the line $x = -\dfrac{d}{c}$, because the function is undefined when the denominator *cx* + *d* is zero **set denominator equal to 0 and solve for x

    ***<u>same</u> as rational functions on Friday!

- The horizontal asymptote is the line $y = \dfrac{a}{c}$

    **dividing leading coefficients

    ***<u>different</u> from rational functions on Friday!

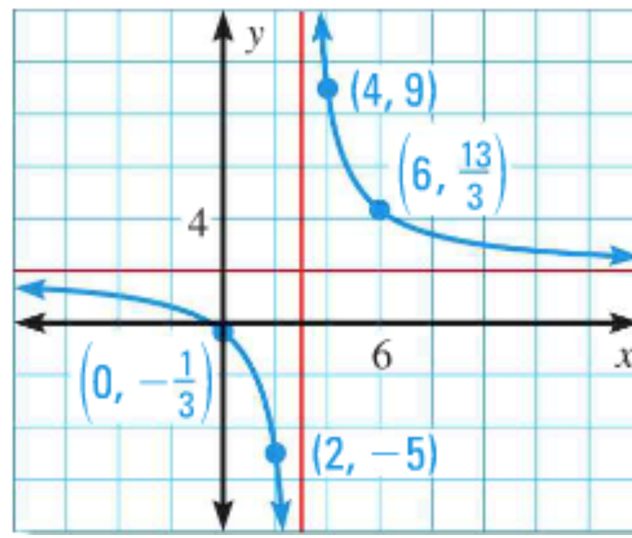
Graph the function $y = \dfrac{2x+1}{x-3}$.

**asymptotes should be drawn in with dotted lines!

Find asymptotes. State the domain and range.

Step 1: Find and draw the asymptotes. Solve $x - 3 = 0$ for x to find the vertical asymptote $x = 3$. The horizontal asymptote is the line $y = \frac{a}{c} = \frac{2}{1} = 2$

Step 2: Create your table of values. Plot points to the left of the vertical asymptote, such as (2, -5) and (0, -1/3), and to the right, such as (4, 9), and (6, 13/3).



Step 3: Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

Domain: (-∞, 3)U(3, ∞)
Range: (-∞, 2)U(2, ∞)

Graph the function $y = \dfrac{x-1}{x+3}$.

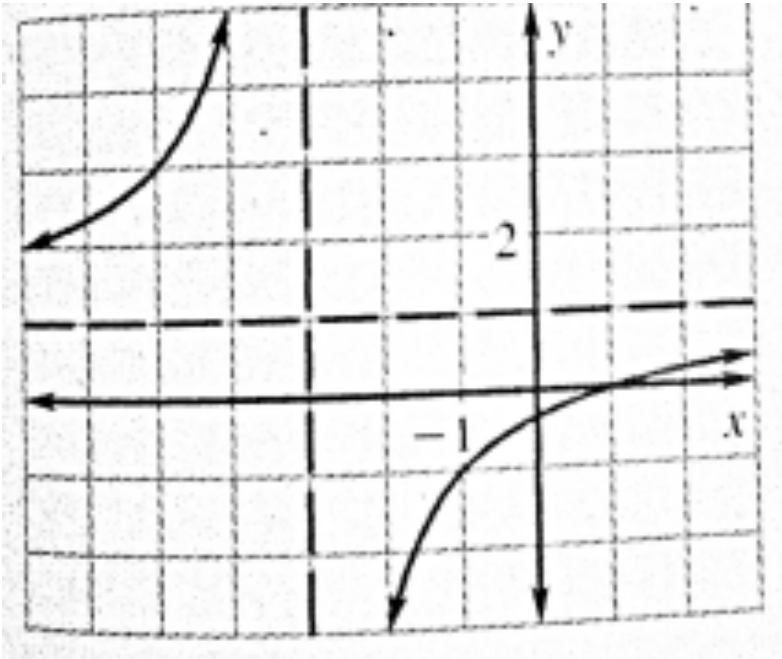
**When creating your table of values, find two points to the right and left of your vertical asymptote

Find asymptotes. State the domain and range.

Asymptotes: x = -3

y = 1

x	y
-6	7/3
-4	5
-2	-3
0	-1/3



Domain: (-∞, -3)U(-3, ∞)

Range: (-∞, 1)U(1, ∞)

Graph the function $y = \dfrac{2x+1}{4x-2}$.

Find asymptotes. State the domain and range.

Asymptotes: x = 1/2

y = 1/2

x	y
-1	1/6
0	-1/2
1	3/2
2	5/6

Domain: (-∞, 1/2)U(1/2, ∞)

Range: (-∞, 1/2)U(1/2, ∞)

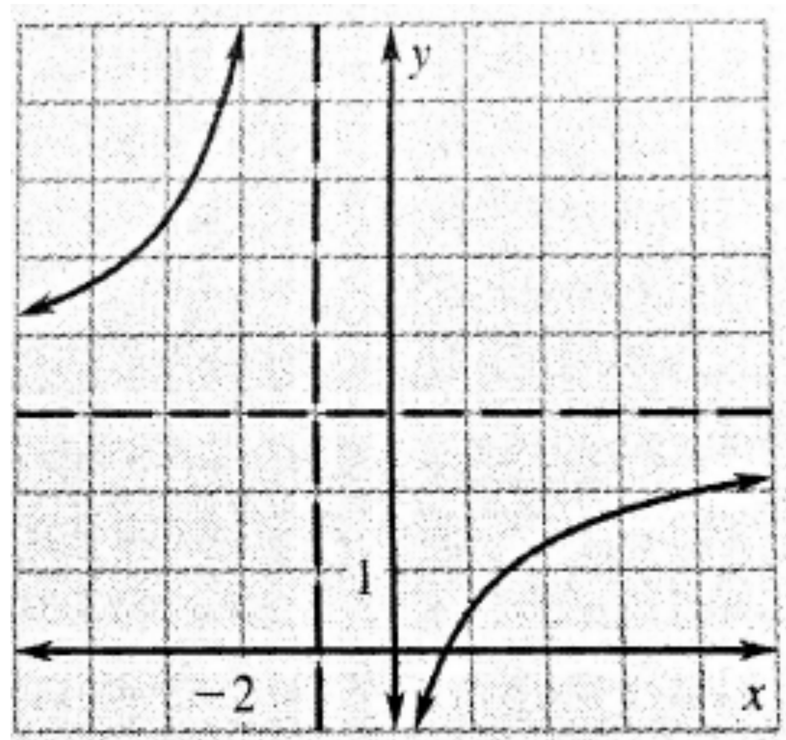
Graph the function $y = \dfrac{-3x+2}{-x-1}$.

Find asymptotes. State the domain and range.

Asymptotes: x = -1

y = 3

x	y
-4	14/3
-2	8
0	-2
2	4/3



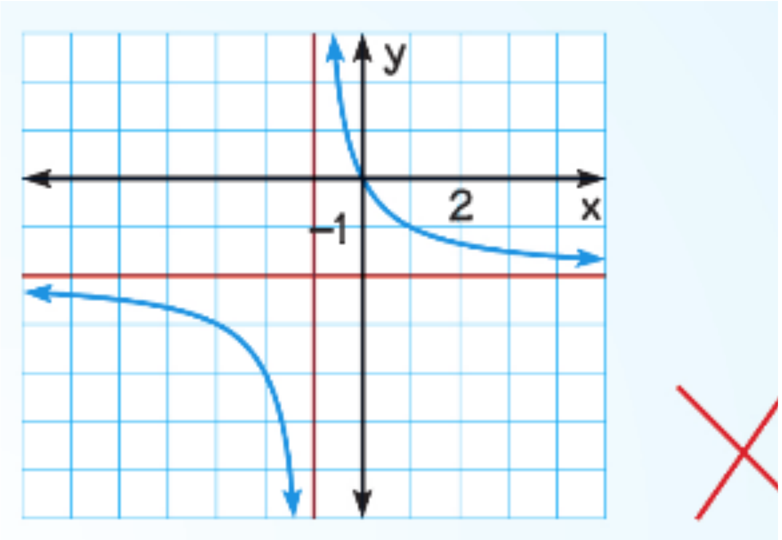
Domain: (-∞, -1)U(-1, ∞)

Range: (-∞, 3)U(3, ∞)

Homework p. 562: 26, 27, 28, 30, 34

ERROR ANALYSIS ***Describe*** **and correct the error in the graph.**

26. $y = \dfrac{2}{x-1} - 2$



GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

27. $y = \dfrac{x+4}{x-3}$

28. $y = \dfrac{x-1}{x+5}$

**Find the asymptote as well

30. $y = \dfrac{8x+3}{2x-6}$

34. $h(x) = \dfrac{5x+3}{-x+10}$

Objective

Students will be able to simplify and multiply rational expressions.

Midterm is on Thursday!

Refresher:

Factor the following problems:

1) $x^2 - 4x - 12$

$(x - 6)(x + 2)$

2) $7z^2 - 20z - 3$

$(7z + 1)(z - 3)$

3) $9y^2 - 64$

$(3y + 8)(3y - 8)$

4) $8x^3 - 125$

**Difference of cubes:
$a^3 - b^3 = (a - \mathrm{b})(a^2 + ab + b^2)$

$(2x - 5)(4x^2 + 10x + 25)$

Simplifying Rational Expressions

$$f(x) = \frac{\cancel{x}+4}{\cancel{x}+1}$$

$$= \frac{4}{1} = 4?$$

NO!!!

Simplify: $\dfrac{x^2 - 2x - 15}{x^2 - 9}$

$$= \frac{(x+3)(x-5)}{(x+3)(x-3)}$$

Factor numerator and denominator.

$$= \frac{\cancel{(x+3)}(x-5)}{\cancel{(x+3)}(x-3)}$$

Divide out common factor.

$$= \frac{x-5}{x-3}$$

Simplified form

Simplify the following expressions, if possible.

**The most simplified form is when everything is multiplied out; there should not be any parentheses left

1) $\dfrac{2(x+1)}{(x+1)(x+3)}$

$\dfrac{2}{x+3}$

2) $\dfrac{40x+20}{10x+30}$

$\dfrac{4x+2}{x+3}$

3) $\dfrac{4}{x(x+2)}$

$\dfrac{4}{x^2+2x}$

Simplify the following expressions, if possible.

4) $\dfrac{x+4}{x^2-16}$

$\dfrac{1}{x-4}$

5) $\dfrac{x^2-2x-3}{x^2-x-6}$

$\dfrac{x+1}{x+2}$

6) $\dfrac{2x^2+10x}{3x^2+16x+5}$

$\dfrac{2x}{3x+1}$

Multiply: $\dfrac{x+2}{x^3-27}\cdot(x^2+3x+9)$

$$=\frac{x+2}{x^3-27}\cdot\frac{x^2+3x+9}{1}$$

Write polynomial as a rational expression.

$$=\frac{(x+2)(x^2+3x+9)}{(x-3)(x^2+3x+9)}$$

Factor denominator.

$$=\frac{(x+2)\cancel{(x^2+3x+9)}}{(x-3)\cancel{(x^2+3x+9)}}$$

Divide out common factors.

$$=\frac{x+2}{x-3}$$

Simplified form

Multiply the following expressions. Simplify the result.

1) $\frac{3x^5y^2}{8xy} \cdot \frac{6xy^2}{9x^3y}$

$$\frac{x^2y^2}{4}$$

2) $\frac{2x^2-10}{x^2-25} \cdot \frac{x+3}{2x^2}$

$$\frac{x+3}{x^2+5x}$$

3) $\frac{x+5}{x^3-1} \cdot (x^2+x+1)$

$$\frac{x+5}{x-1}$$

Homework p.577: 13, 14, 15, 17, 25, 29-33 (odds)

SIMPLIFYING **Simplify the rational expression, if possible.**

13. $\dfrac{x^2 - 36}{x^2 + 12x + 36}$

14. $\dfrac{3x^3 + 6x^2 + 12x}{x^3 - 8}$

15. $\dfrac{8x^2 + 10x - 3}{6x^2 + 13x + 6}$

17. $\dfrac{x^3 - 5x^2 - 3x + 15}{x^2 - 8x + 15}$

MULTIPLYING **Multiply the expressions. Simplify the result.**

25. $\dfrac{48x^5y^3}{y^4} \cdot \dfrac{x^2y}{6x^3y^2}$

29. $\dfrac{x + 5}{4x - 16} \cdot \dfrac{2x^2 - 32}{x^2 - 25}$

31. $\dfrac{x^2 - 3x - 10}{x^2 - 2x - 15} \cdot (x^2 + 10x + 21)$

33. $\dfrac{4x^2 + 20x}{x^3 + 4x^2} \cdot (x^2 + 8x + 16)$

Objective

Students will be able to divide polynomial expressions.

Graphing, Multiplying, and Dividing Rational Expressions Quiz (8.2, 8.4) on Wednesday!

**NO graphing calculator

New hall passes

Dividing Rational Expressions

What can you do when you have a fraction and you divide it by another fraction?

To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression.

Let *a, b, c,* and *d* be expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Property	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$	**Simplify $\frac{ad}{bc}$ if possible.**
Examples	$\frac{2}{5} \div \frac{7}{3} = \frac{2}{5} \cdot \frac{3}{7} = \frac{6}{35}$	
	$\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)}$	

Divide: $\dfrac{7x}{2x-10} \div \dfrac{x^2-6x}{x^2-11x+30}$

$= \dfrac{7x}{2x-10} \cdot \dfrac{x^2-11x+30}{x^2-6x}$ Multiply by reciprocal.

$= \dfrac{7x}{2(x-5)} \cdot \dfrac{(x-5)(x-6)}{x(x-6)}$ Factor.

$= \dfrac{7\cancel{x}\cancel{(x-5)}\cancel{(x-6)}}{2\cancel{(x-5)}\cancel{(x)}\cancel{(x-6)}}$ Divide out common factors.

$= \dfrac{7}{2}$ Simplified form

Divide: $\dfrac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$

$= \dfrac{6x^2 + x - 15}{4x^2} \cdot \dfrac{1}{3x^2 + 5x}$ Multiply by reciprocal.

$= \dfrac{(3x+5)(2x-3)}{4x^2} \cdot \dfrac{1}{x(3x+5)}$ Factor.

$= \dfrac{\cancel{(3x+5)}(2x-3)}{4x^2(x)\cancel{(3x+5)}}$ Divide out common factors.

$= \dfrac{2x-3}{4x^3}$ Simplified form

Divide the following expressions. Simplify the result.

1) $$\frac{4x}{5x-20} \div \frac{x^2-2x}{x^2-6x+8}$$

$$\frac{4}{5}$$

2) $$\frac{2x^2+3x-5}{6x} \cdot (2x^2+5x)$$

$$\frac{x-1}{6x^2}$$

Homework p. 578: 35-43 (odds)

DIVIDING **Divide the expressions. Simplify the result.**

35. $\dfrac{8x^2y^2z}{xz^3} \div \dfrac{10xy}{x^4z}$

37. $\dfrac{8x^2}{x+4} \div \dfrac{x}{2(x-4)}$

39. $\dfrac{x^2-4x-5}{x+5} \div (x^2+6x+5)$

41. $\dfrac{x^2-x-2}{x^2+4x-5} \div \dfrac{x-2}{5x+25}$

43. $\dfrac{x^2+12x+32}{6x+42} \div \dfrac{x^2+4x}{x^2-49}$

Objective

Students will be able to use their knowledge on graphing, multiplying, and dividing rational expressions to study for their quiz tomorrow.

Graphing, Multiplying, and Dividing Rational Expressions Quiz (8.2, 8.4) <u>tomorrow</u>!

**NO graphing calculator

Graphing Rational Expressions Refresher

Two Forms:

$$y = \frac{a}{x-h} + k \qquad\qquad y = \frac{ax+b}{cx+d}$$

V.A: $x = h$ ($x - h = 0$)
H.A: $y = k$

V.A: $x = -d/c$ (c$x + d = 0$)
H.A: $y = a/c$ (L.Cs)

Domain: all real numbers except V.A; (-∞, V.A.)U(V.A., ∞)
Range: all real numbers except H. A; (-∞, H.A.)U(H.A., ∞)

<u>To graph</u>: Sketch and label asymptotes, create a table of values (you need to have at least two points on each side of the vertical asymptote), plot points and draw branches so they approach the asymptotes

Objective

Students will be able to add and subtract rational expressions.

There is a free PreCalculus class this summer at Wright College, if you earn a C or higher, you would be able to take AP Calculus next year. If you are interested, I can email you more information.

Adding or Subtracting with Like Denominators

To add (or subtract) rational expressions with *like* denominators, simply add (or subtract) their numerators. Then place the result over the common denominator.

Let *a*, *b*, and *c* be expressions with $c \neq 0$.

	Addition	**Subtraction**
Properties	$\dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$	$\dfrac{a}{c} - \dfrac{b}{c} = \dfrac{a-b}{c}$
Examples	$\dfrac{3x}{5x^2} + \dfrac{7}{5x^2} = \dfrac{3x+7}{5x^2}$	$\dfrac{9x^3}{x+1} - \dfrac{x^2}{x+1} = \dfrac{9x^3 - x^2}{x+1}$

Perform the indicated operation and simplify.

1) $\dfrac{7}{4x}+\dfrac{3}{4x}$ $=\dfrac{7+3}{4x}$ $=\dfrac{10}{4x}$ $=\dfrac{5}{2x}$

2) $\dfrac{2x}{x+6}-\dfrac{5}{x+6}$ $=\dfrac{2x-5}{x+6}$

Perform the indicated operation and simplify.

3) $\dfrac{7}{12x} - \dfrac{5}{12x}$ $= \dfrac{2}{12x}$ $= \dfrac{1}{6x}$

4) $\dfrac{2}{3x^2} + \dfrac{1}{3x^2}$ $= \dfrac{3}{3x^2}$ $= \dfrac{1}{x^2}$

Perform the indicated operation and simplify.

5) $$\frac{4x}{x-2} - \frac{x}{x-2} = \frac{3x}{x-2}$$

6) $$\frac{2x^2}{x^2+1} + \frac{2}{x^2+1} = \frac{2x^2+2}{x^2+1} = \frac{2(x^2+1)}{x^2+1}$$

$$= 2$$

What if we have unlike denominators?

Need to a common denominator;
find the least common multiple (LCM)!

Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

Step 1: **Factor** each polynomial. Write the numerical factors as a products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = 4(x - 2)(x + 2) = (2^2)(x - 2)(x + 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = 6(x - 2)(x - 2)$$
$$= (2)(3)(x - 2)^2$$

Step 2: **Form** the LCM by writing each factor to the highest power it occurs in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

Find the least common multiple of the polynomials.

1) $5x^3$ and $10x^2 - 15x$

$5x^3(2x - 3)$

2) $8x - 16$ and $12x^2 + 12x - 72$

$24(x - 2)(x + 3)$

Homework p. 586: 3-15 (odds)

LIKE DENOMINATORS **Perform the indicated operation and simplify.**

3. $\dfrac{15}{4x} + \dfrac{5}{4x}$ **5.** $\dfrac{9}{x+1} - \dfrac{2x}{x+1}$ **7.** $\dfrac{5x}{x+3} + \dfrac{15}{x+3}$

FINDING LCMS **Find the least common multiple of the polynomials.**

9. $3x$ and $3(x-2)$ **11.** $2x$ and $2x(x-5)$

13. $x^2 - 25$, x, and $x - 5$

15. ★ **MULTIPLE CHOICE** What is the least common multiple of the polynomials $3x^2 - 9x$ and $6x^2$?

Ⓐ $3x(x-3)$ Ⓑ $6x^2$ Ⓒ $6x(x-3)$ Ⓓ $6x^2(x-3)$

Objective

Students will be able to add and subtract rational expressions.

Add and Subtract Rational Expressions (8.5) Quiz on Friday!!!!

Homework Quiz tomorrow!

There is a free PreCalculus class this summer at Wright College, if you earn a C or higher, you would be able to take AP Calculus next year. If you are interested, I can email you more information.

Adding or Subtracting with Unlike Denominators

To add (or subtract) rational expressions with unlike denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add (or subtract).

Let *a*, *b*, *c*, and *d* be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

Subtraction

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

You can always find a common denominator of two rational expressions by multiplying the denominators, as shown above. However, if you use the **LCM** (LCD), you may have to do less simplifying.

Add: $\dfrac{7}{9x^2}+\dfrac{x}{3x^2+3x}$

$9x^2 = 3^2x^2$

Find the LCM (LCD) of denominators: $3x^2 + 3x = 3x(x+1)$

LCD: $3^2x^2(x+1) = 9x^2(x+1)$

$$=\frac{7}{9x^2}+\frac{x}{3x(x+1)}$$

Factor denominators

$$=\frac{7}{9x^2}\cdot\frac{x+1}{x+1}+\frac{x}{3x(x+1)}\cdot\frac{3x}{3x}$$

LCD is $9x^2(x+1)$; figure out what to multiply each part by so that it becomes the LCD

$$=\frac{7x+7}{9x^2(x+1)}+\frac{3x^2}{9x^2(x+1)}$$

Multiply

$$=\frac{3x^2+7x+7}{9x^2(x+1)}=\frac{3x^2+7x+7}{9x^3+9x^2}$$

Add numerators

Factor numerator out to see if anything cancels out

Subtract: $\dfrac{x+2}{2x-2}-\dfrac{-2x-1}{x^2-4x+3}$

$$=\frac{x+2}{2(x-1)}-\frac{-2x-1}{(x-1)(x-3)}$$

Factor denominators

$$=\frac{x+2}{2(x-1)}\cdot\frac{x-3}{x-3}-\frac{-2x-1}{(x-1)(x-3)}\cdot\frac{2}{2}$$

LCD is 2(x – 1)(x – 3)

$$=\frac{x^2-x-6}{2(x-1)(x-3)}-\frac{-4x-2}{2(x-1)(x-3)}$$

Multiply

$$=\frac{x^2-x-6-(-4x-2)}{2(x-1)(x-3)}$$

Subtract numerators

$$=\frac{x^2+3x-4}{2(x-1)(x-3)}$$

Simplify numerators

$$=\frac{\cancel{(x-1)}(x+4)}{2\cancel{(x-1)}(x-3)}=\frac{x+4}{2(x-3)}=\frac{x+4}{2x-6}$$

Factor numerator

Simplify

Homework p. 586: 17-27 (odds)

UNLIKE DENOMINATORS **Perform the indicated operation and simplify.**

17. $\dfrac{8}{3x^2} - \dfrac{5}{4x}$

19. $\dfrac{12}{x^2 + 5x - 24} + \dfrac{3}{x - 3}$

21. $\dfrac{9}{x - 3} + \dfrac{2x}{x + 1}$

23. $\dfrac{-15x}{x^2 - 8x + 16} + \dfrac{12}{x - 4}$

25. ERROR ANALYSIS *Describe* and correct the error in adding the rational expressions.

$$\frac{x}{x + 2} + \frac{4}{x - 5} = \frac{x + 4}{(x + 2)(x - 5)}$$

UNLIKE DENOMINATORS **Perform the indicated operation(s) and simplify.**

27. $\dfrac{x}{x^2 - 9} + \dfrac{x + 1}{x^2 + 6x + 9}$

Objective

Students will be able to add and subtract rational expressions.

Add and Subtract Rational Expressions (8.5) Quiz tomorrow!!!!

Rational Functions (Chapter 8) Test on Thursday, May 4th

I send each of you an email about your AP exams. Make sure you fill it out! It will take 2 minutes!

Simplifying Complex Fractions

A <u>complex fraction</u> is a fraction that contains a fraction in its numerator or denominator.

Example: $$\frac{\frac{4}{9}}{\frac{9}{25}+\frac{5}{9}} = \frac{\frac{4}{9}\cdot\frac{25}{25}}{\frac{9}{25}\cdot\frac{9}{9}+\frac{5}{9}\cdot\frac{25}{25}}$$

$$= \frac{\frac{100}{225}}{\frac{81}{225}+\frac{125}{225}} = \frac{\frac{100}{225}}{\frac{206}{225}} = \frac{100}{225}\cdot\frac{225}{206} = \frac{100}{206} = \frac{50}{103}$$

1) Simplify: $\dfrac{\dfrac{4}{x}-3}{1+\dfrac{x}{2}}$

Find the LCD of all fractions in numerator and denominator

LCD is 2x; figure out what to multiply each part by so that it becomes the LCD

$$=\frac{\frac{4}{x}\left(\frac{2}{2}\right)-3\left(\frac{2x}{2x}\right)}{1\left(\frac{2x}{2x}\right)+\frac{x}{2}\left(\frac{x}{x}\right)}$$

Multiply

$$=\frac{\frac{8}{2x}-\frac{6x}{2x}}{\frac{2x}{2x}+\frac{x^2}{2x}}$$

Write numerator and denominator as a single fraction

$$=\frac{\frac{8-6x}{2x}}{\frac{2x+x^2}{2x}}$$

Divide numerator by denominator (multiply by reciprocal of denominator)

$$=\frac{8-6x}{\cancel{2x}}\cdot\frac{\cancel{2x}}{2x+x^2}$$

Simply

$$=\frac{8-6x}{2x+x^2}$$

2) Simplify: $$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}}$$

The LCD of all the fractions in the numerator and denominator is $x(x+4)$.

Multiply numerator and denominator by the LCD

$$=\frac{\frac{5}{x+4}\left(\frac{x}{x}\right)}{\frac{1}{x+4}\left(\frac{x}{x}\right)+\frac{2}{x}\left(\frac{x+4}{x+4}\right)}=\frac{\frac{5x}{x(x+4)}}{\frac{x}{x(x+4)}+\frac{2x+8}{x(x+4)}}=\frac{\frac{5x}{x(x+4)}}{\frac{3x+8}{x(x+4)}}$$

$$=\frac{5x}{\cancel{x(x+4)}}\cdot\frac{\cancel{x(x+4)}}{3x+8}=\frac{5x}{3x+8}$$

Homework

p. 587: 31-33

SIMPLIFYING COMPLEX FRACTIONS **Simplify the complex fraction.**

31. $\dfrac{\frac{x}{3} - 6}{10 + \frac{4}{x}}$

32. $\dfrac{15 - \frac{2}{x}}{\frac{x}{5} + 4}$

33. $\dfrac{\frac{16}{x - 2}}{\frac{4}{x + 1} + \frac{6}{x}}$

STUDY FOR QUIZ TOMORROW!!!

Objective

Students will be able to solve rational expressions.

Rational Functions (Chapter 8) Test on Thursday, May 4th

If you have an AP test that day, you can either still take it that day, or you can take it on Friday during your lunch.

Refresher

Solve for x.

1) $\frac{x}{4} = \frac{3}{2}$

x = 6

2) $\frac{3}{24} = \frac{2}{x^2}$

x = $\pm$ 4

You can use <u>cross multiplying</u> to solve a rational equation when each side of the equation is a single rational expression.

Solve: $\dfrac{3}{x+1} = \dfrac{9}{4x+5}$

$$3(4x+5) = 9(x+1)$$ Cross Multiply

$$12x+15 = 9x+9$$ Distribute

$$3x+15 = 9$$ Subtract 9x from each side

$$3x = -6$$ Subtract 15 from each side

$$x = -2$$ Divide each side by 3

**Remember to always check your solution!

When a rational expression is not expressed as a proportion, you can solve it by multiplying each side of the equation by the least common denominator of each rational expression.

Solve: $$1-\frac{8}{x-5}=\frac{3}{x}$$

$$1\left(\frac{x(x-5)}{x(x-5)}\right)-\frac{8}{x-5}\left(\frac{x}{x}\right)=\frac{3}{x}\left(\frac{x-5}{x-5}\right)$$

Multiply each side by LCD, x(x – 5)

$$\frac{x^2-5x}{x(x-5)}-\frac{8x}{x(x-5)}=\frac{3x-15}{x(x-5)}$$

Multiply

$$\frac{x^2-13x}{x(x-5)}=\frac{3x-15}{x(x-5)}$$

Simplify

Cross Multiply

$$(x^2-13x)\cancel{(x(x-5))}=(3x-15)\cancel{(x(x-5))}$$

Simplify

$$x^2-13x=3x-15$$

Write in standard form

$$x^2-16x+15=0$$

Factor

$$(x-1)(x-15)=0$$

Zero product property

$x=1$ or $x=15$

So, once you multiply by your LCD, you do not need to worry about the denominators!

Again solve: $$1-\frac{8}{x-5}=\frac{3}{x}$$

$$1\left(\frac{x(x-5)}{x(x-5)}\right)-\frac{8}{x-5}\left(\frac{x}{x}\right)=\frac{3}{x}\left(\frac{x-5}{x-5}\right)$$
Multiply each side by LCD, x(x – 5)

$$x^2-5x-(8x)=3x-15$$
Multiply

$$x^2-16x+15=0$$
Simplify/ Write in standard form

$$(x-1)(x-15)=0$$
Factor

$x=1$ or $x=15$
Zero product property

**Remember to always check your solutions!

Solve: $$\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$$

$$\frac{6}{x-3} = \frac{8x^2}{(x-3)(x+3)} - \frac{4x}{x+3}$$

Write each denominator in factored form

LCD: (x + 3)(x – 3)

$$\frac{6}{x-3}\left(\frac{x+3}{x+3}\right) = \frac{8x^2}{(x-3)(x+3)} - \frac{4x}{x+3}\left(\frac{x-3}{x-3}\right)$$

$$6(x+3) = 8x^2 - (4x(x-3))$$

$$6x+18 = 8x^2 - 4x^2 + 12x$$

$$0 = 4x^2 + 6x - 18$$

$$0 = 2(2x^2 + 3x - 9)$$

$$0 = 2(2x-3)(x+3)$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -3$$

**Remember to always check your solutions!

CHECK!

$$\frac{6}{x-3}=\frac{8x^2}{x^2-9}-\frac{4x}{x+3}$$

$$x=\frac{3}{2} \quad \text{or} \quad x=-3$$

$$\frac{6}{\frac{3}{2}-3}=\frac{8\left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2-9}-\frac{4\left(\frac{3}{2}\right)}{\frac{3}{2}+3}$$

$$\frac{6}{\frac{3}{2}-\frac{6}{2}}=\frac{8\left(\frac{9}{4}\right)}{\frac{9}{4}-\frac{36}{4}}-\frac{4\left(\frac{3}{2}\right)}{\frac{3}{2}+\frac{6}{2}}$$

$$\frac{6}{-\frac{3}{2}}=\frac{18}{-\frac{27}{4}}-\frac{6}{\frac{9}{2}} \qquad -\frac{12}{3}=-\frac{72}{27}-\frac{12}{9} \qquad -4=-\frac{12}{3}$$

$$-4=-\frac{8}{3}-\frac{4}{3} \qquad -4=-4 \; ✔$$

$$\frac{6}{-3-3}=\frac{8(-3)^2}{(-3)^2-9}-\frac{4(-3)}{-3+3}$$

$$\frac{6}{-6}=\frac{8(9)}{9-9}-\frac{-12}{0}$$

undefined

-3 is an extraneous solution

Homework

p. 593: 5-11 (odds), 15, 17, 20

CROSS MULTIPLYING **Solve the equation by cross multiplying. Check for extraneous solutions.**

5. $\frac{9}{3x} = \frac{4}{x+2}$

7. $\frac{8}{3x-2} = \frac{2}{x-1}$

9. $\frac{x-3}{x+5} = \frac{x}{x+2}$

11. $\frac{4(x-4)}{x^2+2x-8} = \frac{4}{x+4}$

LEAST COMMON DENOMINATOR **Solve the equation by using the LCD. Check for extraneous solutions.**

15. $\frac{2}{3x} + \frac{1}{6} = \frac{4}{3x}$

17. $\frac{1}{2x} + \frac{3}{x+7} = \frac{-1}{x}$

20. $\frac{x+1}{x+6} + \frac{1}{x} = \frac{2x+1}{x+6}$