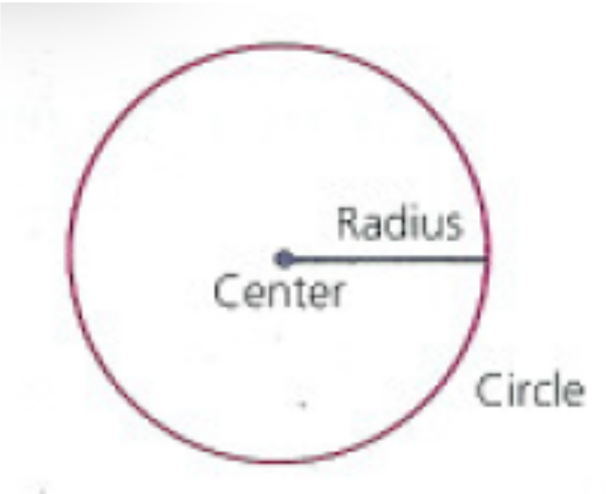
Objective

Students will be able to identify the characteristics of circles, recognize chords and diameters of circles, and recognize special relationships between radii and chords.

FINAL DRAFT OF DATA IS DUE FRIDAY!!!!

Circles

A circle is the set of all points in a plane that are a given distance from a given point in the plane. The given point is the center of the circle, and the given distance is the radius. A segment that joins the center to a point on the circle is also called a radius.

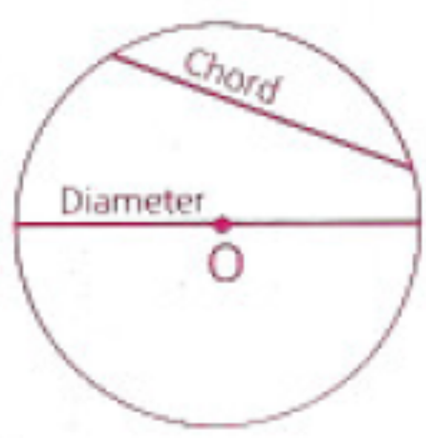


Chords and Diameters

Points on a circle can be connected by segments called chords.

A chord of a circle is a segment joining any two points on the circle.

A diameter of a circle is a chord that passes through the center of the circle.



Circumference and Area of a Circle

Area of a circle:

$$A = \pi r^2$$

Circumference (perimeter) of a circle:

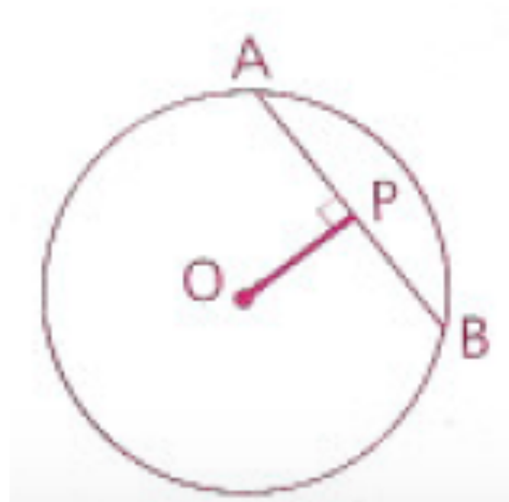
$$C = \pi d$$

where *r* is the circle's radius, *d* is its diameter, and $\pi \approx 3.14$

Radius-Chord Relationships

OP is the distance from O to chord $\overline{AB}$.

The distance from the center of a circle to a chord is the measure of the perpendicular segment from the center to the chord.

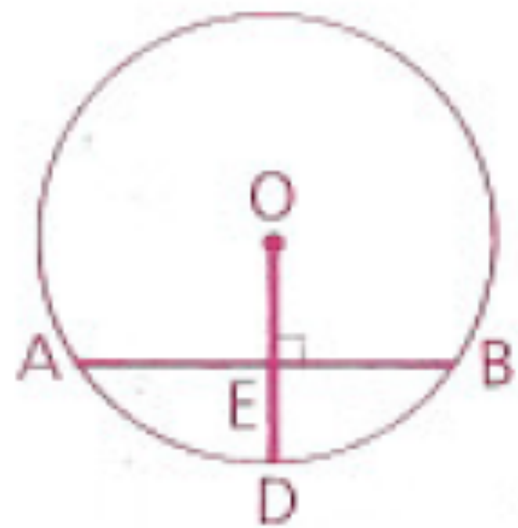


Radius-Chord Relationships

If a radius is perpendicular to a chord, then it bisects the chord.

Given: $\odot O$,
$\overline{OD} \perp \overline{AB}$

Prove: $\overline{OD}$ bisects $\overline{AB}$.

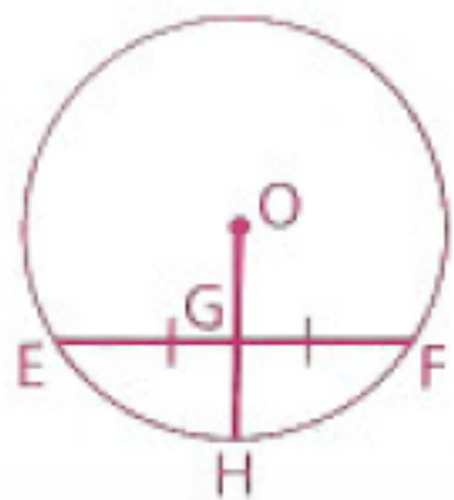


Radius-Chord Relationships

If a radius of a circle bisects a chord that is not a diameter, then it is perpendicular to that chord.

Given: $\odot O$;
$\overline{OH}$ bisects $\overline{EF}$.

Prove: $\overline{OH} \perp \overline{EF}$

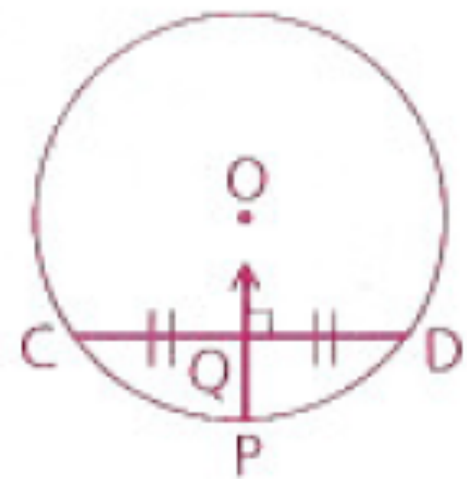


Radius-Chord Relationships

The perpendicular bisector of a chord passes through the center of the circle.

Given: $\overleftrightarrow{PQ}$ is the $\perp$ bisector of $\overline{CD}$.

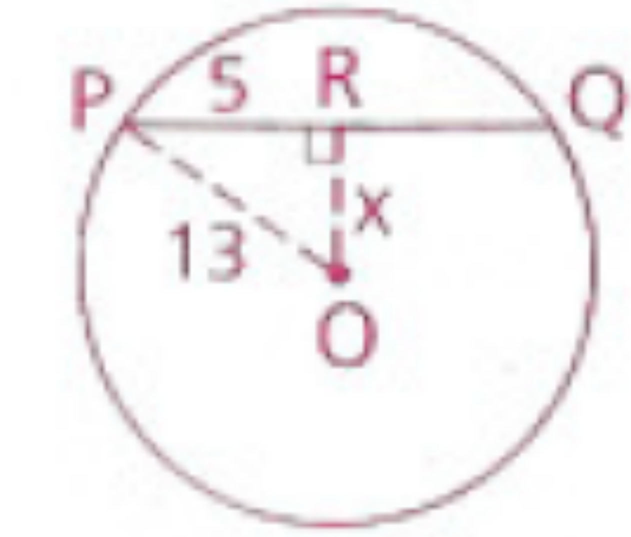
Prove: $\overleftrightarrow{PQ}$ passes through O.



Example:

The radius of a circle O is 13 mm. The length of chord $\overline{PQ}$ is 10 mm. Find the distance from chord $\overline{PQ}$ to the center, O.

Draw $\overline{OR}$ perpendicular to $\overline{PQ}$. Draw radius $\overline{OP}$ to complete a right triangle.



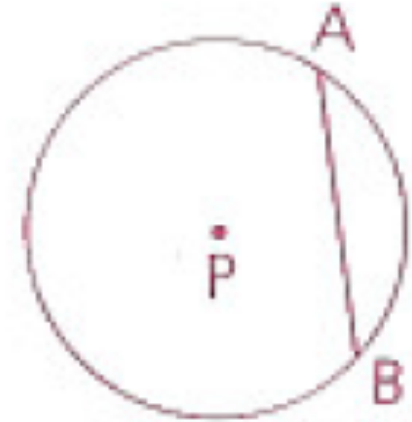
Since a radius perpendicular to a chord bisects the chord, PR = ½(PQ) = ½(10) = 5

By the Pythagorean Theorem, $x^2 + 5^2 = 13^2$, so OR = 12

Homework

p. 443: 5, 6, 11, 14, 17

5 Chord $\overline{AB}$ measures 12 mm and the radius of $\odot P$ is 10 mm. Find the distance from $\overline{AB}$ to P.



6 Find the length of a chord that is 15 cm from the center of a circle with a radius of 17 cm.

11 Find the distance from the center of a circle to a chord 30 m long if the diameter of the circle is 34 m.

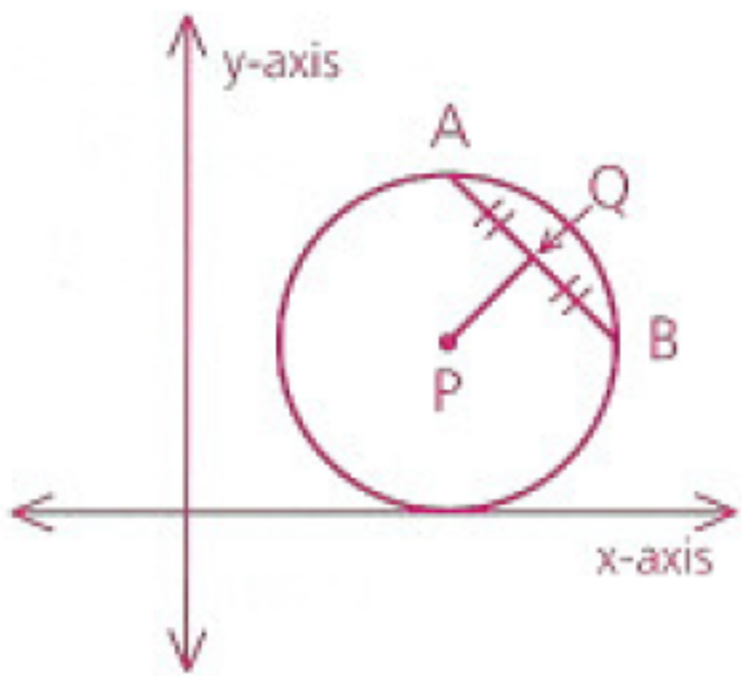
14 Two circles intersect and have a common chord 24 cm long. The centers of the circles are 21 cm apart. The radius of one circle is 13 cm. Find the radius of the other circle.

17 $\odot$P just touches (is tangent to) the x-axis. P = (15, 13) and Q = (19, 16).

a Find the radius of $\odot$P.

b Find PQ.

c Find the length of $\overline{AB}$.

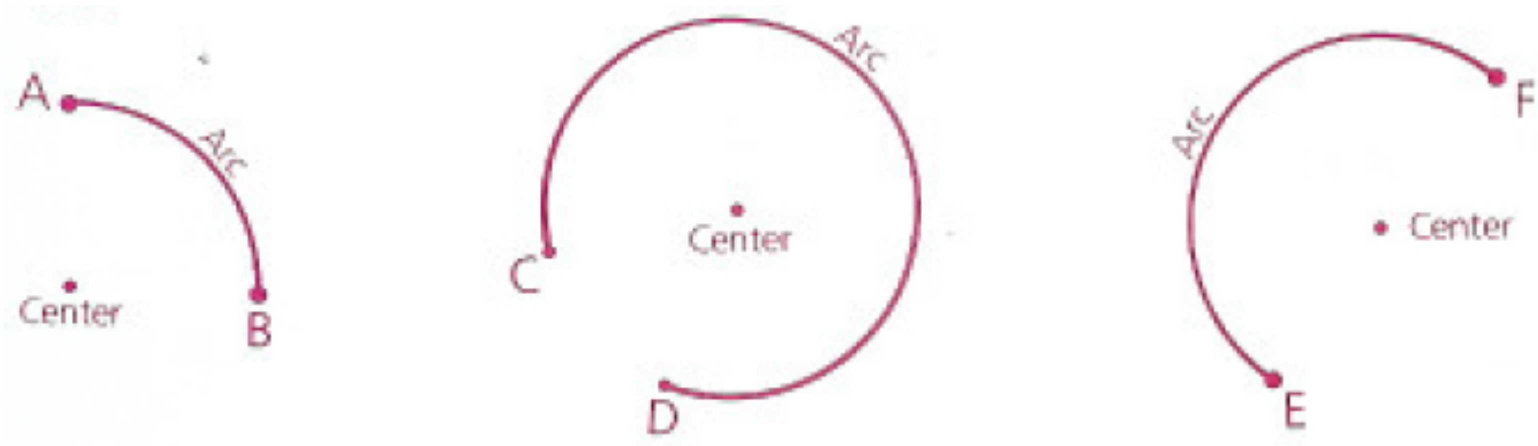


Objective

Students will be able to identify the different types of arcs, determine the measure of an arc, and recognize congruent arcs.

FINAL DRAFT OF DATA IS DUE FRIDAY!!!!

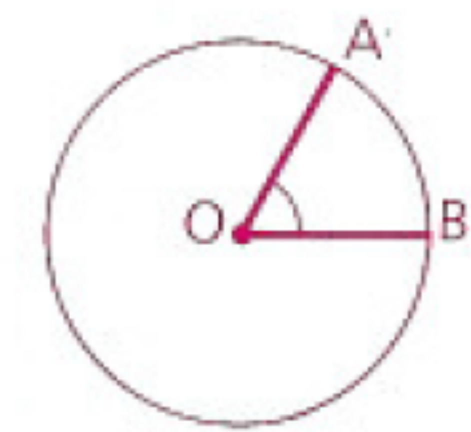
Types of Arcs



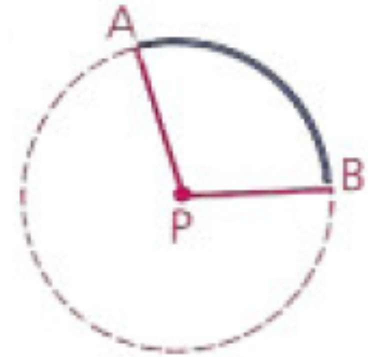
An arc consists of two points on a circle and all points on the circle needed to connect the points by a single path.

The center of an arc is the center of the circle of which the arc is a part.

A central angle is an angle whose vertex is at the center of a circle

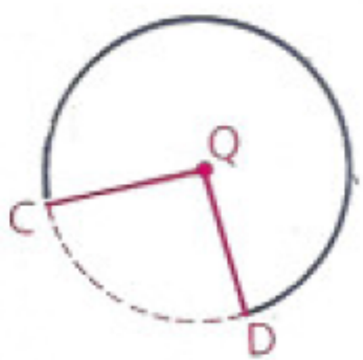


Types of Arcs



A <u>minor arc</u> is an arc whose points are on or between the sides of a central angle.

Central angle APB determines minor arc AB.



A <u>major arc</u> is an arc whose points are on or outside the sides of a central angle.

Central angle CQD determines major arc CD.

A <u>semicircle</u> is an arc whose endpoints are the endpoints of a diameter.

Arc EF is a semicircle

Types of Arcs

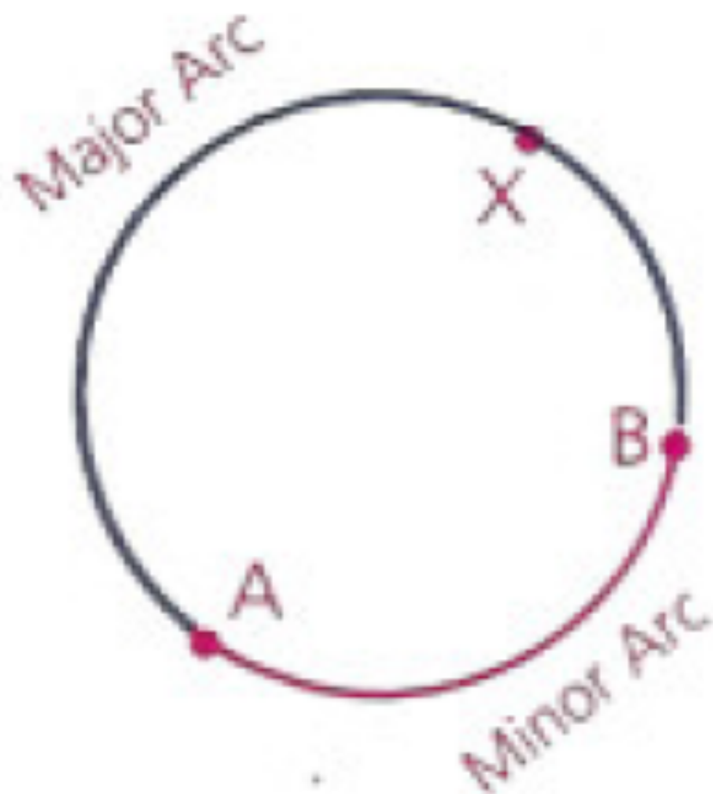
The symbol $\frown$ is used to label arcs.

Minor arc joining A to B?

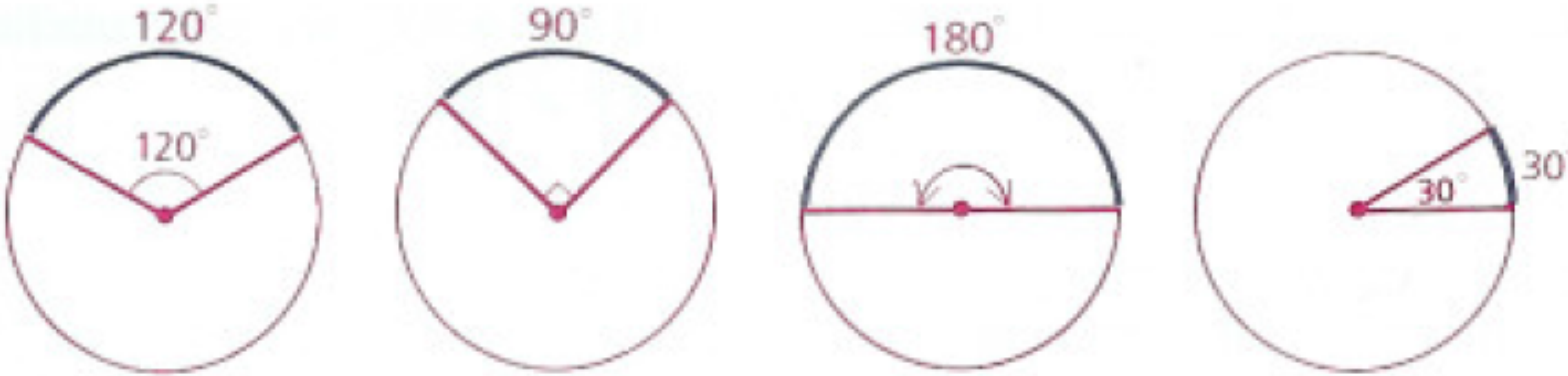
$\overset{\frown}{AB}$

Major arc joining A to B?

$\overset{\frown}{AXB}$



The Measure of an Arc

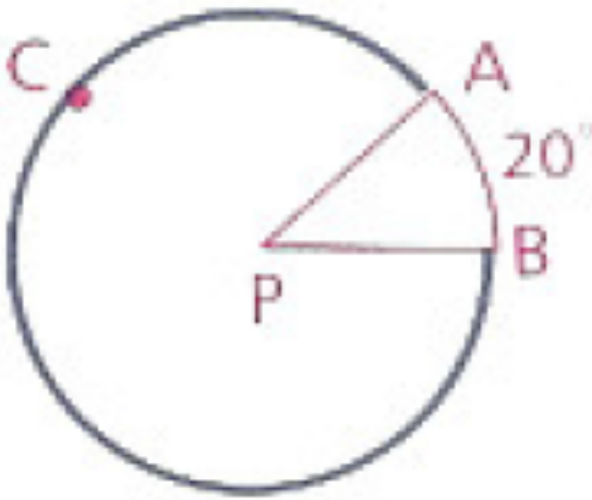


The <u>measure of a minor arc</u> or a semicircle is the same as the measure of the central angle that intercepts the arc.

The <u>measure of a major arc</u> is 360 minus the measure of the minor arc with the same endpoints.

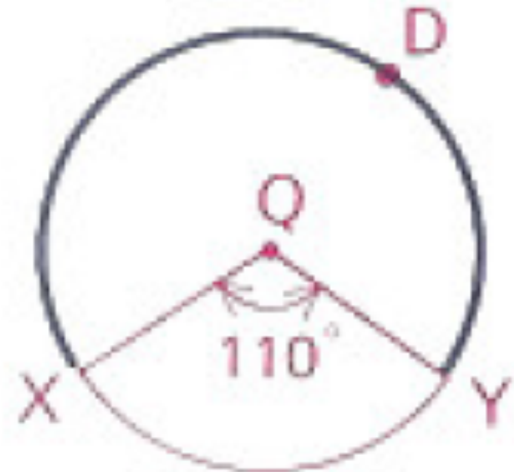
The Measure of an Arc

a Given: $m\overparen{AB} = 20$
Find: $m\overparen{ACB}$



$$m\overparen{ACB} = 360 - 20 = 340$$

b Given: $m\angle XQY = 110$
Find: $m\overparen{XDY}$

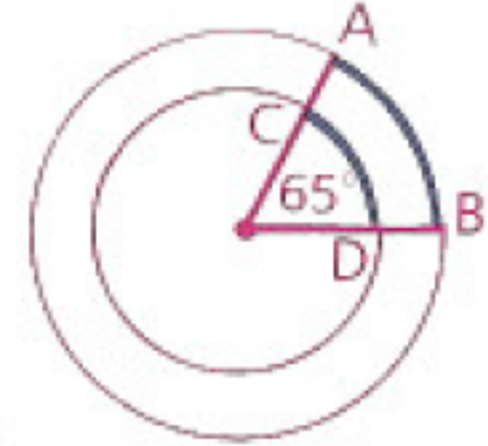


$$m\overparen{XY} = m\angle XQY = 110$$

Therefore, $m\overparen{XDY} = 360 - 110 = 250$

Congruent Arcs

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\widehat{AB} = 65$ and $m\widehat{CD} = 65$, but $\widehat{AB}$ and $\widehat{CD}$ are *not* congruent. Under what conditions, do you think, will two arcs be congruent?



**Two or more coplanar circles with the same center are called <u>concentric</u> circles.

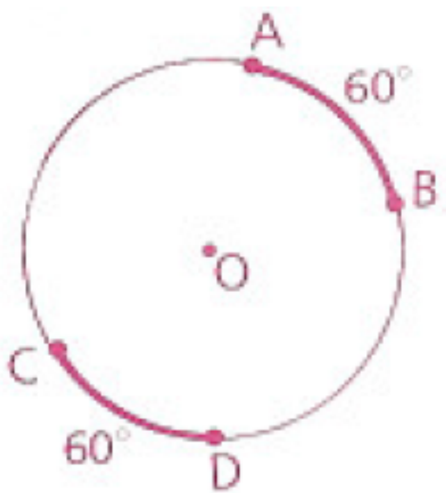
Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.

Congruent Arcs

Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.

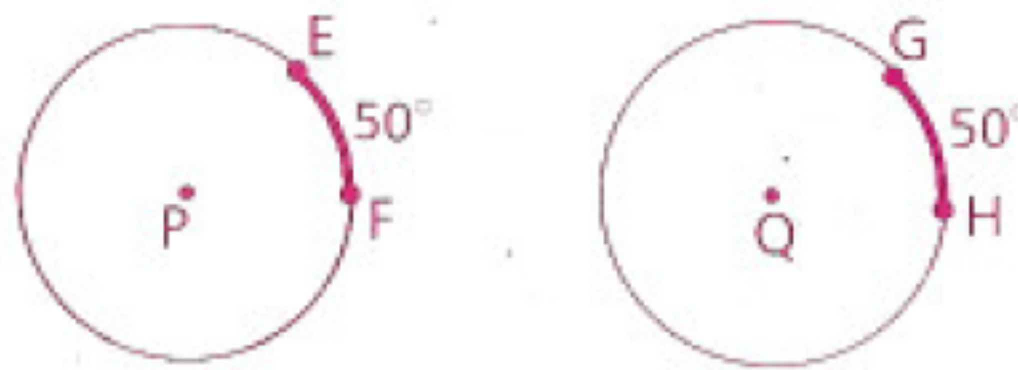
What can we conclude?

1)



We may conclude that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

2)

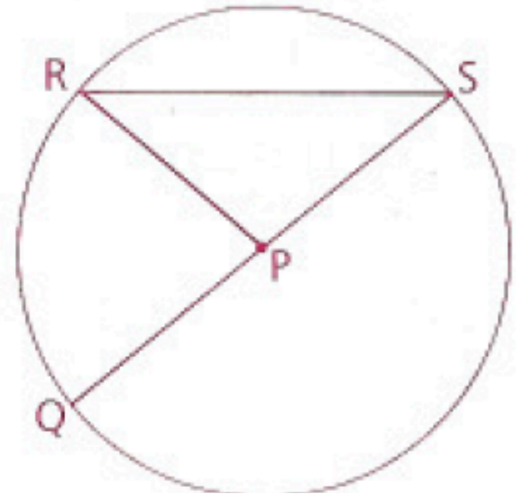


If $\odot P \cong \odot Q$, we may conclude that $\overset{\frown}{EF} \cong \overset{\frown}{GH}$.

Homework p. 454: 1 - 4

1 Match each item in the left column with the correct term in the right column.

a $\overset{\frown}{QRS}$	1 Radius
b $\overline{QS}$	2 Diameter
c $\overset{\frown}{RQS}$	3 Chord
d $\overset{\frown}{RS}$	4 Minor arc
e $\overline{RS}$	5 Major arc
f $\angle RPQ$	6 Semicircle
g $\overline{PS}$	7 Central angle



2 Given: Two concentric circles with center O;
$\angle BOC$ is acute.

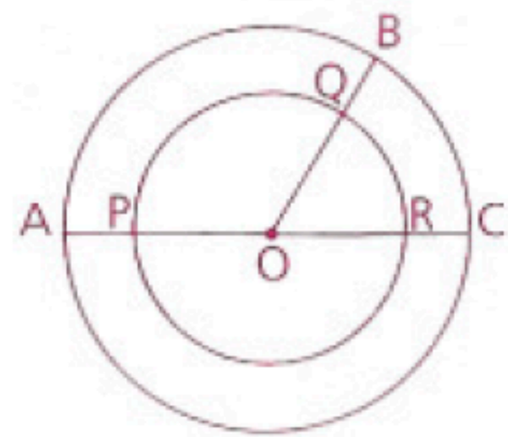
a Name a major arc of the smaller circle.

b Name a minor arc of the larger circle.

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$?

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$?

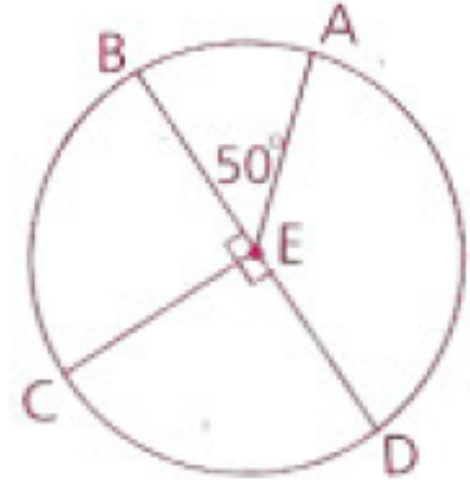
e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?



3 In circle E, find each of the following.

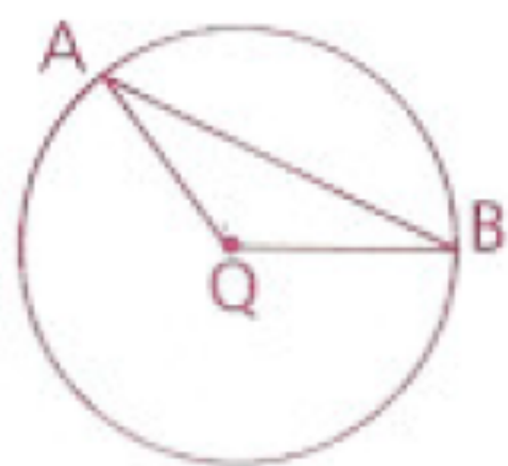
a $m\widehat{BC}$ **c** $m\widehat{ACD}$ **e** $m\widehat{ADC}$

b $m\widehat{AD}$ **d** $m\widehat{BAD}$



4 Given: $\odot Q$, $\angle A = 25°$

Find: $m\widehat{AB}$



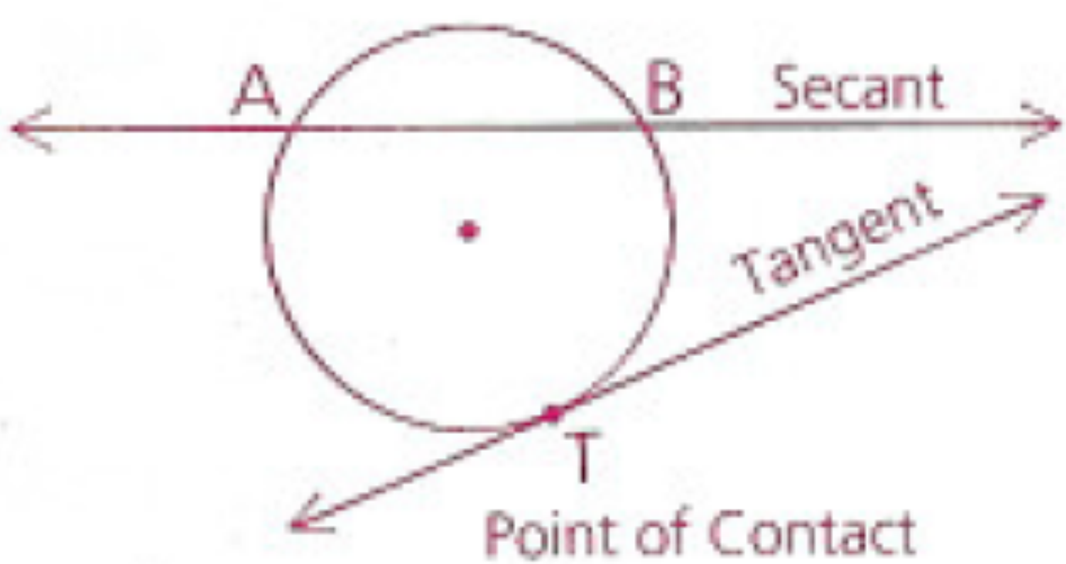
Objective

Students will be able to identify secant and tangent lines and segments, distinguish between two types of tangent circles, and recognize common internal and common external tangents.

FINAL DRAFT OF DATA IS DUE TOMORROW!!!!

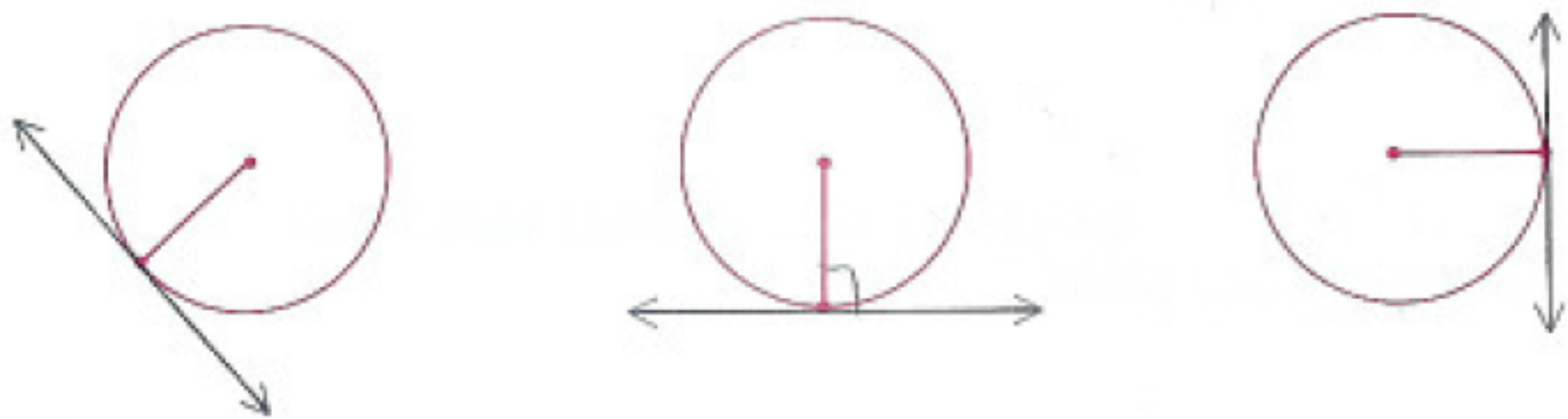
Secant and Tangent Lines

A secant is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)



Secant and Tangent Lines

A <u>tangent</u> is a line that intersects a circle at exactly one point. This point is called the <u>point of tangency</u> or <u>point of contact</u>.

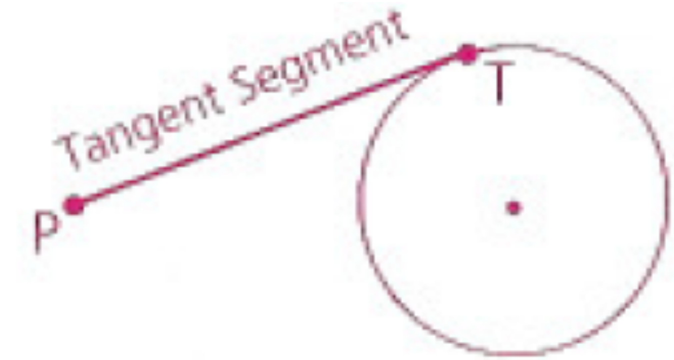


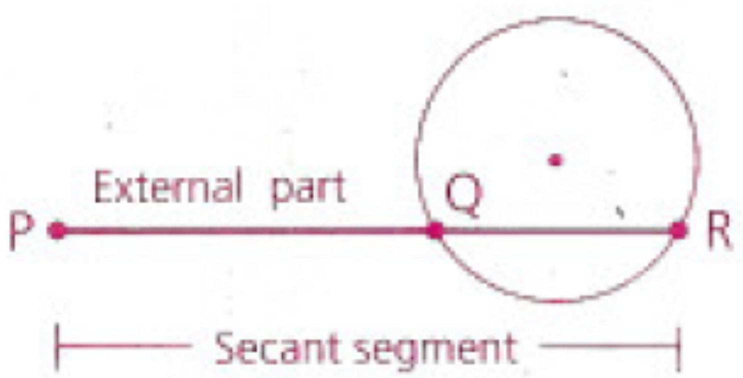
A tangent line is perpendicular to the radius drawn to the point of contact.

If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.

Secant and Tangent Segments

A <u>tangent segment</u> is the part of a tangent line between the point of contact and a point outside the circle.





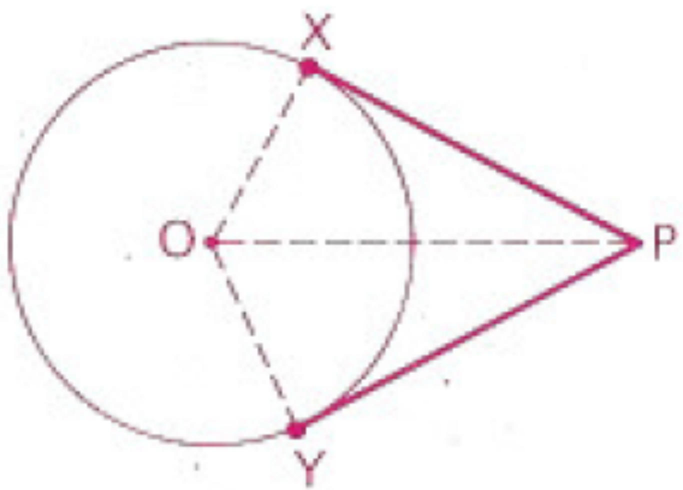
A <u>secant segment</u> is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

A <u>external part</u> of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

Secant and Tangent Segments

Two-Tangent Theorem: If two tangent segments are drawn to a circle from an external point, then those segments are congruent.

Given: $\odot O$;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.
Prove: $\overline{PX} \cong \overline{PY}$

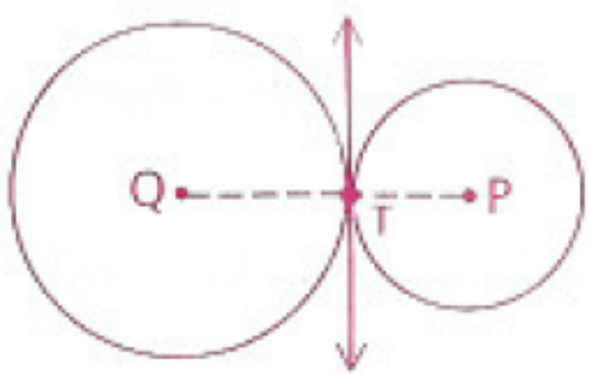


What could we use to prove this?

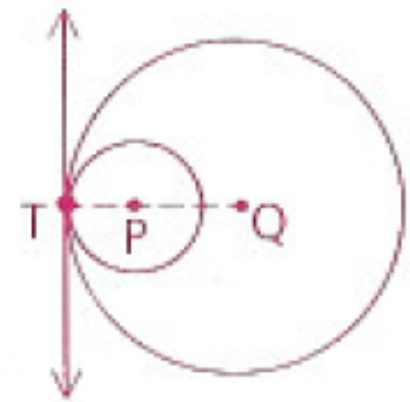
Hypotenuse- Leg Theorem!

Tangent Circles

<u>Tangent circles</u> are circles that intersect each other at exactly one point.



externally tangent



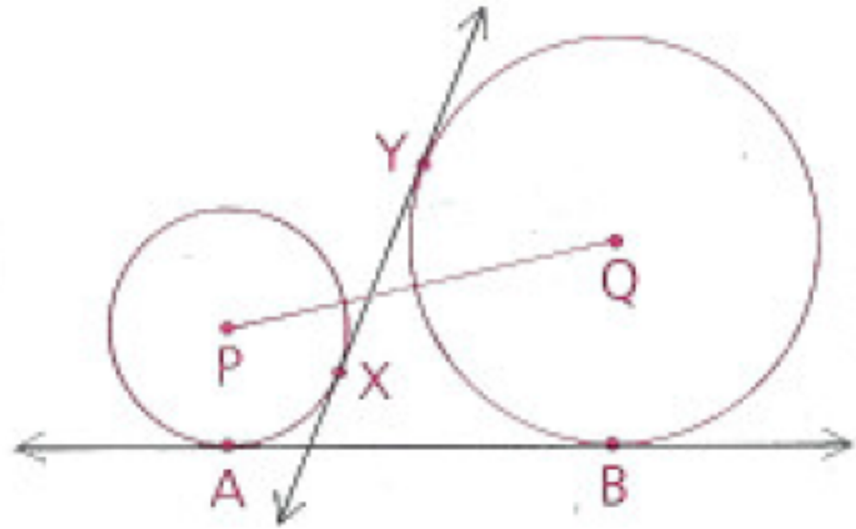
internally tangent

Two circles are <u>externally tangent</u> if each of the tangent circles lies outside each other.

Two circles are <u>internally tangent</u> if one of the tangent circles lies inside each other.

Common Tangents

$\overleftrightarrow{PQ}$ is the line of centers.
$\overleftrightarrow{XY}$ is a ***common internal tangent.***
$\overleftrightarrow{AB}$ is a ***common external tangent.***

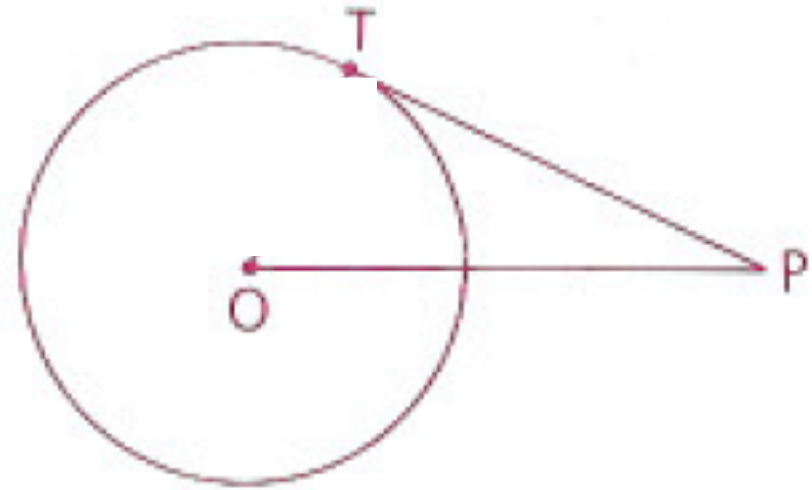


A <u>common tangent</u> is a line tangent to two circles (not necessarily at the same point).

A <u>common internal tangent</u> is if it lies between the circles (intersects the segment joining the centers).

A <u>common external tangent</u> is if it is not between the circles (does not intersect the segment joining the centers).

$\overleftrightarrow{TP}$ *is tangent to circle* O *at* T.
The radius of circle O *is* 8 *mm.*
Tangent segment $\overline{TP}$ *is* 6 *mm long.*
Find the length of $\overline{OP}$.

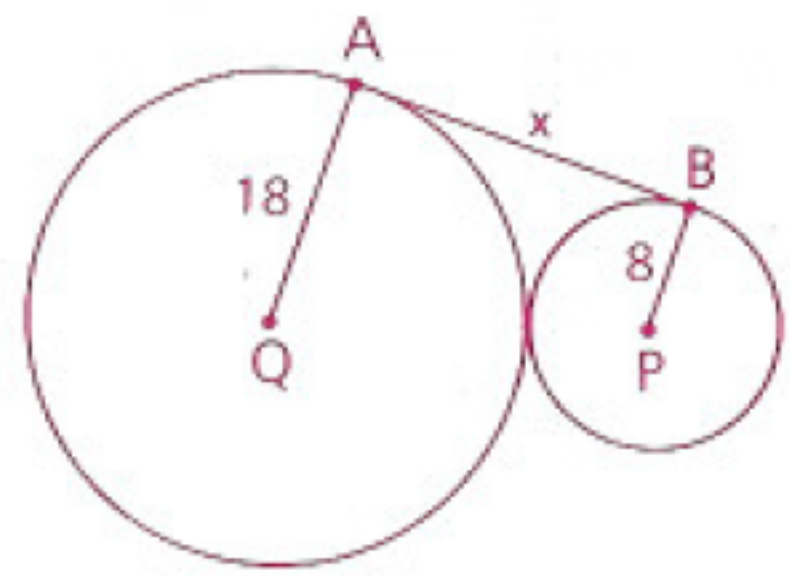


Draw radius $\overline{OT}$ to form right triangle OTP.

$$
\begin{aligned}
(TP)^2 + (TO)^2 &= (OP)^2 \\
6^2 + 8^2 &= (OP)^2 \\
\pm 10 &= OP \qquad \text{(Reject } -10.)
\end{aligned}
$$

Thus, OP = 10 mm.

A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.



Common-Tangent Procedure

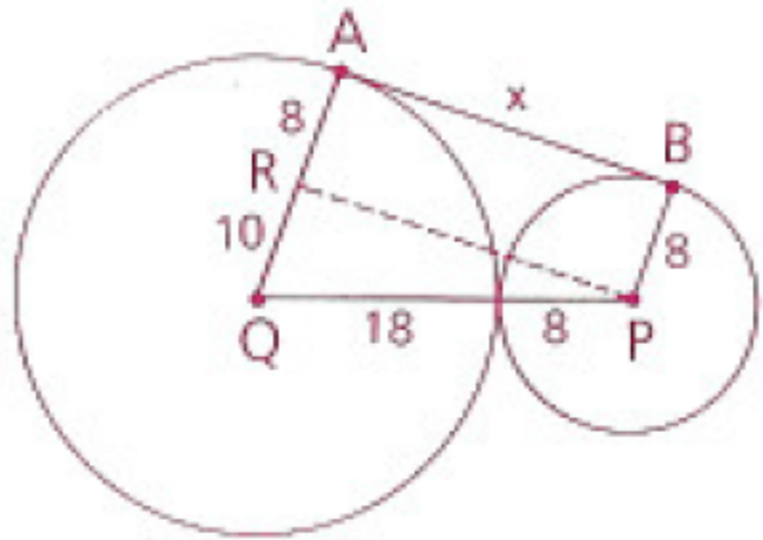
1. Draw the segment joining the centers.
2. Draw the radii to the points of contact.
3. Through the center of the smaller circle, draw a line parallel to the common tangent.
4. Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.
5. Use the Pythagorean Theorem and properties of a rectangle.

A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.

In $\triangle RPQ$,

$$(QR)^2 + (RP)^2 = (PQ)^2$$
$$10^2 + (RP)^2 = 26^2$$
$$RP = \pm 24$$

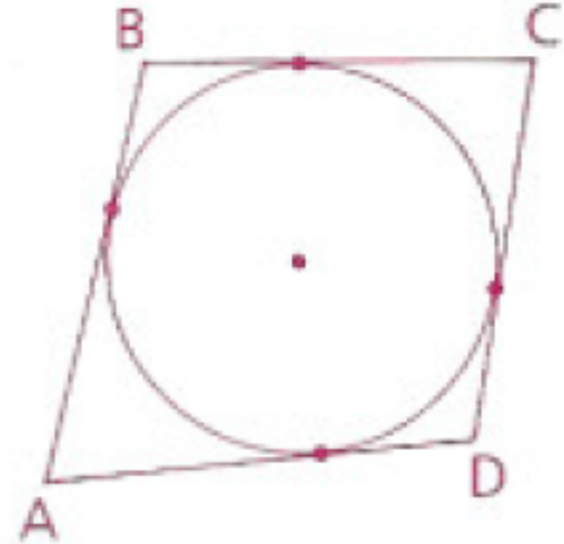
Thus, AB = 24 cm.



A walk-around problem:

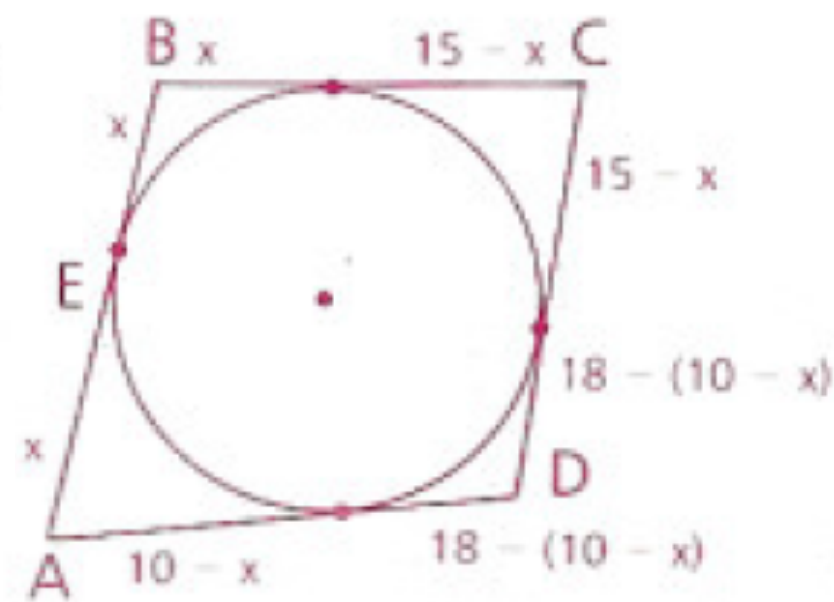
Given: *Each side of quadrilateral* ABCD *is tangent to the circle.* AB = 10, BC = 15, AD = 18

Find: CD



Let BE = x and "walk around" the figure, using the given information and the Two-Tangent Theorem.

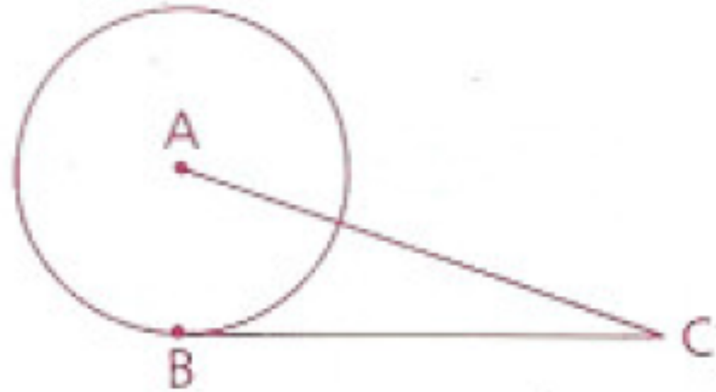
$$\begin{aligned} CD &= 15 - x + 18 - (10 - x) \\ &= 15 - x + 18 - 10 + x \\ &= 23 \end{aligned}$$



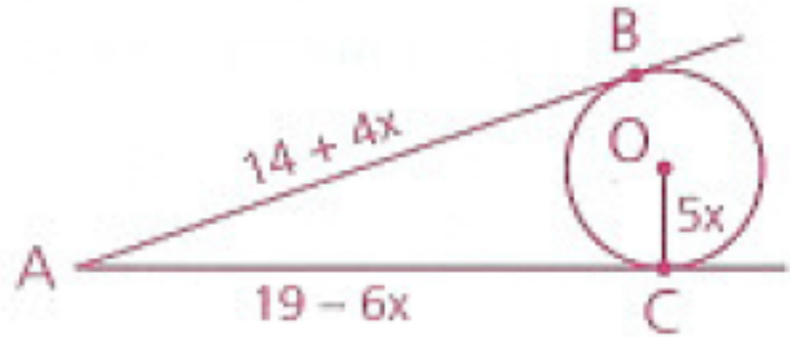
Homework

p. 463: 1, 6, 10, 11, 16

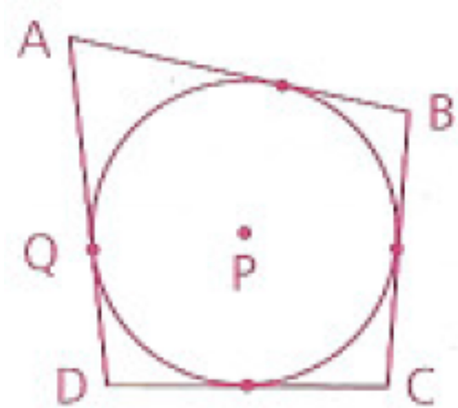
1 The radius of ⊙A is 8 cm.
Tangent segment $\overline{BC}$ is 15 cm long.
Find the length of $\overline{AC}$.



6 $\overline{AB}$ and $\overline{AC}$ are tangents to ⊙O, and OC = 5x. Find OC.

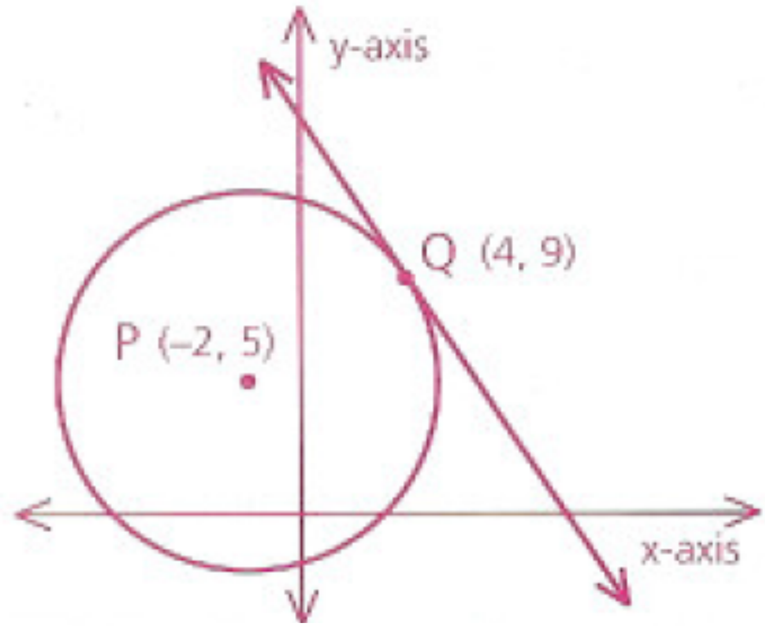


10 ⊙P is tangent to each side of ABCD. AB = 20, BC = 11, and DC = 14. Let AQ = x and find AD.



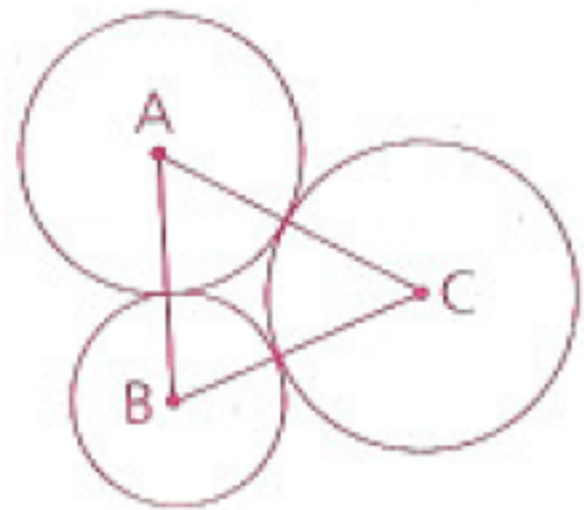
11 **a** Find the radius of ⊙P.

b Find the slope of the tangent to ⊙P at point Q.



16 Given: Tangent ⊛ A, B, and C,
AB = 8, BC = 13, AC = 11

Find: The radii of the three ⊛ (Hint: This is a walk-around problem.)



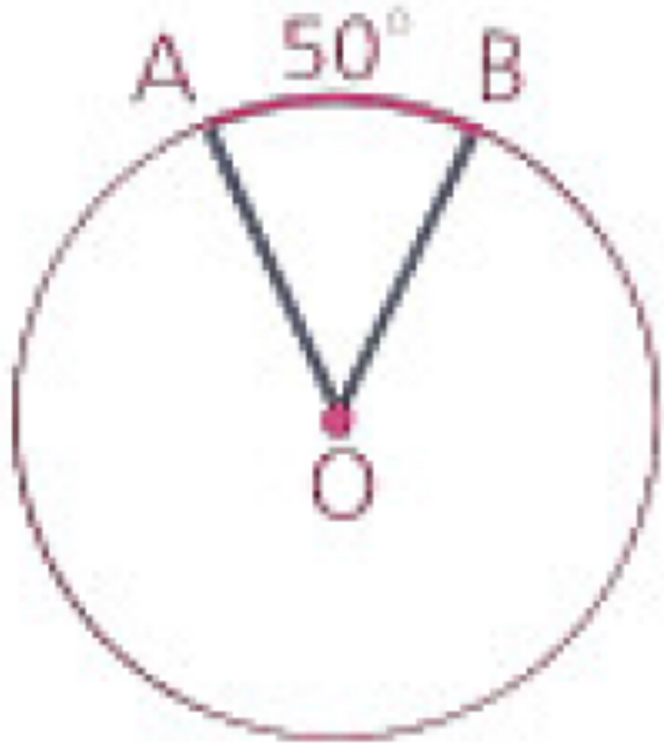
Objective

Students will be able to determine the measures of central angles, inscribed and tangent-chord angles, chord-chord angles, and secant-secant, secant-tangent, and tangent-tangent angles.

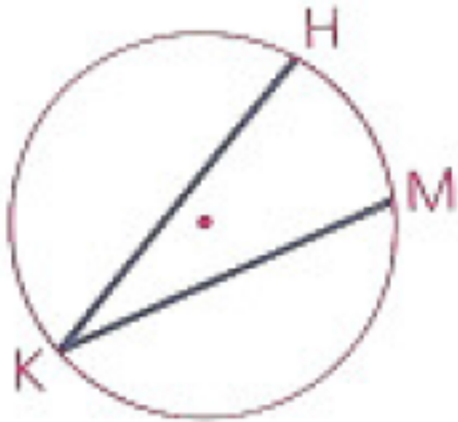
Circle's Quiz on Tuesday!

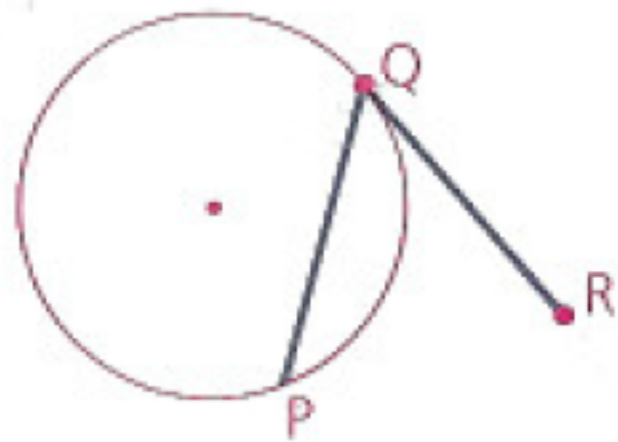
Who is taking the AP Human Geography Exam on May 12th?

What is the measure of ∠AOB?



Angles with Vertices on a Circle





∠HKM is an <u>inscribed angle</u>

∠PQR is a <u>tangent-chord angle</u>

An <u>inscribed angle</u> is an angle whose vertex is on a circle and whose sides are determined by two chords.

An <u>tangent-chord angle</u> is an angle whose vertex is on a circle and whose sides are determined by a tangent and a chord that intersect at the tangent's point of contact.

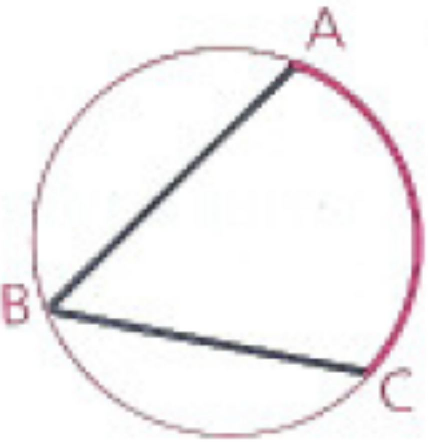
Angles with Vertices on a Circle

The measure of an inscribed angle or a tangent-chord angle (vertex on a circle) is one-half the measure of its intercepted arc.

Examples:

Given: $m\widehat{AC} = 112$

Find: $m\angle B$

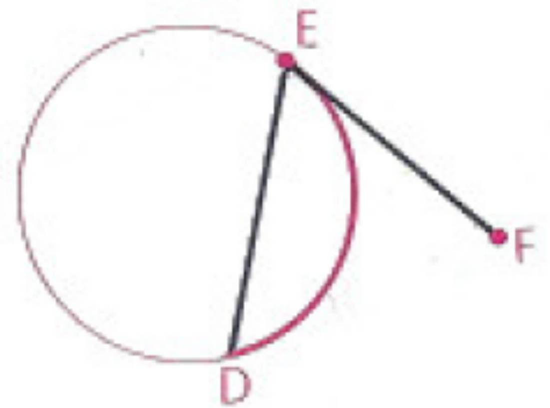


$$m\angle B = \tfrac{1}{2}(m\widehat{AC}) = \frac{1}{2} \cdot 112 = 56$$

Given: $\overline{FE}$ *is tangent at* E.

$m\widehat{DE} = 80$

Find: $m\angle DEF$

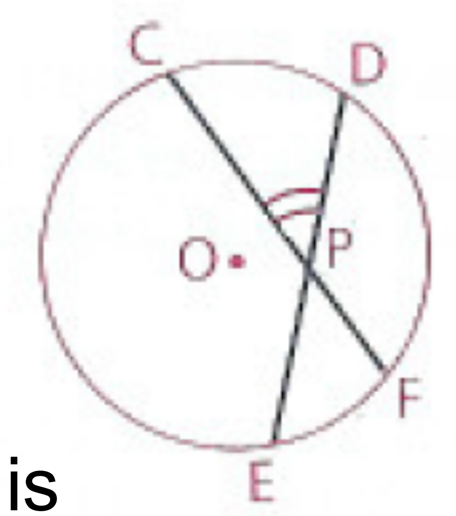


$$m\angle DEF = \tfrac{1}{2}(m\widehat{DE}) = \frac{1}{2} \cdot 80 = 40$$

Angles with Vertices Inside, but Not at the Center of, a Circle

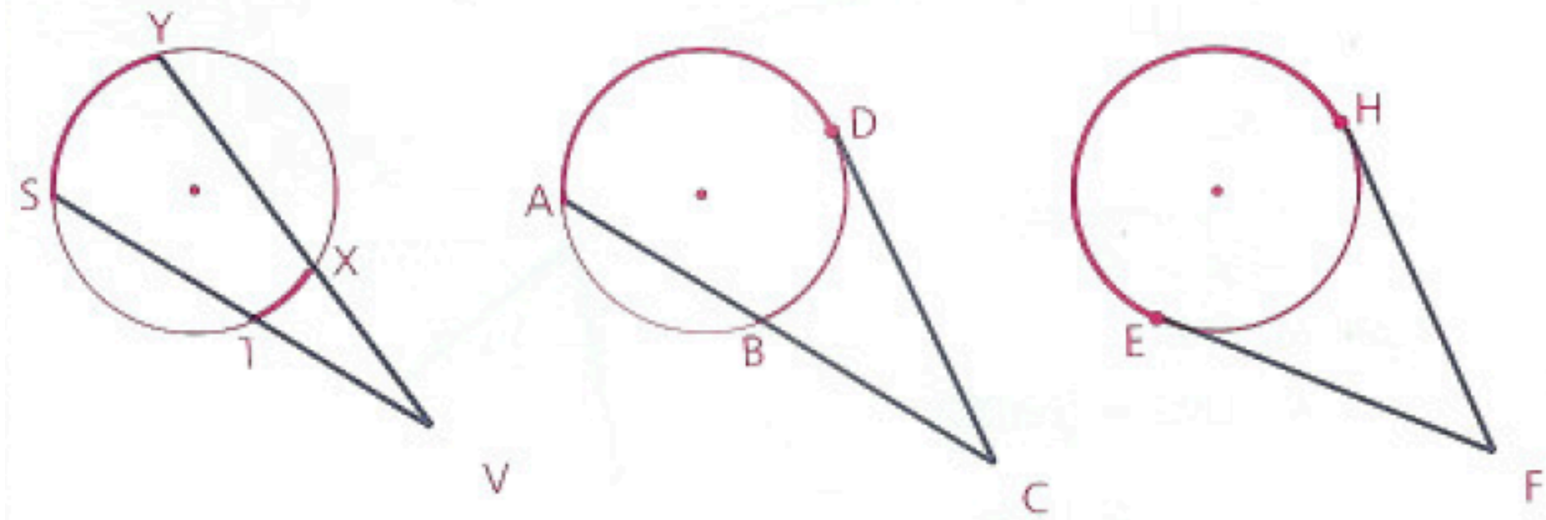
A <u>chord-chord angle</u> is an angle formed by two chords that intersect inside a circle but not at the center.

$\angle CPD$ is one of four chord-chord angles formed by chords $\overline{CF}$ and $\overline{DE}$ in circle O.



The measure of a chord-chord angle is one-half the sum of the measure of the arcs intercepted by the chord-chord angle and its vertical angle.

Angles with Vertices Outside a Circle



$\angle V$ is a ***secant-secant angle.*** $\angle C$ is a ***secant-tangent angle.*** $\angle F$ is a ***tangent-tangent angle.***

A <u>secant-secant angle</u> is an angle whose vertex is outside a circle and whose sides are determined by two secants.

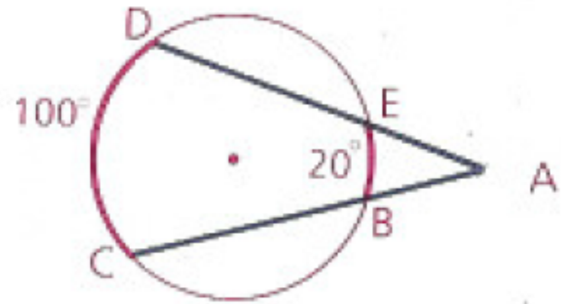
A <u>secant-tangent angle</u> is an angle whose vertex is outside a circle and whose sides are determined by a secant and a tangent.

A <u>tangent-tangent angle</u> is an angle whose vertex is outside a circle and whose sides are determined by two tangents.

Angles with Vertices Outside a Circle

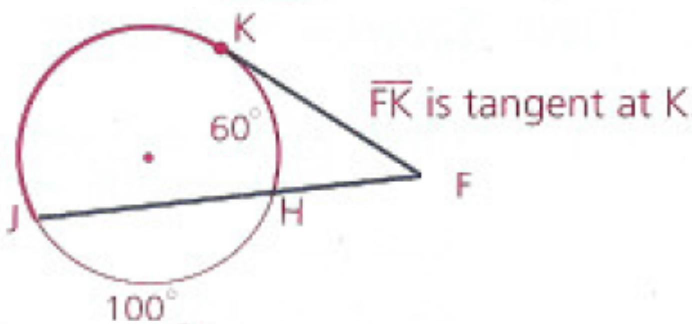
The measure of a secant-secant angle, a secant-tangent angle, or a tangent-tangent angle (vertex outside a circle) is one-half the difference of the measures of the intercepted arcs.

Find m∠A.



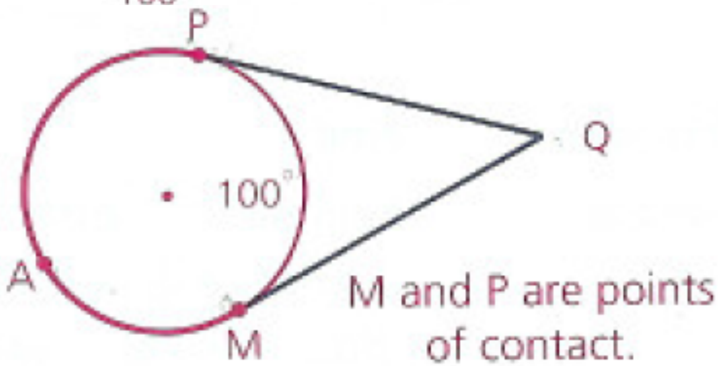
$$\begin{aligned} m\angle A &= \tfrac{1}{2}(m\overparen{CD} - m\overparen{BE}) \\ &= \tfrac{1}{2}(100 - 20) \\ &= 40 \end{aligned}$$

Find m∠F.



$$\begin{aligned} m\overparen{JK} &= 360 - 100 - 60 \\ &= 200 \\ m\angle F &= \tfrac{1}{2}(m\overparen{JK} - m\overparen{HK}) \\ &= \tfrac{1}{2}(200 - 60) \\ &= 70 \end{aligned}$$

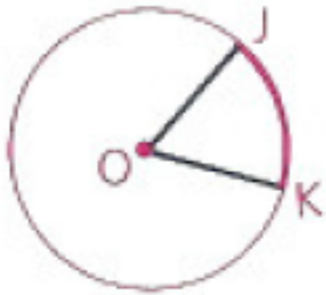
Find m∠Q.



$$\begin{aligned} m\overparen{MAP} &= 360 - 100 = 260 \\ m\angle Q &= \tfrac{1}{2}(m\overparen{MAP} - m\overparen{MP}) \\ &= \tfrac{1}{2}(260 - 100) \\ &= 80 \end{aligned}$$

Summary

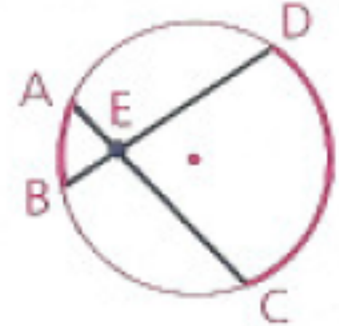
Central Angle



$m\angle KOJ = m\widehat{JK}$

Vertex at center $\Rightarrow$ equal

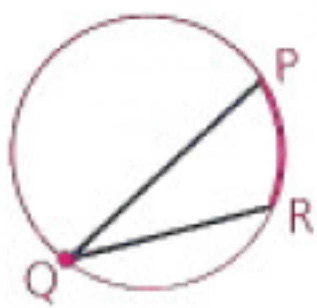
Chord-Chord Angle



$m\angle DEC = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$

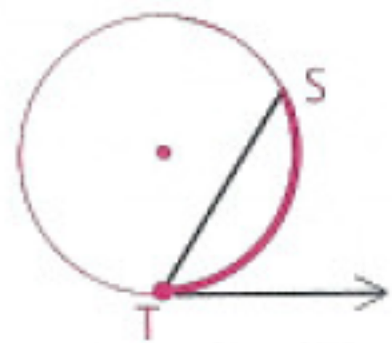
Vertex inside $\Rightarrow$ half the sum

Inscribed Angle



$m\angle Q = \frac{1}{2}(m\widehat{PR})$

Tangent-Chord Angle

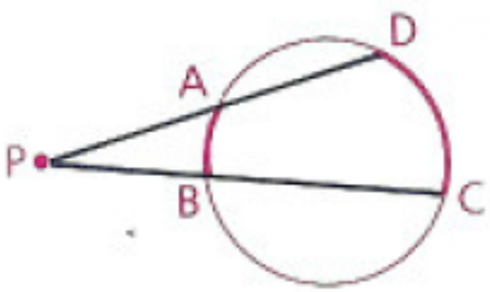


$m\angle T = \frac{1}{2}(m\widehat{ST})$

Vertex on circle $\Rightarrow$ half the arc

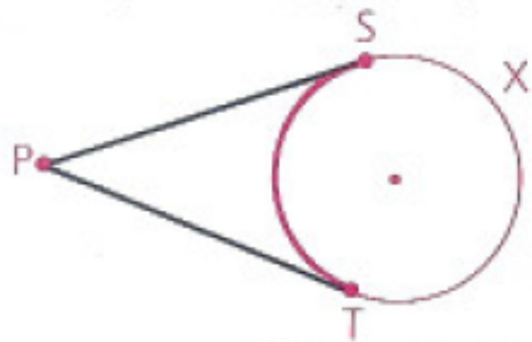
Summary Continued

Secant-Secant Angle



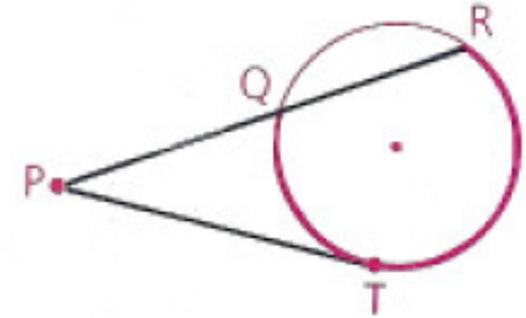
$m\angle P = \frac{1}{2}(m\overset{\frown}{CD} - m\overset{\frown}{AB})$

Tangent-Tangent Angle



$m\angle P = \frac{1}{2}(m\overset{\frown}{SXT} - m\overset{\frown}{ST})$

Secant-Tangent Angle



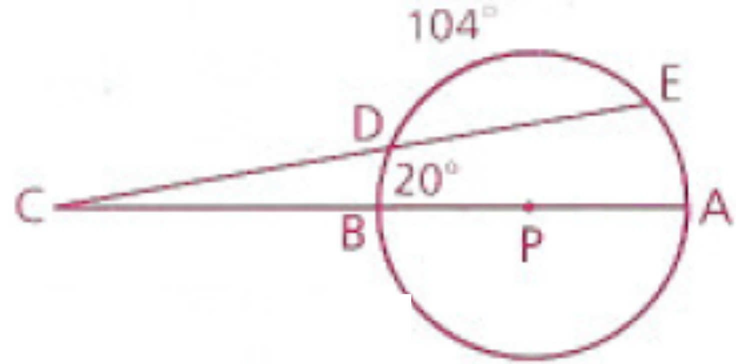
$m\angle P = \frac{1}{2}(m\overset{\frown}{RT} - m\overset{\frown}{QT})$

Vertex outside circle $\Rightarrow$ half the difference

Examples

Problem 1 *Given:* $\overline{AB}$ *is a diameter of* $\odot P$.
$\overset{\frown}{BD} = 20°$, $\overset{\frown}{DE} = 104°$

Find: $m\angle C$

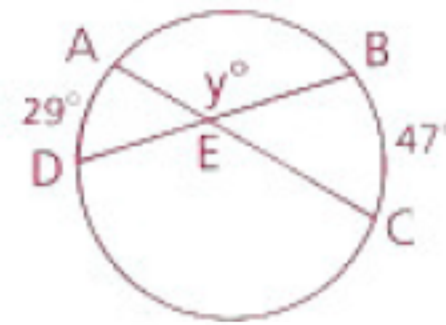


Solution First find $m\overset{\frown}{EA}$.

$m\overset{\frown}{AEB} = 180$, so $m\overset{\frown}{EA} = 180 - (104 + 20) = 56$.

Thus, $m\angle C = \frac{1}{2}(m\overset{\frown}{EA} - m\overset{\frown}{DB}) = \frac{1}{2}(56 - 20) = 18$.

Problem 2 *Find y.*

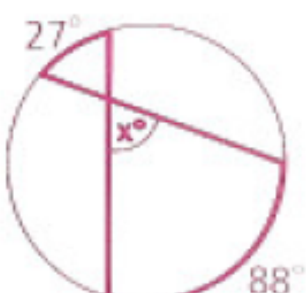


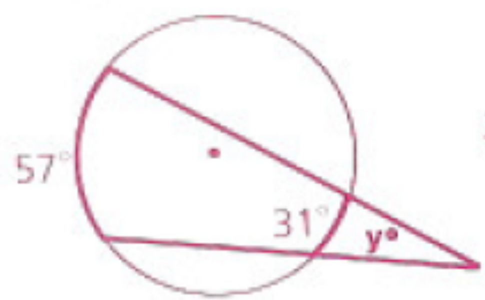
Solution Find $m\angle BEC$ first.

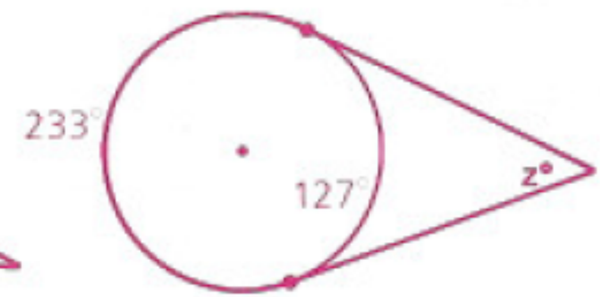
$m\angle BEC = \frac{1}{2}(29 + 47) = 38$

Thus, $y = 180 - m\angle BEC = 142$.

Problem 3 **a** *Find x.* **b** *Find y.* **c** *Find z.*







Solution

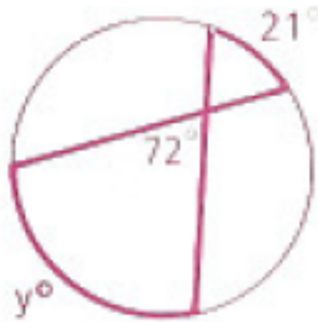
a $x = \frac{1}{2}(88 + 27) = 57\frac{1}{2}$

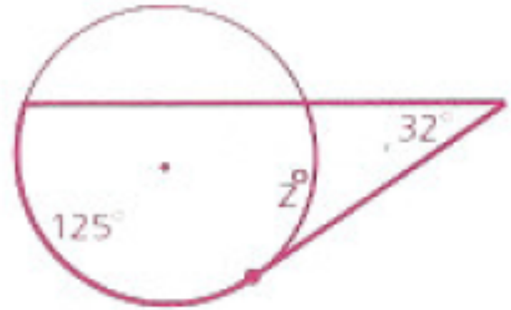
b $y = \frac{1}{2}(57 - 31) = 13$

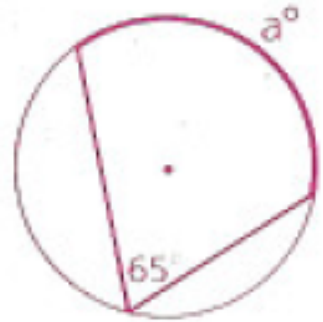
c $z = \frac{1}{2}(233 - 127) = 53$

Examples

Problem 4 **a** *Find y.* **b** *Find z.* **c** *Find a.*







Solution

a $\frac{1}{2}(21 + y) = 72$

$21 + y = 144$

$y = 123$

b $\frac{1}{2}(125 - z) = 32$

$125 - z = 64$

$z = 61$

c $\frac{1}{2}a = 65$

$a = 130$

Problem 5 *Find* $m\widehat{AB}$ *and* $m\widehat{CD}$.

Solution Let $m\widehat{AB} = x$ and $m\widehat{CD} = y$.

Then $\frac{1}{2}(x + y) = 65$ and $\frac{1}{2}(x - y) = 24$.

So $x + y = 130$ and $x - y = 48$.

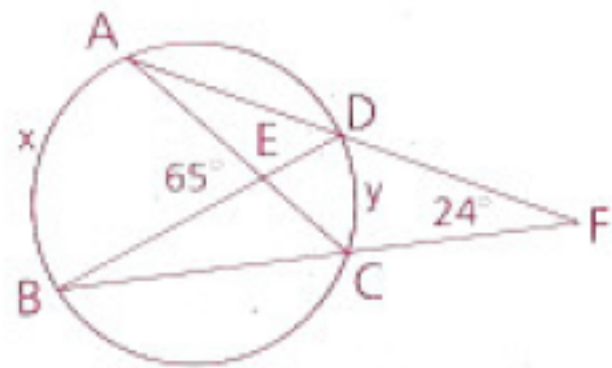
$$\begin{array}{r} x + y = 130 \\ \underline{x - y = 48} \\ 2x = 178 \end{array}$$ Add the equations.

$x = 89$

$89 + y = 130$

$y = 41$

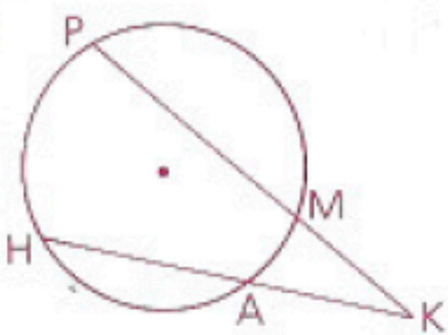
Thus, $m\widehat{AB} = 89$ and $m\widehat{CD} = 41$.



Homework p. 474: 4, 5, 6, 18, 21

4 *Vertex outside:*

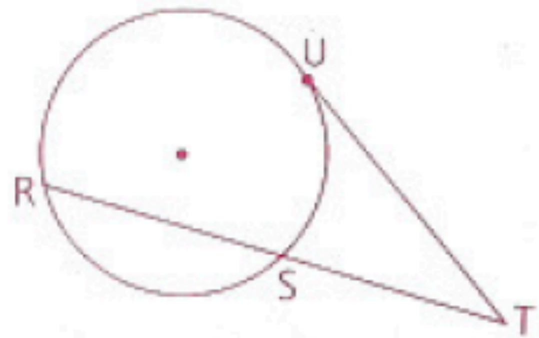
a



Given: $\overset{\frown}{HP} = 120°$,
$\overset{\frown}{AM} = 36°$

Find: $m\angle K$

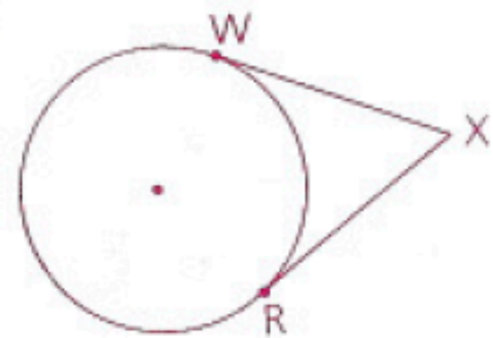
b



Given: $\overline{TU}$ is tangent at U.
$\overset{\frown}{RU} = 160°$,
$\overset{\frown}{SU} = 60°$

Find: $m\angle T$

c

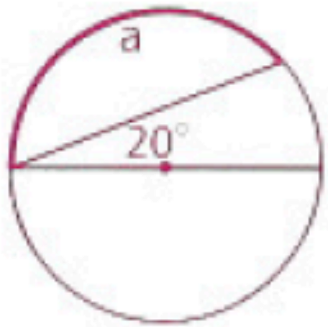


Given: W and R are points of contact.
$\overset{\frown}{WR} = 140°$

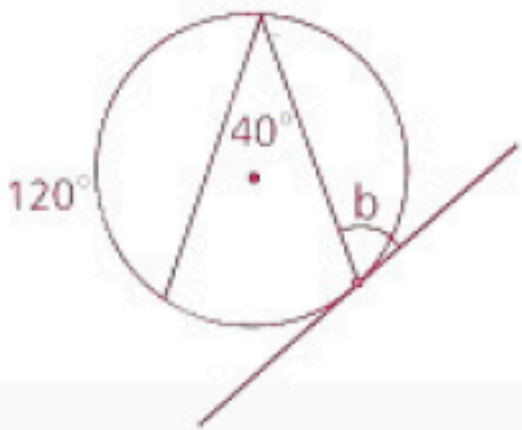
Find: $m\angle X$

5 Find the measure of each angle or arc that is labeled with a letter.

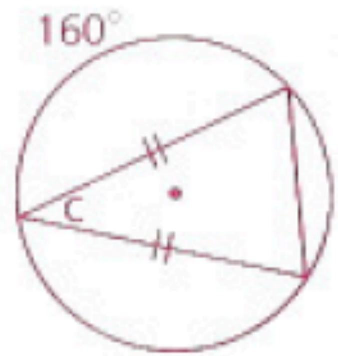
a



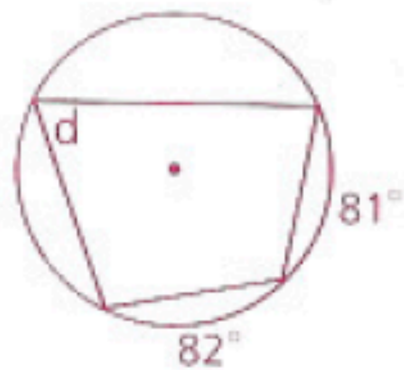
b



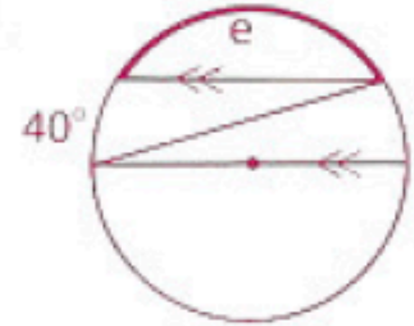
c



d

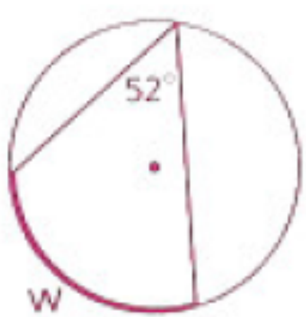


e

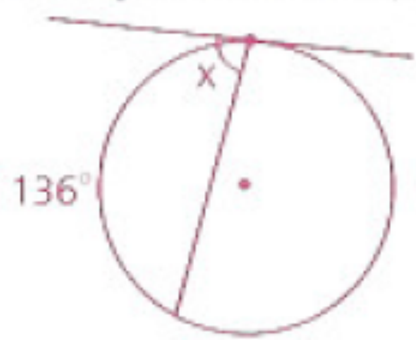


6 Find the measure of each angle or arc that is labeled with a letter.

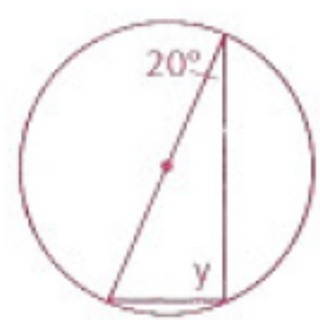
a



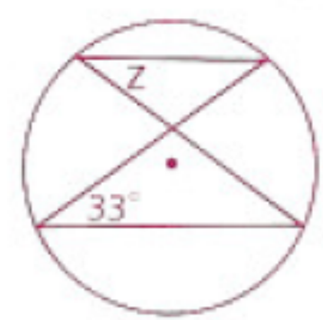
b



c



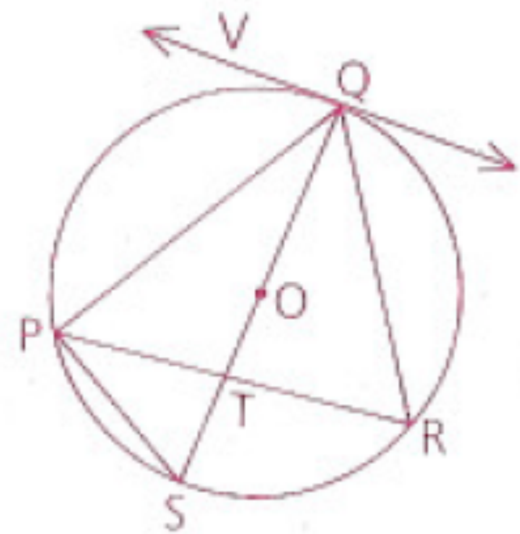
d



18 Given: $\overleftrightarrow{VQ}$ is tangent to $\odot O$ at Q.
$\overline{QS}$ is a diameter of $\odot O$.
$\widehat{PQ} = 115°$; $\angle RPS = 36°$

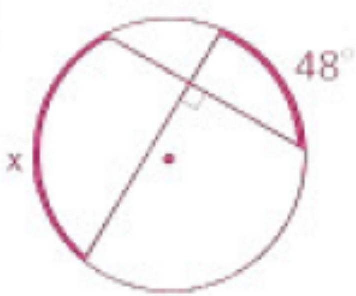
Find:

a $\angle R$	**e** $\angle QPR$	**i** $\widehat{PRQ}$
b $\angle S$	**f** $\angle QPS$	**j** $\widehat{RSP}$
c $\widehat{SR}$	**g** $\angle QTP$	**k** $\angle VQS$
d $\widehat{QR}$	**h** $\angle PQV$	**l** $\angle QOP$

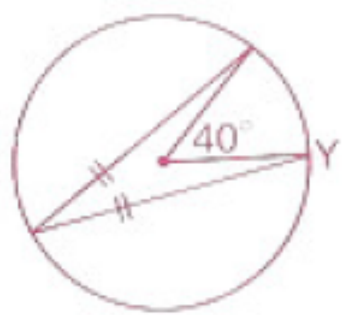


21 Find the measure of each arc or angle labeled with a letter.

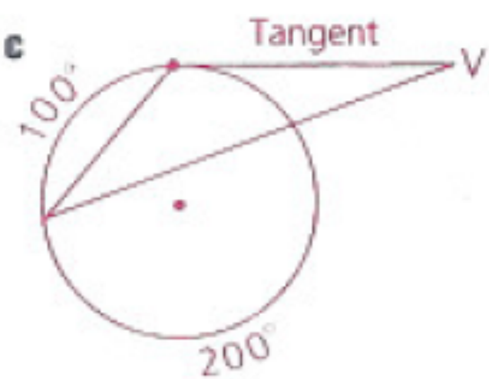
a



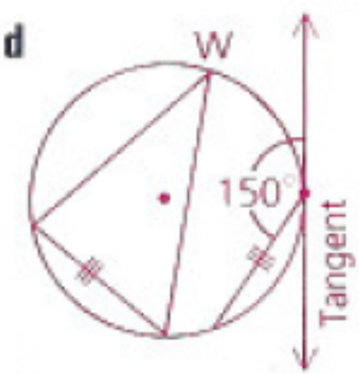
b



c



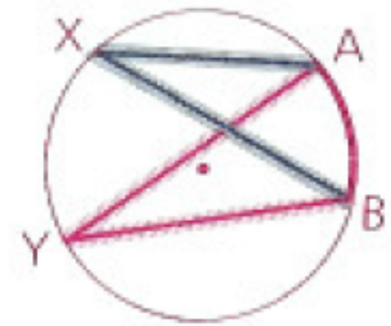
d



Objective

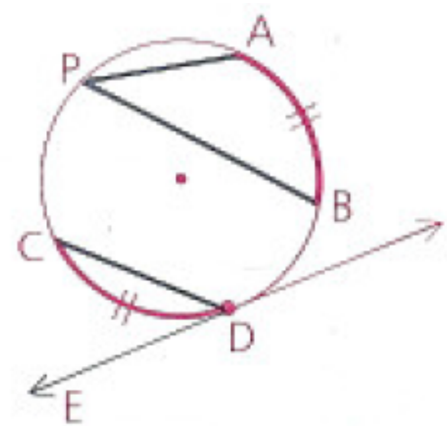
Students will be able to recognize congruent inscribed and tangent-chord angles, determine the measure of an angle inscribed in a semicircle, and apply the relationship between the measures of a tangent-tangent angle and its minor arc.

Congruent Inscribed and Tangent-Chord Angles



$\angle X \cong \angle Y$

If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.



$\angle P \cong \angle CDE$

If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.

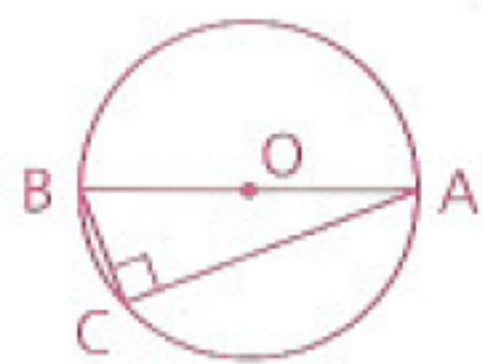
Angles Inscribed in Semicircles

All angles inscribed in semicircles have the same measure. What do you think that measure might be?

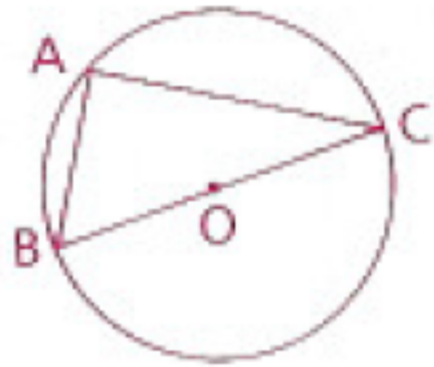
An angle inscribed in a semicircle is a right angle.

Given: $\overline{AB}$ is a diameter of $\odot O$.

Prove: $\angle C$ is a right angle.



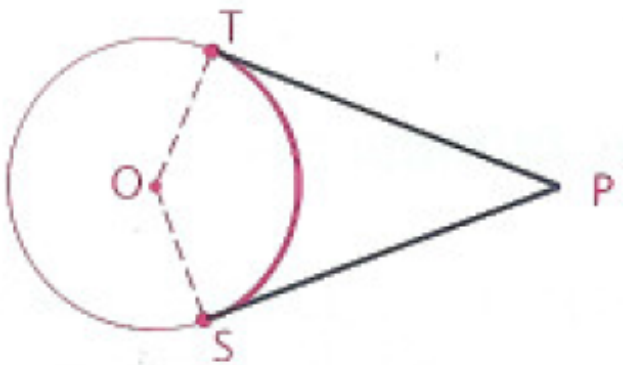
In circle O, $\overline{BC}$ is a diameter and the radius of the circle is 20.5 mm.
Chord $\overline{AC}$ has a length of 40 mm. Find AB.



Since $\angle A$ is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

$$(AB)^2 + (AC)^2 = (BC)^2$$
$$(AB)^2 + 40^2 = 41^2$$
$$AB = 9 \text{ mm}$$

A Special Theorem About Tangent-Tangent Angles

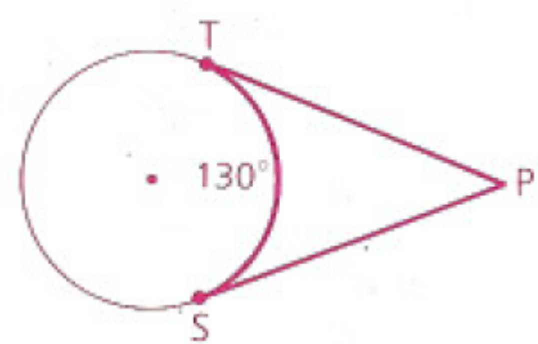


What can you tell me about the sum of ∠P and ∠O?

The sum of the measures of a tangent-tangent angle and its minor arc is 180.

EX: $\overleftrightarrow{PT}$ *and* $\overleftrightarrow{PS}$ *are tangents at T and S. Find m∠P.*

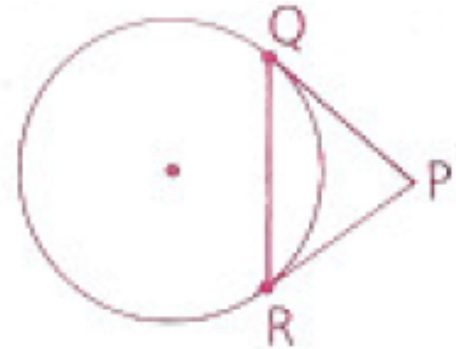
$$m\angle P + m\widehat{TS} = 180$$
$$m\angle P + 130 = 180$$
$$m\angle P = 50$$



Homework p. 481: 4, 5, 9, 12, 16, 17

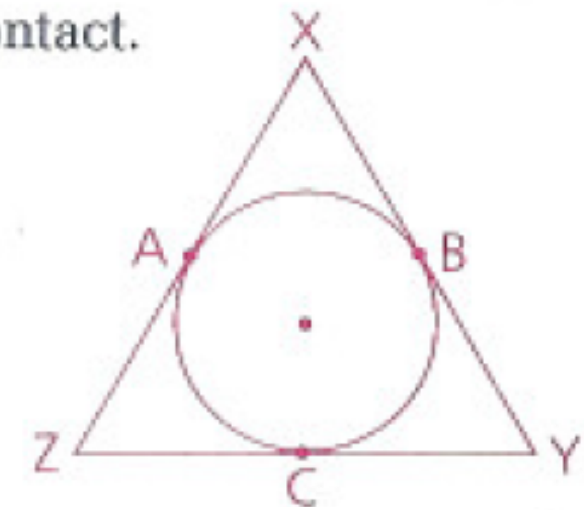
4 Given: $\overline{PQ}$ and $\overline{PR}$ are tangent segments.
$\widehat{QR} = 163°$

Find: **a** $\angle P$
b $\angle PQR$

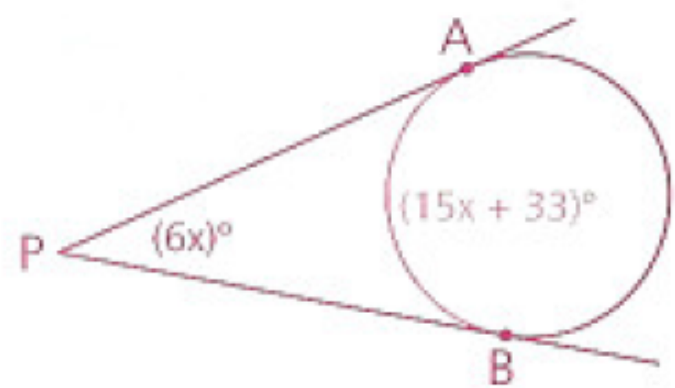


5 Given: A, B, and C are points of contact.
$\widehat{AB} = 145°$, $\angle Y = 48°$

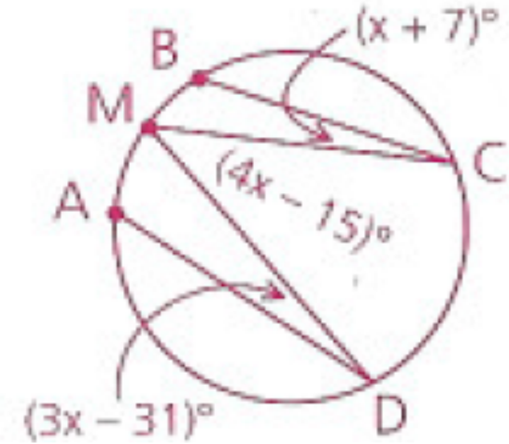
Find: $\angle Z$



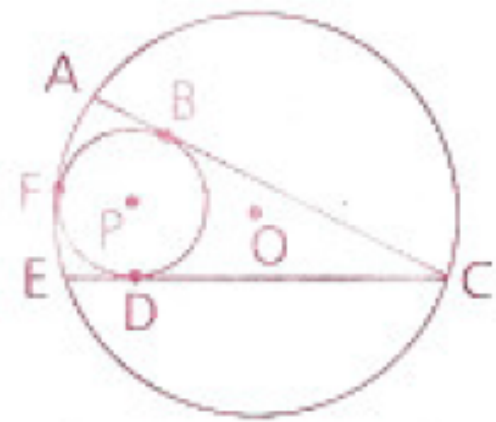
9 In the figure shown, find m$\angle P$.



12 M is the midpoint of $\widehat{AB}$. Find $m\widehat{CD}$.



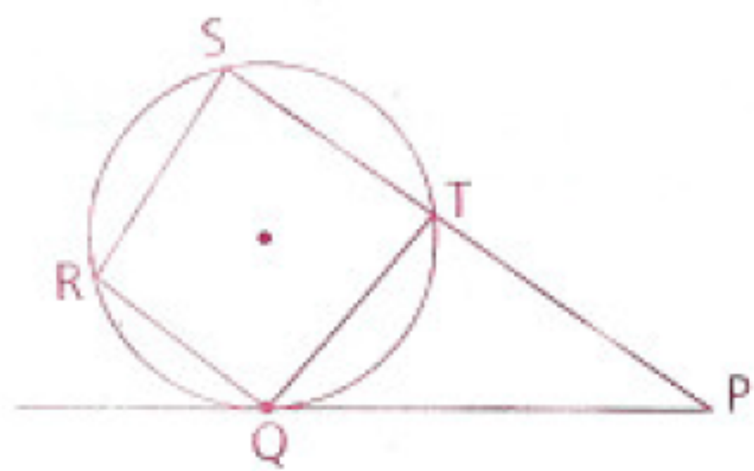
16 Circles O and P are tangent at F. $\overline{AC}$ and $\overline{CE}$ are tangent to $\odot P$ at B and D. If $\widehat{DFB} = 223°$, find $\widehat{AE}$.



17 Given: $\angle S = 88°$, $\widehat{QT} = 104°$, $\widehat{ST} = 94°$, tangent $\overline{PQ}$

Find: **a** $\angle P$

b $\angle STQ$



Objective

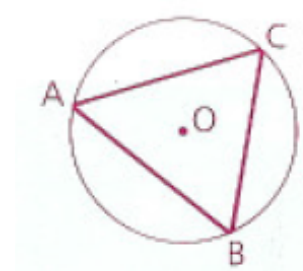
Students will be able to recognize inscribed and circumscribed polygons, apply the relationship between opposite angles of an inscribed quadrilateral, and identify the characteristics of an inscribed parallelogram.

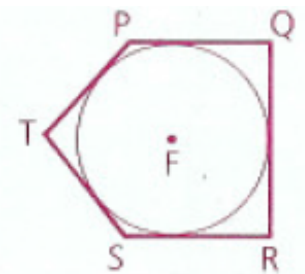
Circles Test on Thursday, May 11th

Inscribed and Circumscribed Polygons

We have seen problems where polygons are inside or around circles. These types of polygons have particular names.

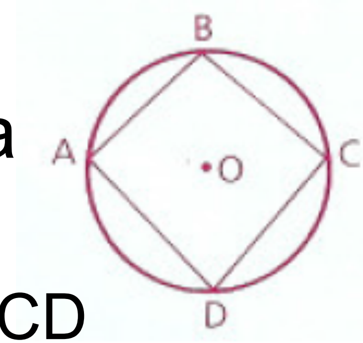
A polygon is <u>inscribed in</u> a circle if all its vertices lie on the circle.



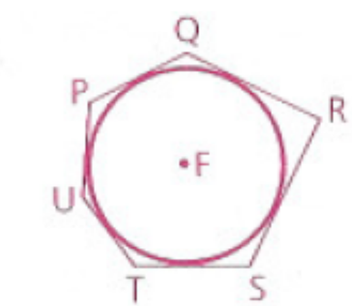


A polygon is <u>circumscribed about</u> a circle if each of its sides is tangent to the circle.

The center of a circle circumscribed about a polygon is the <u>circumcenter</u> of the polygon.



O is the circumcenter of ABCD



The center of a circle inscribed in a polygon is the <u>incenter</u> of the polygon.

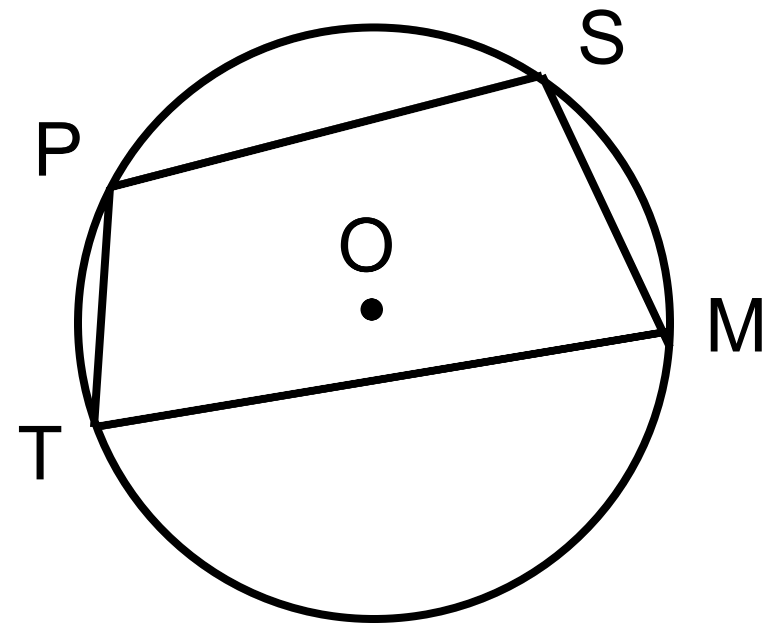
F is the incenter of hexagon PQRSTU

Given: $\odot O$,

$\overset{\frown}{SM} = 80^\circ$,

$\overset{\frown}{PS} = 90^\circ$,

$\overset{\frown}{PT} = 70^\circ$



Find: Measures of arc TM, ∠P, ∠S, ∠M, ∠T, ∠S + ∠T, and ∠P + ∠M

arc TM = 120°

∠P = ½(80 + 120) = 100°

∠S = 95°

∠M = 80°

∠T = 85°

∠S + ∠T = 180°

∠P + ∠M = 180°

If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.

If a parallelogram is inscribed in a circle, it must be a rectangle.

Example: Find:

a) $\angle C$ b) $\angle D$ c) $\overset{\frown}{AD}$ d) $\overset{\frown}{BC}$

a) 97°

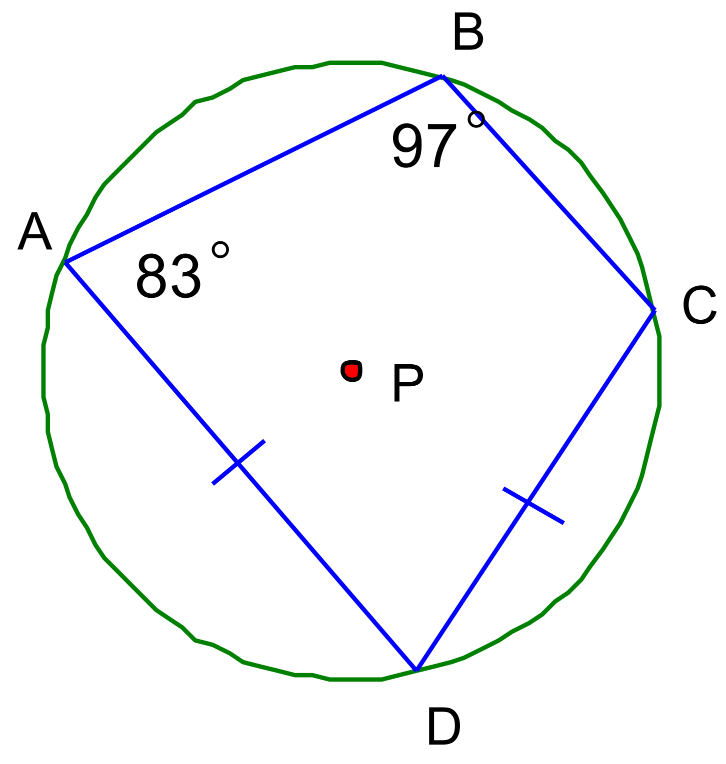
b) 83°

c) 97°

d) 69°

*opposite angles of a polygon inscribed in a circle are supplementary

$$\overset{\frown}{AD} \cong \overset{\frown}{DC}$$



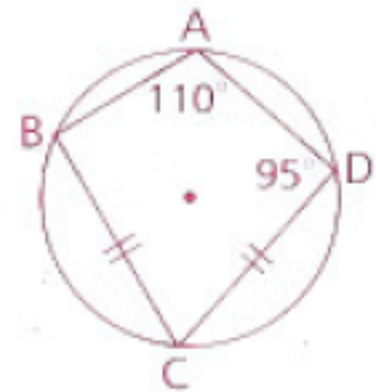
Homework

p. 489: 3, 6, 9, 15, 20

3 Given: $\angle A = 110°$, $\overline{BC} \cong \overline{CD}$, $\angle D = 95°$

Find: **a** $\angle C$ **c** $\angle B$

b $\widehat{BC}$ **d** $\widehat{AB}$

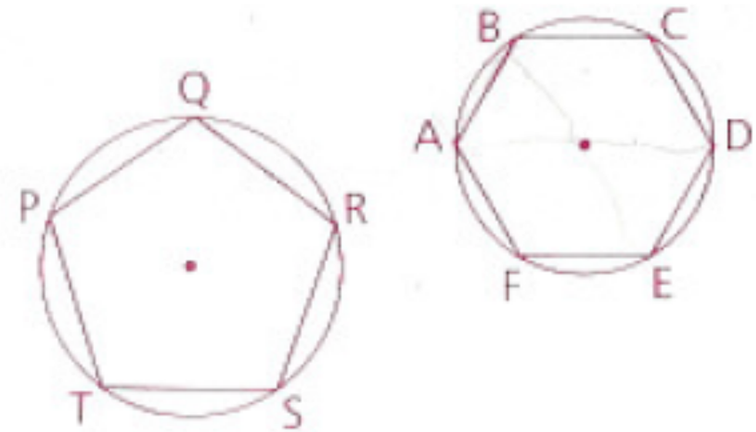


6 Given: PQRST is a regular pentagon.
ABCDEF is a regular hexagon.

Find: **a** $m\widehat{PQ}$ **d** $m\widehat{BD}$

b $m\widehat{RT}$ **e** $m\widehat{DEA}$

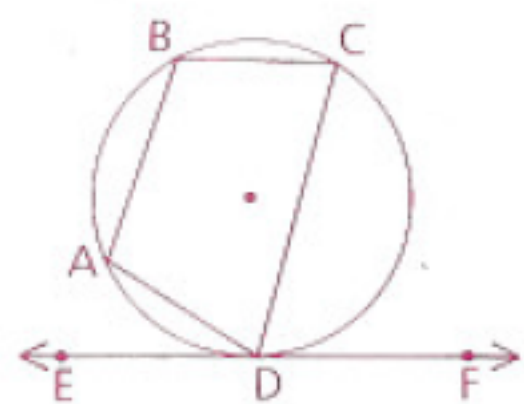
c $m\widehat{AB}$



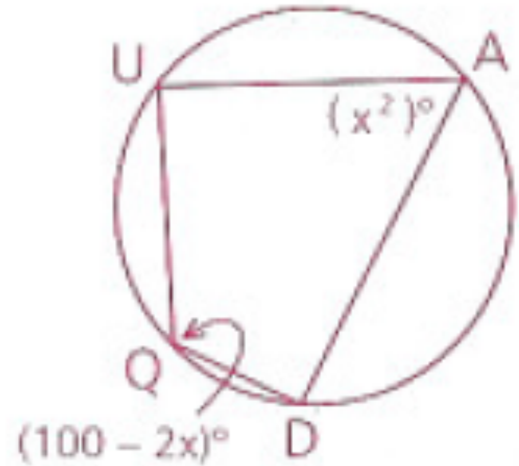
9 Given: $\angle B = 115°$, $\widehat{AD} = 60°$, $\overline{BC} \parallel \overline{EF}$

Find: **a** $\angle ADC$ **c** $\angle C$

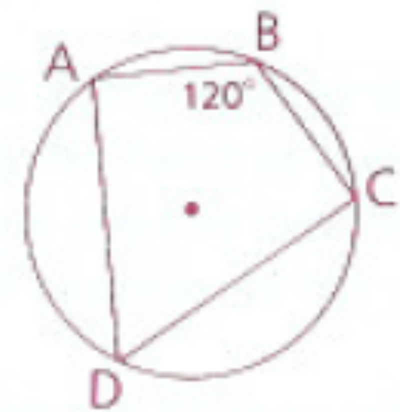
b $\angle CDF$ **d** $\angle A$



15 Given the figure shown, find m∠Q.



20 ABCD is a kite, with $\overline{AB} \cong \overline{BC}$, $\overline{AD} \cong \overline{CD}$, and m∠B = 120. The radius of the circle is 3. Find the perimeter of ABCD.



Objective

Students will be able to determine the circumference of a circle and the length of an arc.

Circles Test on Thursday, May 11th

Circumference

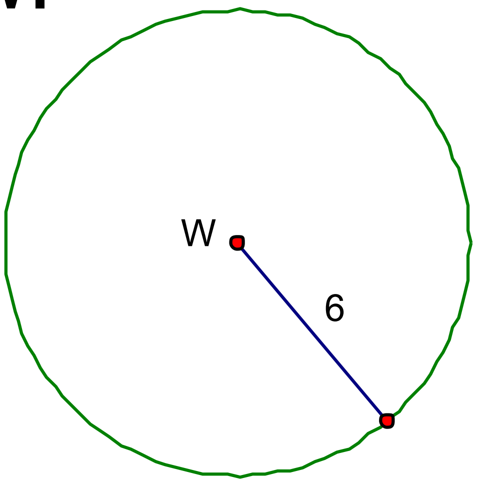
The <u>circumference</u> of a circle is its perimeter.

$$C = 2\pi r \text{ or } C = d\pi$$

Find the circumference of circle W.

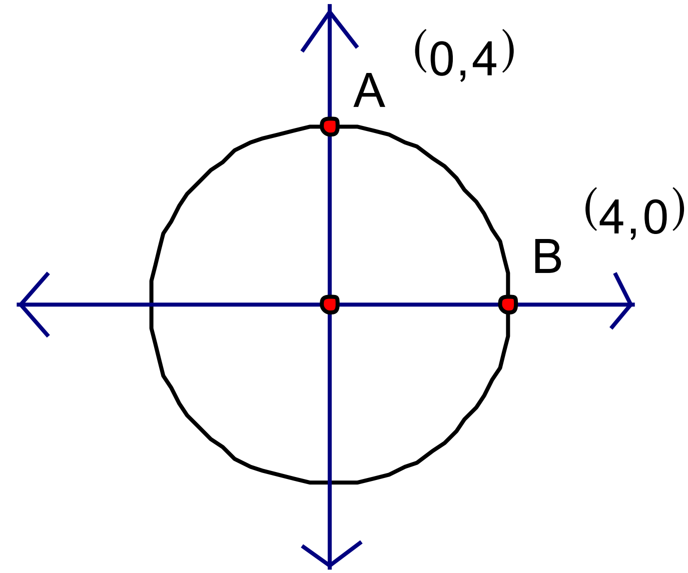
$$C = 2\pi(6)$$

$$= 12\pi$$



Find the measure and length of $\overparen{AB}$

$$m\overparen{AB} = \frac{1}{4}360^{\circ} = 90^{\circ}$$



We know that the arc is a portion of the circumference.

To find the length of the arc, we can determine the circumference of the circle, and take a portion of it.

$$C = 2\pi r \quad = 2\pi(4) \quad = 8\pi$$

Since the arc is one-fourth of the circle, the length of the arc is:

$$\frac{1}{4}(8\pi) \quad = 2\pi$$

Length of an Arc

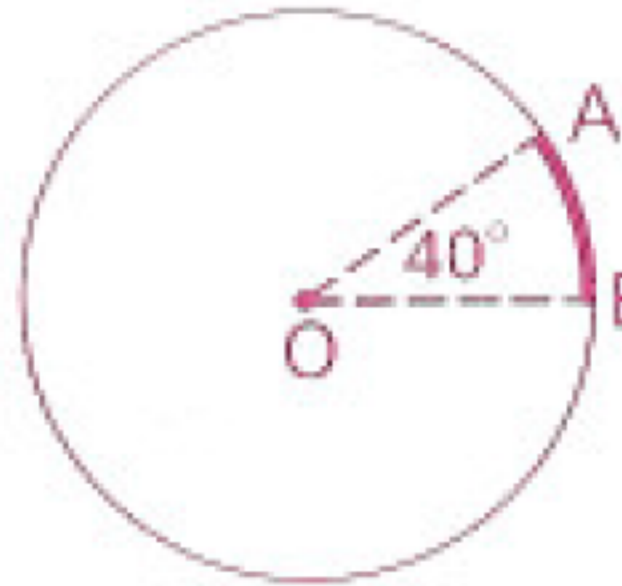
The length of an arc is equal to the circumference of its circle times the fractional part of the circle determined by the arc.

$$\text{Length of } \overparen{PQ} = \left(\frac{m\overparen{PQ}}{360}\right)\pi d$$

where *d* is the diameter and $\overparen{PQ}$ is measured in degrees.

Find the length of a 40º arc of a circle with an 18-cm radius.

$$C = 2\pi r = 2\pi(18) = 36\pi$$



40º arc is 40/360 or 1/9 of the circle

Length of arc AB = 1/9(circumference)

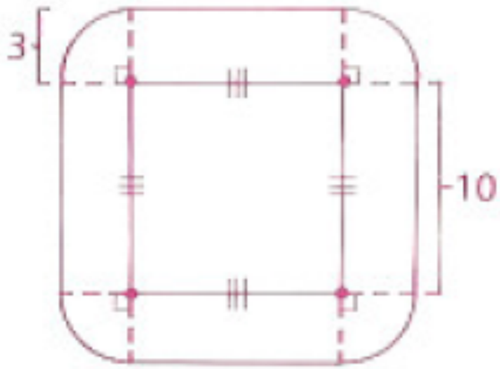
$$= \frac{1}{9}(36\pi) \quad = 4\pi$$

Homework

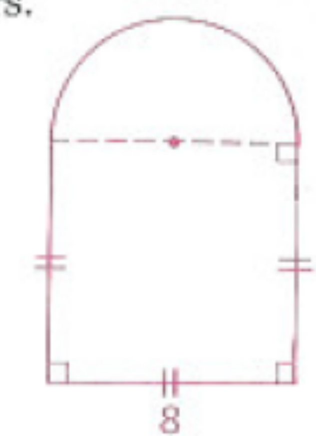
p.501: 5, 6, 9, 12, 15

5 Find the complete perimeter of each figure. Leave your answers in terms of π and whole numbers.

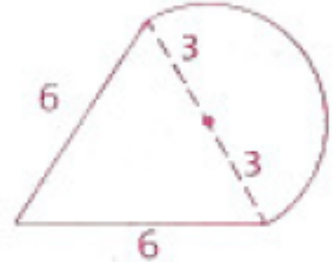
a



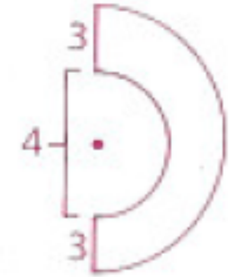
b



c

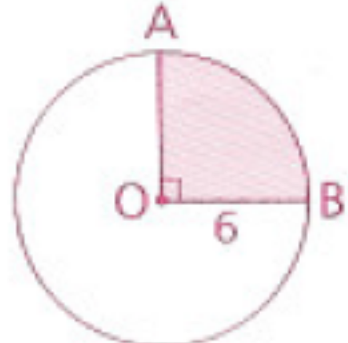


d



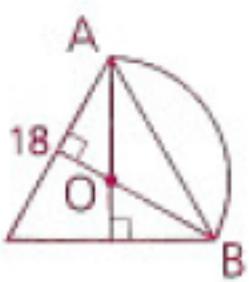
6 **a** Find the length of $\overset{\frown}{AB}$.

b Find the perimeter of sector AOB. (The shaded region is a sector.)

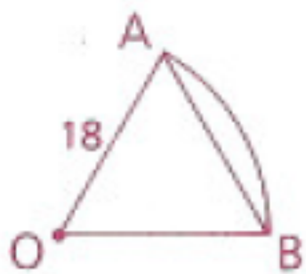


9 Given arcs mounted on equilateral triangles as shown, find the length of each arc. In each case $\overline{OA}$ is a radius of $\overset{\frown}{AB}$.

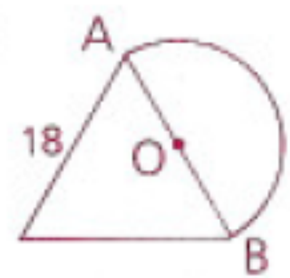
a



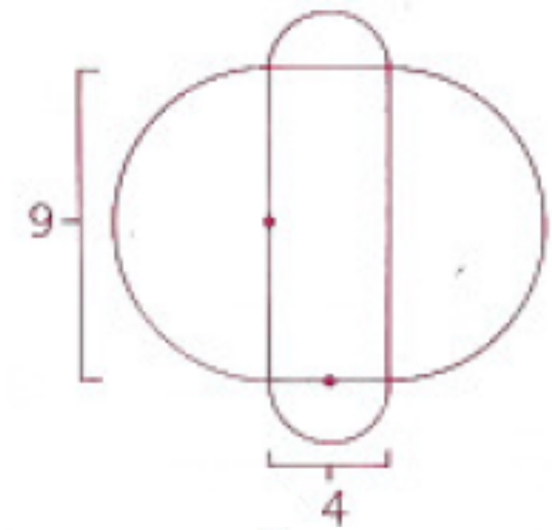
b



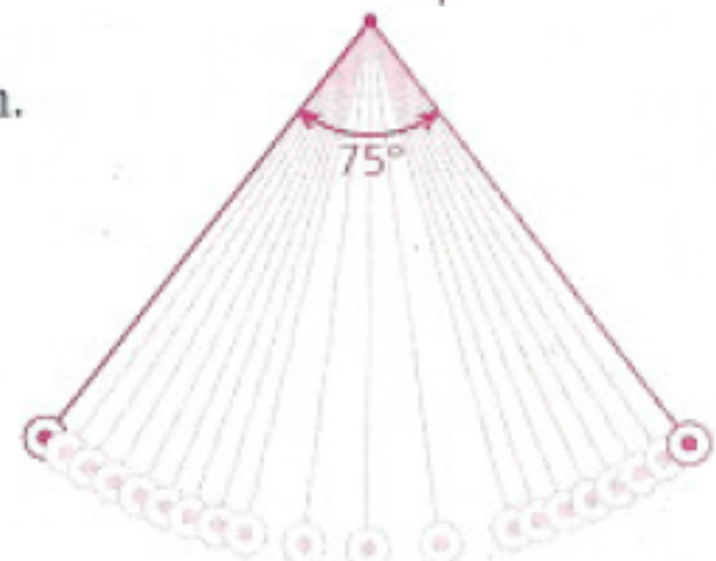
c



12 Find the outer perimeter of the figure, which is composed of semicircles mounted on the sides of a rectangle.



15 Find the distance traveled in one back-and-forth swing by the weight of a 12-in. pendulum that swings through a 75° angle.



Objective

Students will be able to find the area of circles, sectors, and segments.

Circles Test on Thursday!!!

Area of a Circle

$$A = \pi r^2$$

where r is the radius

1) Find the area of a circle whose diameter is 10.

The radius is half the diameter, so $r = 5$

$$A = \pi(5)^2$$
$$= 25\pi \text{ sq units}$$

2) Find the circumference of a circle whose area is 49π sq units.

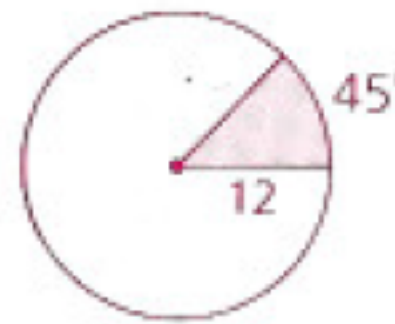
$$A = \pi(\mathrm{r})^2$$
$$49\pi = \pi(\mathrm{r})^2$$
$$7 = \mathrm{r}$$

$$C = 2\pi r$$
$$= 2\pi(7)$$
$$= 14\pi$$

Area of a Sector

A <u>sector</u> of a circle is a region bounded by two radii and an arc of the circle.

Find the area of a sector with a radius of 12 and a 45^o arc.



**Hint: think about how we find arc length

$A = \pi(12)^2$ 45^o arc is 45/360 or 1/8
$= 144\pi$ of the circle

$$A = \frac{1}{8}(144\pi) = 18\pi \text{ sq units}$$

Area of a Sector

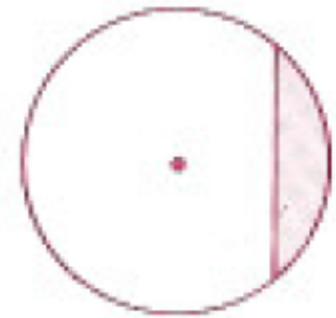
The area of a sector of a circle is equal to the area of the circle times the fractional part of the circle determined by the sector's arc.

$$\text{Area of sector} = \left(\frac{m\overset{\frown}{HP}}{360}\right)\pi r^2$$

where *r* is the radius and $\overset{\frown}{HP}$ is measured in degrees.

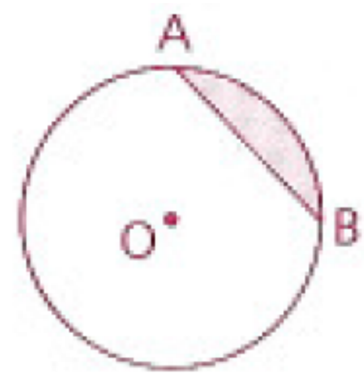
Area of a Segment

A <u>segment</u> of a circle is a region bounded by a chord of the circle and its corresponding arc.



The measure of the arc of the segment $\overset{\frown}{AB}$ is 90. The radius of the circle is 10. Find the area of the segment.

Draw radii to the endpoints of AB, forming sector AOB.



Area of segment = area of sector AOB – area of △AOB

$$=\left(\frac{m\overset{\frown}{AB}}{360}\right)\pi r^2-\frac{1}{2}bh=\left(\frac{90}{360}\right)\pi(10^2)-\frac{1}{2}(10)(10)=25\pi-50$$

Homework p.539: 1, 7, 9, 11, 13, 14 (a-e)

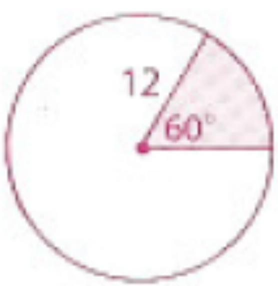
1 Find the areas and circumferences of circles with the following radii.

a 1 **b** 8 **c** 15

7 Find the total area of the region shown.

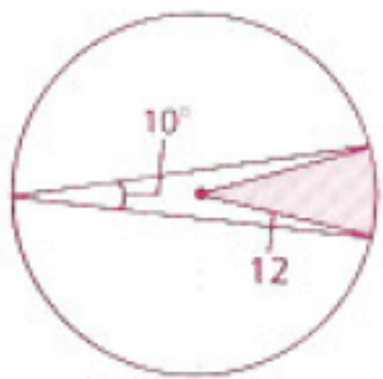
9 Find the area of each sector.

a



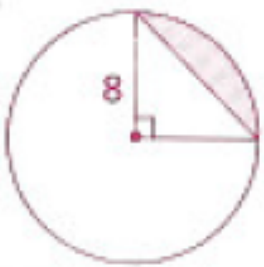
b

c

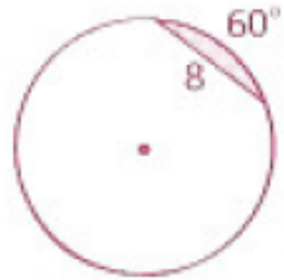


11 Find the area of each segment.

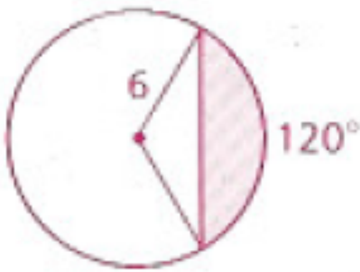
a



b

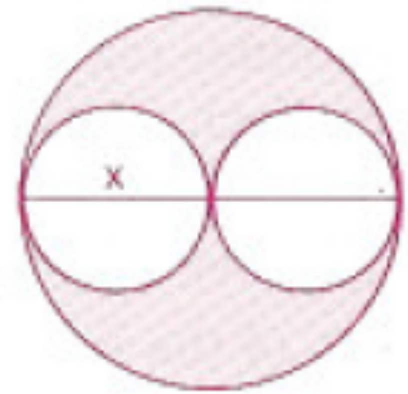


c



13 a What is the area of the shaded region if x = 6? If x = 10? If x = 7?

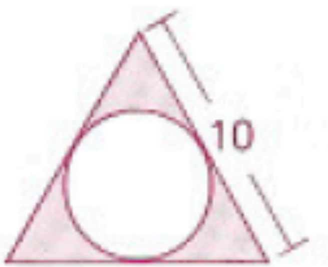
b What observation can you make about the shaded region's area?



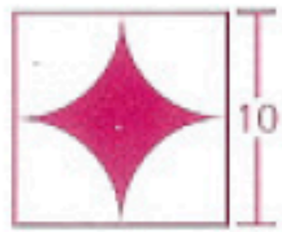
14 Find the area of the shaded part of each figure. (Assume regular polygons.)

a

b



c



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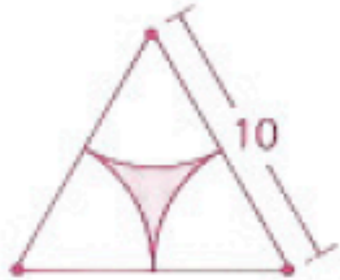

e