Objective

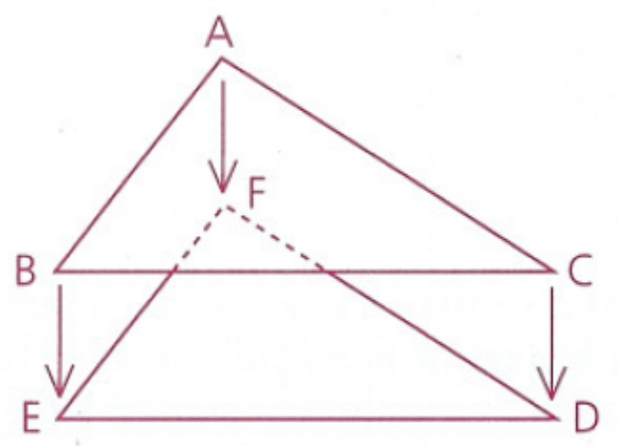
Students will be able to use their knowledge of transformations in order show two polygons are congruent by mapping them on to themselves.

Congruent Figures

Figures are congruent if one of them could be placed on top of the other and fit exactly, point for point, side for side, and angle for angle.

Congruent figures have the same size and shape.

Congruent triangles- all pairs of corresponding parts are congruent



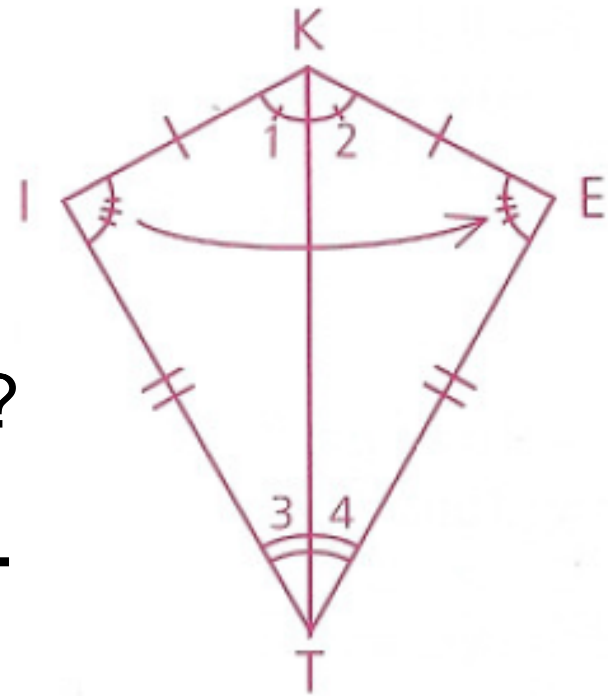
Would the statement $\triangle ABC \cong \triangle DEF$ be true?

No!

Congruent polygons- all pairs of corresponding parts are congruent

Notice that $\triangle KET$ is a ***reflection*** of $\triangle KIT$ over $\overline{KT}$.

$\angle I$ reflects onto $\angle E$.
$\angle 1$ reflects onto $\angle 2$.
$\angle 3$ reflects onto $\angle 4$.
$\overline{KI}$ reflects onto $\overline{KE}$.
$\overline{IT}$ reflects onto $\overline{ET}$.



What else do you notice?

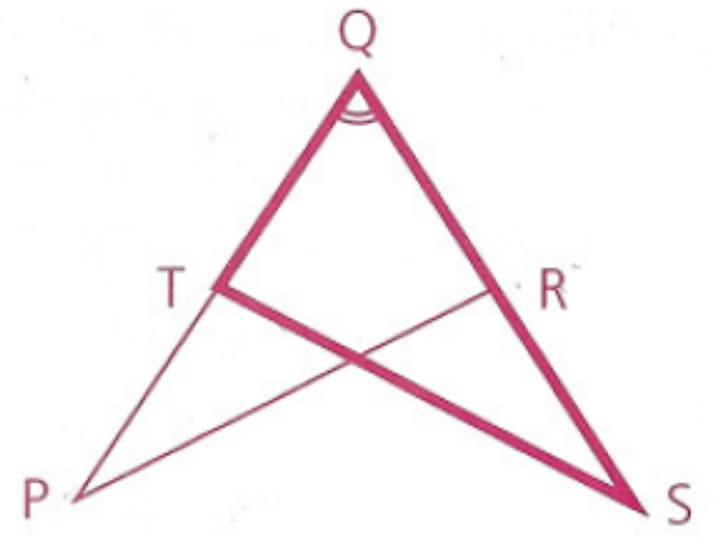
$\overline{KT}$ reflects onto itself.

Whenever a side or an angle is shared by two figures, we can say that the side or angle is congruent to itself, which is the Reflexive Property

Postulate: Any segment or angle is congruent to itself. (Reflexive Property)

$\angle PQR$, in ΔPQR, is congruent to $\angle SQT$, in ΔSQT, by the Reflexive Property.

Notice that $\angle SQT$ and $\angle PQR$ are actually different names for the same angle. We used different names so that you could see that the angle belonged to two different triangles.



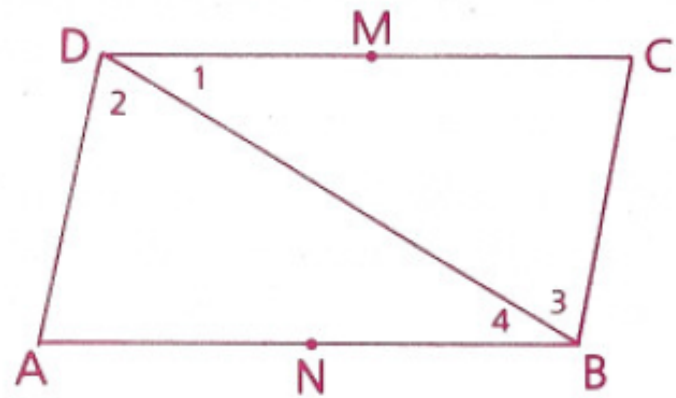
In the following problems, use the properties we learned last chapter to justify each conclusion.

Problem 1

Given: M and N are midpoints.
$\overline{DC} \cong \overline{AB}$, $\overline{AB} \cong \overline{DB}$,
$\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$

Conclusions:
a $\angle ADC \cong \angle ABC$
b $\overline{CM} \cong \overline{AN}$
c $\overline{BD} \cong \overline{DB}$
d $\overline{DC} \cong \overline{DB}$

a Addition Property
b Division Property
c Reflexive Property
d Transitive Property

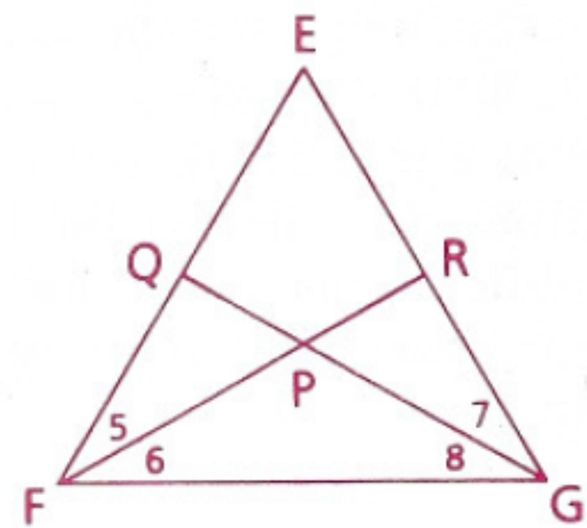


Problem 2

Given: $\overrightarrow{FP}$ and $\overrightarrow{GP}$ are angle bisectors.
$\angle 5$ is an acute angle.
$\angle 5 \cong \angle 7$, $\overline{PF} \cong \overline{PG}$, $\overline{QG} \cong \overline{FR}$

Conclusions:
a $\angle QFG \cong \angle RGF$
b $\overline{QP} \cong \overline{PR}$
c $\angle 7$ is an acute angle.
d $\angle FER \cong \angle GEQ$

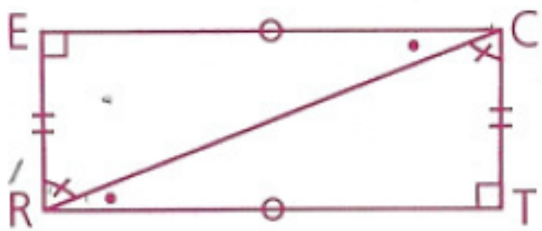
a Multiplication Property
b Subtraction Property
c Substitution
d Reflexive Property



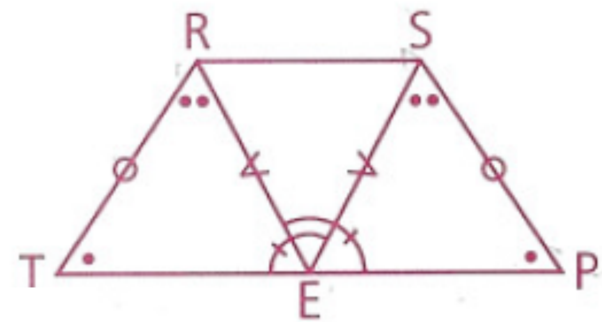
Homework p.114: 1-5

In problems 1–3, indicate which triangles are congruent. Be sure to have the correspondence of letters correct.

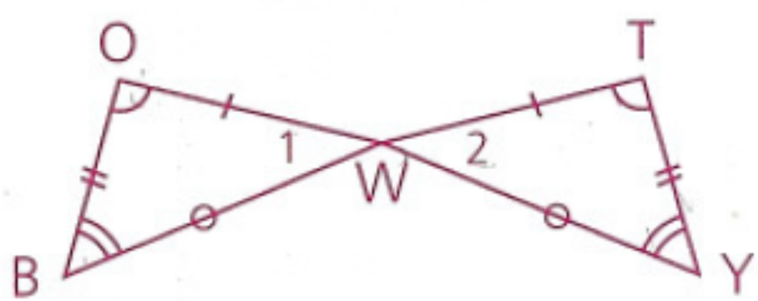
1 $\triangle ERC \cong$ ___?___
Why is $\overline{RC} \cong \overline{RC}$?



2 E is the midpt. of $\overline{TP}$.
$\triangle SPE \cong$ ___?___



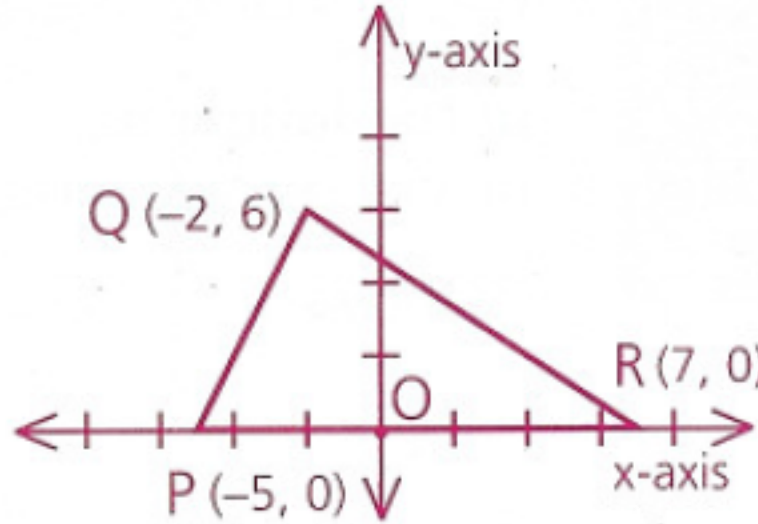
3 $\triangle BOW \cong$ ___?___
Why is $\angle 1 \cong \angle 2$?



4 **a** Copy $\triangle PQR$. Draw its reflection over the x-axis and give the coordinates of the vertices.

b Copy $\triangle PQR$. Draw its reflection over the y-axis and give the coordinates of the vertices.

c Copy $\triangle PQR$. Slide it 3 units to the left and give the coordinates of the vertices.



5 **a** Draw the rotation of $\triangle PQR$ 180° clockwise about O. Label its vertices with their coordinates.

b Draw the slide of $\triangle PQR$ along ray $\overrightarrow{PR}$ so that P is at O, and label its vertices with their coordinates.

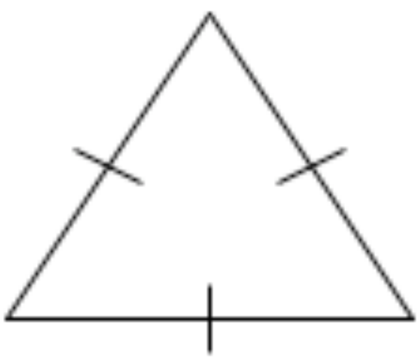
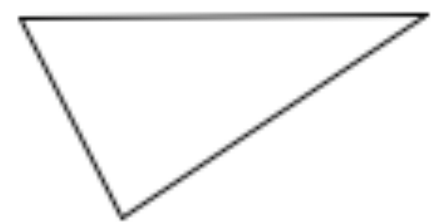
c Draw the reflection of $\triangle PQR$ over the y-axis and label its vertices with their coordinates.

Objective

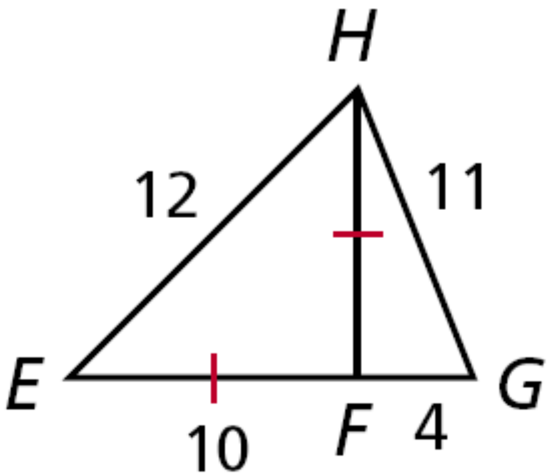
Students will be able to name the various types of triangles and their parts.

A triangle can be classified 2 ways:

1. By Sides:

Equilateral Δ	Isosceles Δ	Scalene Δ
		
3 ≅ sides	At least 2 ≅ sides	no ≅ sides

Examples:



Classify △*EHF* by its side lengths.

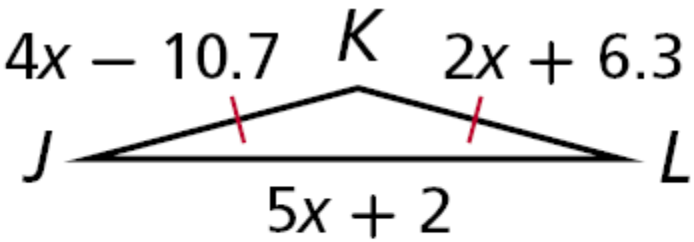
Isosceles

Classify △*EHG* by its side lengths.

Scalene

Example:

Find the side lengths of △*JKL*.



$4x - 10.7 = 2x + 6.3$

$2x = 17.0$

$x = 8.5$

$JK = 4(8.5) - 10.7 = 23.3$

$KL = 23.3$

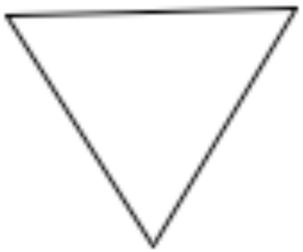
$JL = 5(8.5) + 2 = 44.5$

2. By Angles:

Acute Δ

3 acute ∠s

Equiangular Δ



3 ≅ ∠s
(also acute)

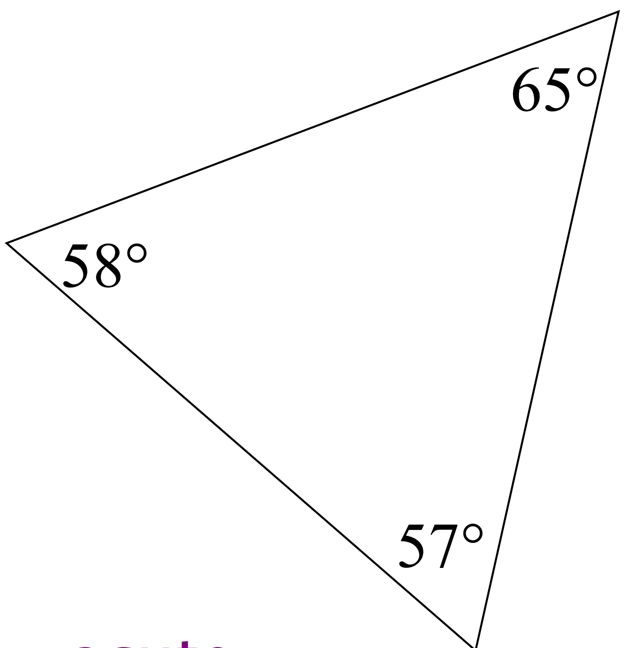
Right Δ

1 right ∠

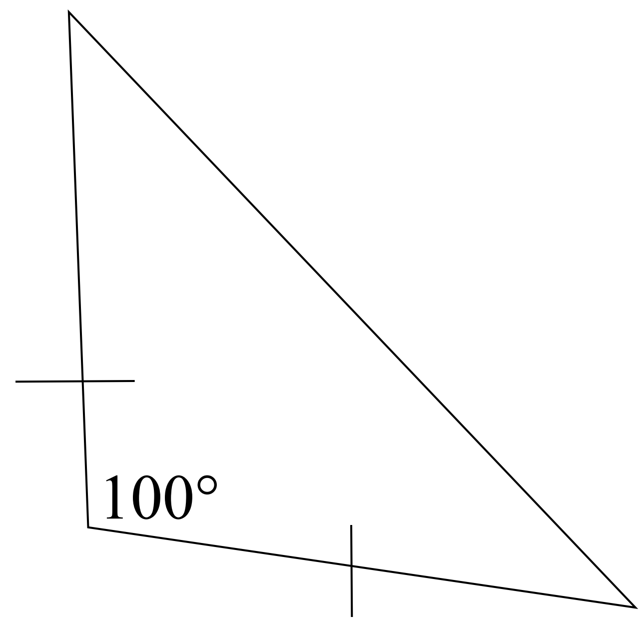
Obtuse Δ

1 obtuse ∠

Ex: How would you classify these triangles?

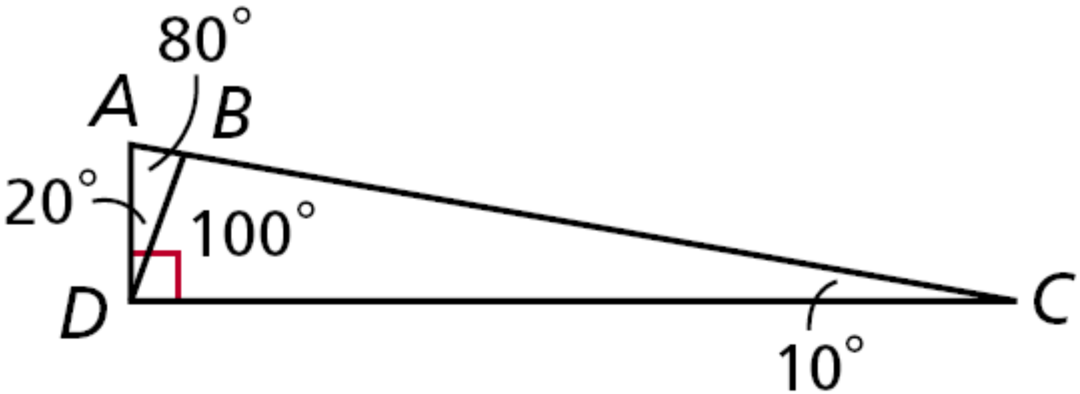


acute
scalene



obtuse
isosceles

More Examples:



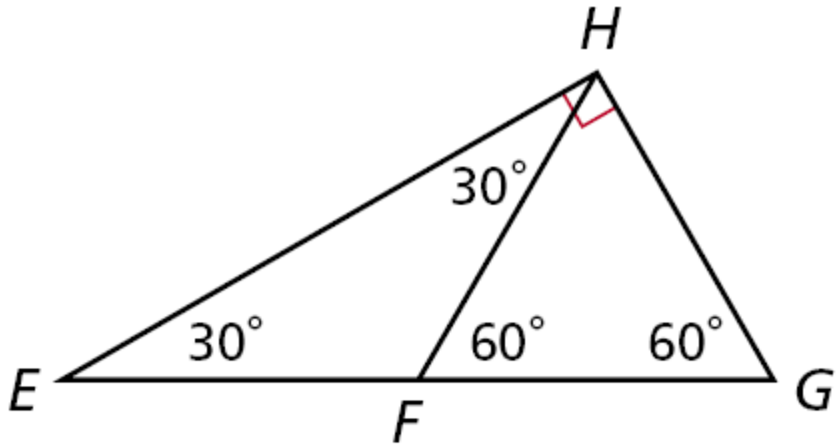
Classify △*BDC* by its angle measures.

Obtuse triangle

Classify △*ABD* by its angle measures.

Acute triangle

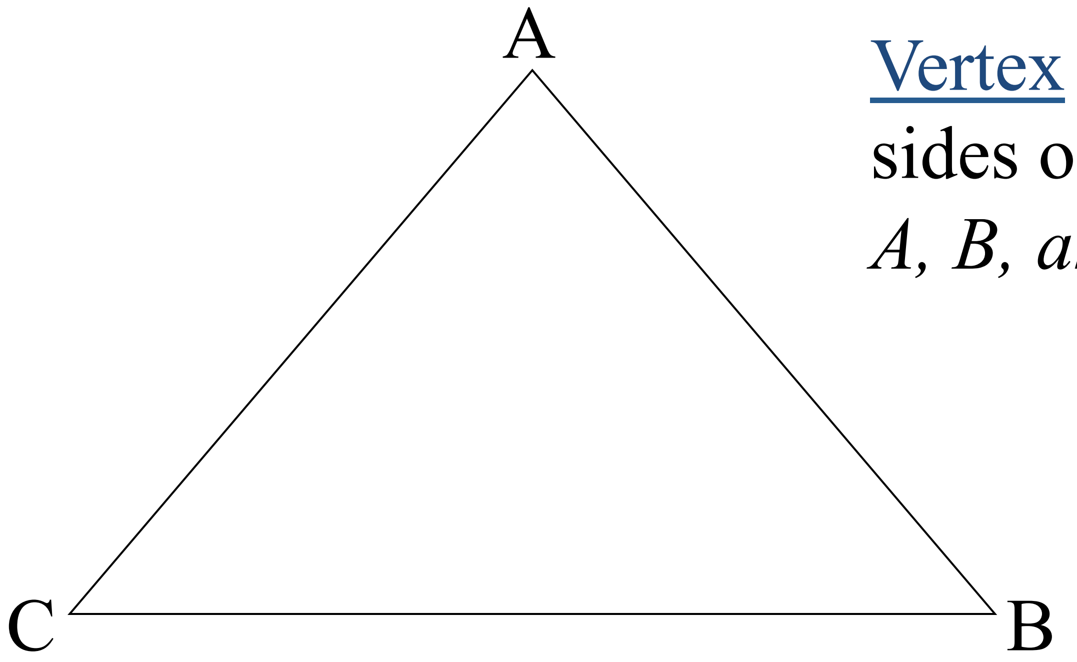
More Examples:



Classify △*FHG* by its angle measures.

Equiangular triangle

Anatomy of a Triangle



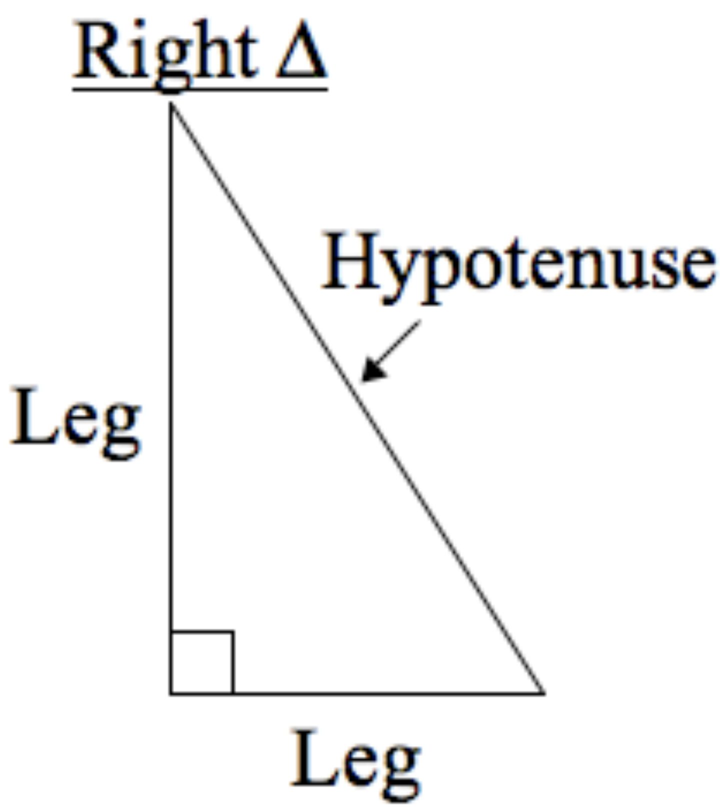
Vertex - each point joining the sides of the triangle

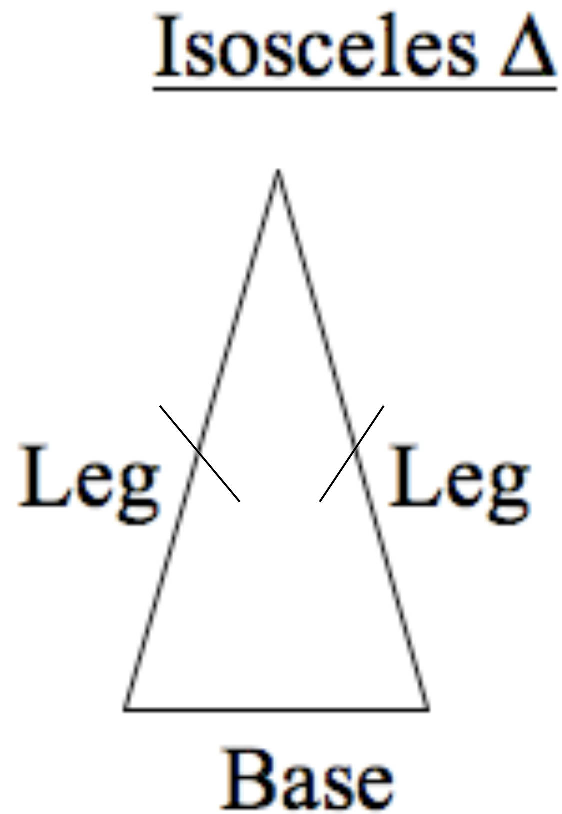
A, B, and C are vertices.

Adjacent Sides - 2 sides that share a vertex (next to each other)

What are the adjacent sides here?

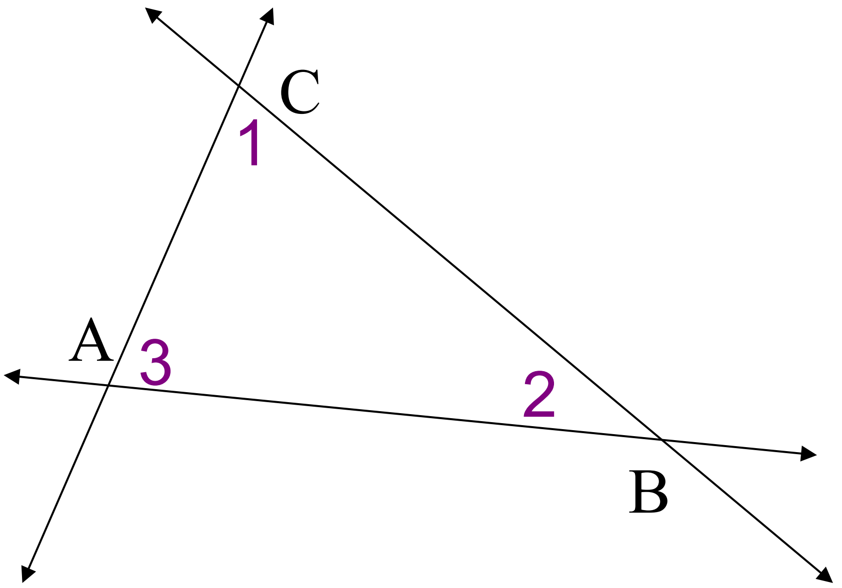
Isosceles and Right △s





Interior Angles

<u>Triangle Sum Theorem</u>: the sum of the interior ∠s of a △ is 180°

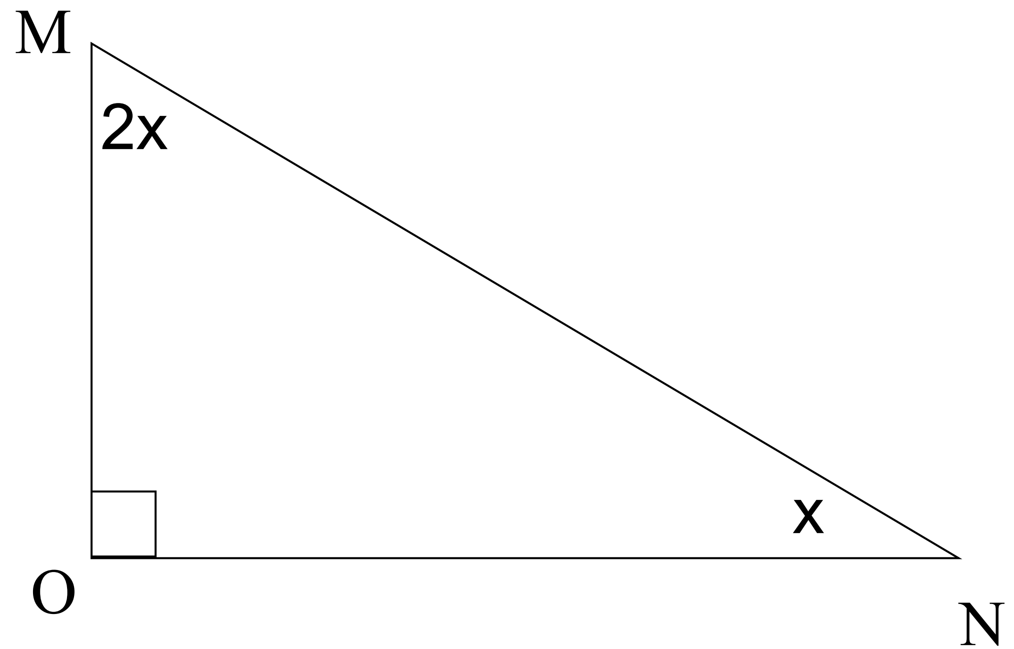


$\angle 1 + \angle 2 + \angle 3 = 180°$

Ex: 1 acute ∠ of a right △ is 2 times the other acute ∠. Find the measure of each acute ∠.

$$2x + x + 90 = 180$$
$$2x + x = 90$$
$$3x = 90$$
$$x = 30$$

$$\angle M = 2(30) = 60$$
$$\angle N = (30) = 30$$



*In a right △, the 2 acute ∠s are complements.

Homework

Finding Congruent Triangles Worksheet

You will need a ruler, compass, and protractor to help you complete this worksheet and for the next two days!

Think about each statement and decide whether it is true. Give good reasons for your answers. You might want to also sketch or construct a diagram to help explain your reasoning.

Objective

Students will be able to use the concepts of congruency and similarity to prove and evaluate proofs in a geometry context and be able to identify and understand the significance of a counter-example.

Must the Two Triangles be Congruent?

1.

One side of Triangle A is the same length as one side of Triangle B.

In your notebook, draw two triangles, Triangle A and Triangle B.

Make sure one side of Triangle A is the same length as one side of Triangle B.

Make up some other side lengths and angles on your example.

Is there a way to draw these triangles so they're not congruent?
Must Triangle A be congruent to Triangle B?

The triangles **can** be congruent, but do not **have** to be.

Must the Two Triangles be Congruent?

7.

One side of Triangle A is the same length as one side of Triangle B

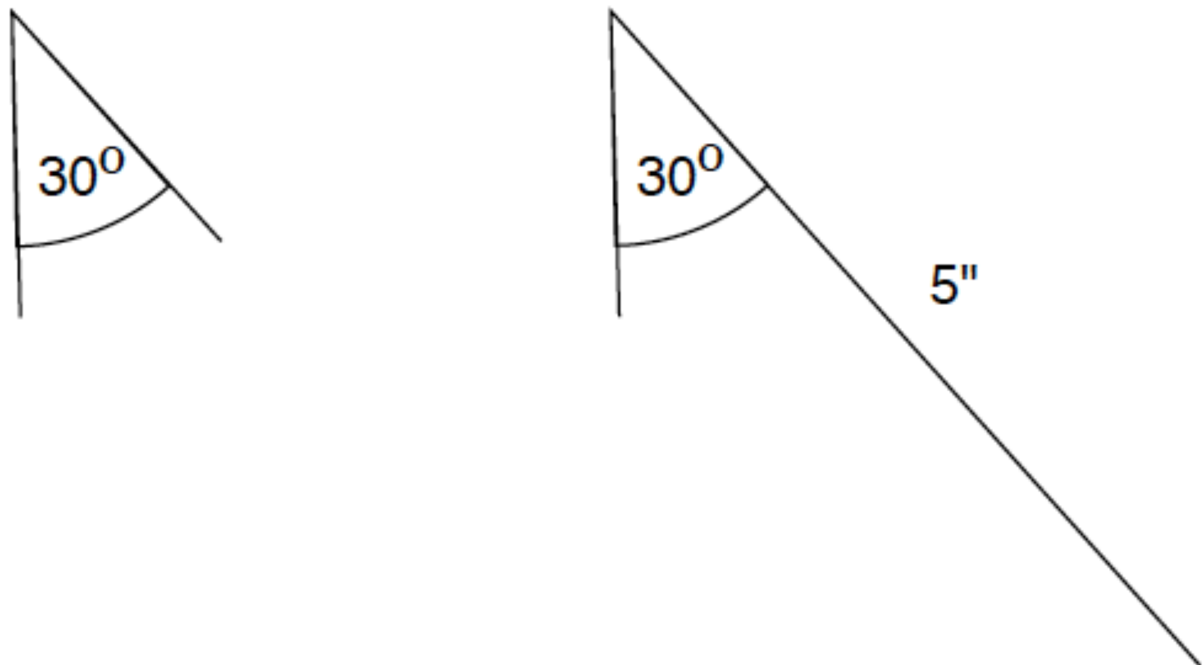
and

two angles in Triangle A are the same sizes as two angles in Triangle B.

In your notebook, draw two triangles with these properties.

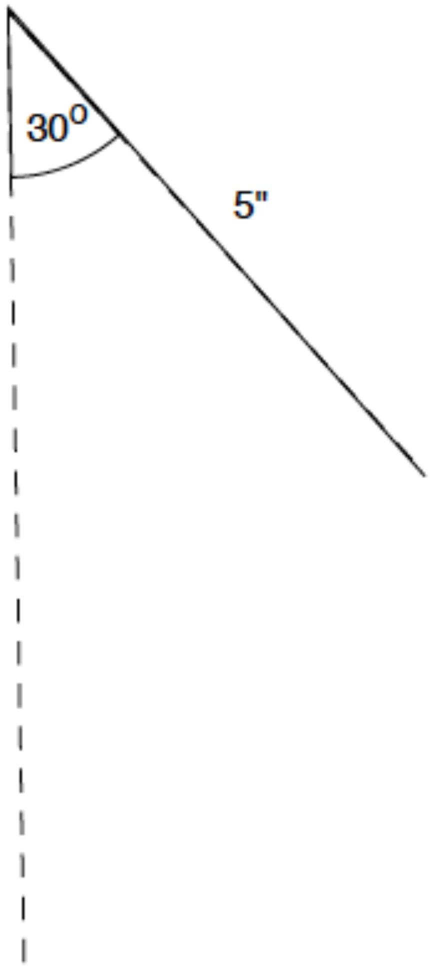
Is there a way to draw these triangles so they're not congruent?

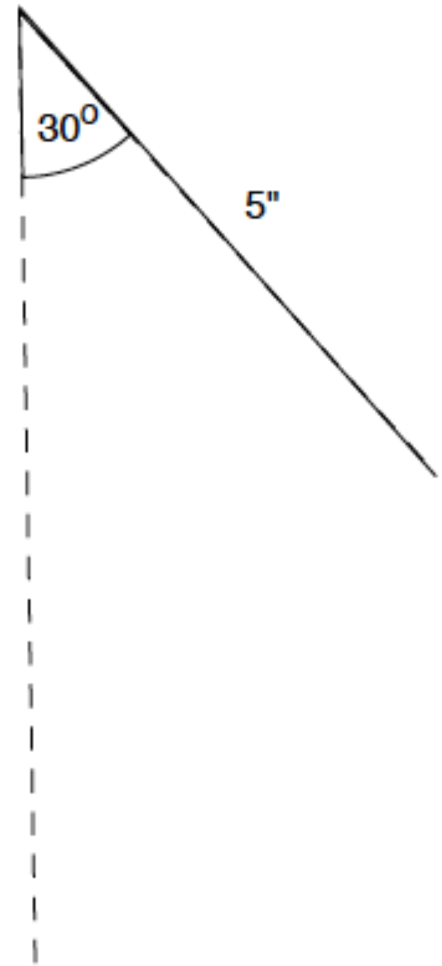
Card 7: Constructing Triangles



Suppose I choose angles 30°, 40° and a side 5" long.
Is there a way to make two triangles with these properties so they are not congruent?

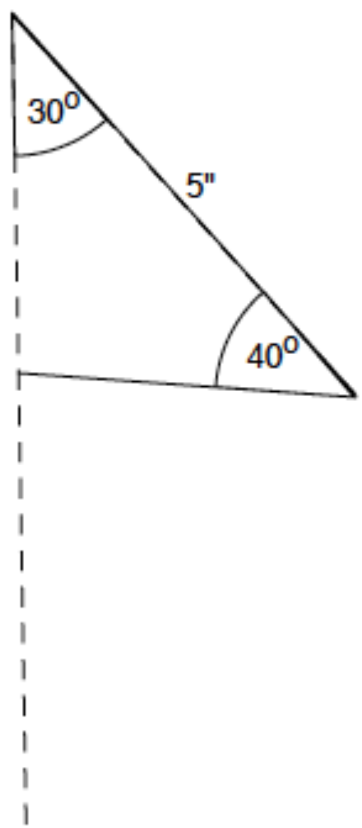
Card 7: Constructing Non-Congruent



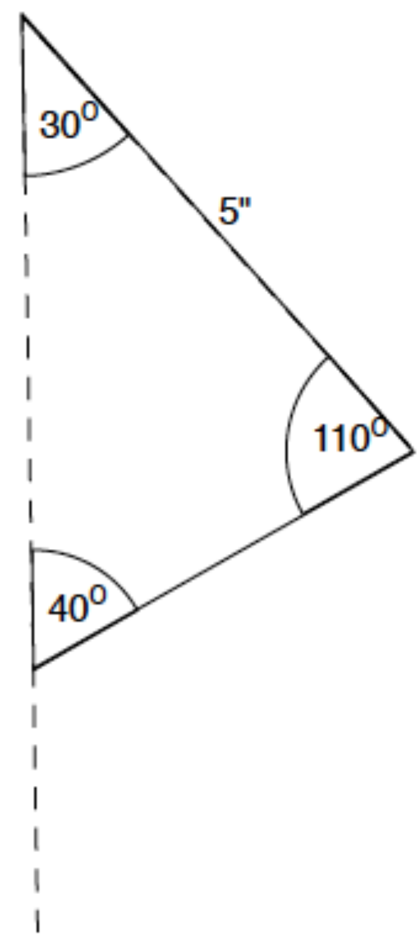


Where could I construct the 40° angle?

Card 7: Constructing Non-Congruent Triangles



Triangle 1



Triangle 2

Cards: Must the Two Triangles be Congruent?

1. One side of Triangle A is the same length as one side of Triangle B.	2. Two sides of Triangle A are the same lengths as two sides of Triangle B.	3. Three sides of Triangle A are the same lengths as three sides of Triangle B.
4. One side of Triangle A is the same length as one side of Triangle B and one angle in Triangle A is the same size as one angle in Triangle B.	5. Two sides of Triangle A are the same lengths as two sides of Triangle B and one angle in Triangle A is the same size as one angle in Triangle B.	6. Three sides of Triangle A are the same lengths as three sides of Triangle B. and one angle in Triangle A is the same size as one angle in Triangle B.
7. One side of Triangle A is the same length as one side of Triangle B and two angles in Triangle A are the same sizes as two angles in Triangle B.	**8.** Two sides of Triangle A are the same lengths as two sides of Triangle B and two angles in Triangle A are the same sizes as two angles in Triangle B.	9. Three sides of Triangle A are the same lengths as three sides of Triangle B. and two angles in Triangle A are the same sizes as two angles in Triangle B.

Must the Two Triangles be Congruent?

You need a ruler, compass, and protractor

For at least two other cards (not card 1 or 7):

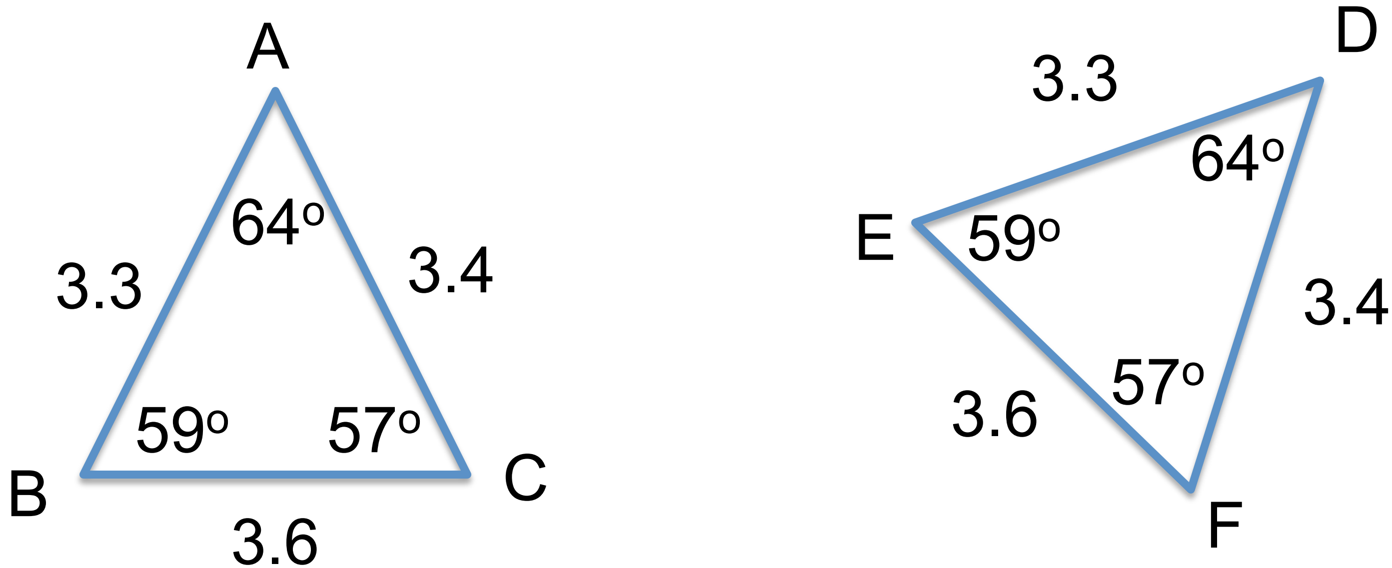
- Draw examples of pairs of triangles A and B that have the properties stated on the card
- Decide whether the two triangles **must** be congruent, and record your decision
- If you decide that the triangles **do not** have to be congruent, draw examples and explain why
- If you decide that triangles *must* be congruent, try to write a convincing proof

Make sure that you go through Card 5

Objective

Students will be able to determine whether two triangles are congruent when three parts of one triangle are congruent to three parts of another triangle.

Triangle Congruence



Is △ABC ≅ △DEF?

Triangle Congruency

If the corresponding sides and corresponding angles of two triangles are congruent, then the triangles are congruent. This gives an exact method for proving triangles are congruent, but checking the six pairs of corresponding parts is a great deal of work.

You can *sometimes* determine whether two triangles are congruent when three parts of one triangle are congruent to three parts of another triangle.

Three Parts of a Triangle

Three Parts	Abbreviation Triplet	Meaning for a Triangle	Example for $\triangle ABC$
Three angles	AAA	Three angles of the triangle	$\angle A, \angle B, \angle C$
Two angles, one side	ASA	Two angles and the side between them	$\angle A, \overline{AB}, \angle B$
	AAS	Two angles and a side not between the angles	$\angle A, \angle B, \overline{BC}$
Two sides, one angle	SAS	Two sides and the angle between them	$\overline{AC}, \angle A, \overline{AB}$
	SSA	Two sides and an angle not between the sides	$\overline{AC}, \overline{AB}, \angle B$
Three sides	SSS	Three sides of the triangle	$\overline{AC}, \overline{AB}, \overline{BC}$

In-Class Experiment

Use a ruler and protractor and work with a partner to determine which of the triplets below guarantee triangle congruence. For each triplet, try to build two noncongruent triangles with the given angle measures and side lengths.

4. **ASA** $m\angle A = 40°, AB = 2 \text{ in.}, m\angle B = 70°$

5. **AAS** $m\angle A = 40°, m\angle B = 70°, BC = 2 \text{ in.}$

6. **SAS** $AC = 2 \text{ in.}, m\angle A = 60°, AB = 3 \text{ in.}$

7. **SSA** $AC = 2 \text{ in.}, AB = 4 \text{ in.}, m\angle B = 20°$

8. **SSS** $AC = 2 \text{ in.}, AB = 3 \text{ in.}, BC = 4 \text{ in.}$

Objective

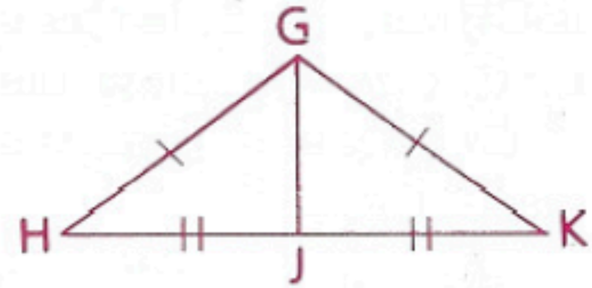
Students will be able to apply postulates of congruent triangles.

The SSS Postulate

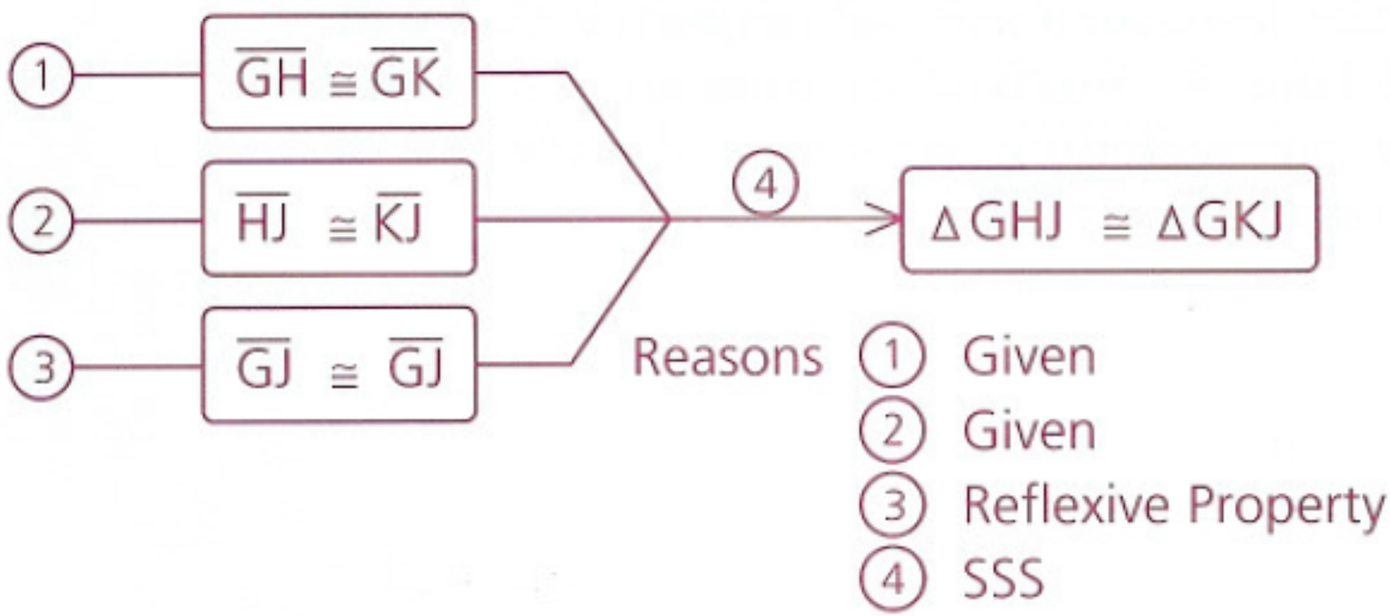
If there exists a correspondence between the vertices of two triangles such that three sides of one triangle are congruent to the corresponding sides of the other triangle, the two triangles are congruent (SSS).

The SSS relationship can be proved by methods that are not part of this course; we shall assume it and use the abbreviation SSS in proofs.

In the figure, is $\triangle GHJ$ congruent to $\triangle GKJ$ by SSS? The tick marks give us *two* pairs of congruent sides, but that is not enough. However, since $\overline{GJ}$ is a common side of both triangles, $\overline{GJ} \cong \overline{GJ}$ by the Reflexive Property. So we actually do have SSS!



The following diagram illustrates the flow of logic that proves that $\triangle GHJ$ and $\triangle GKJ$ are congruent.



The SAS Postulate

If there exists a correspondence between the vertices of two triangles such that two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle, the two triangles are congruent (SAS).

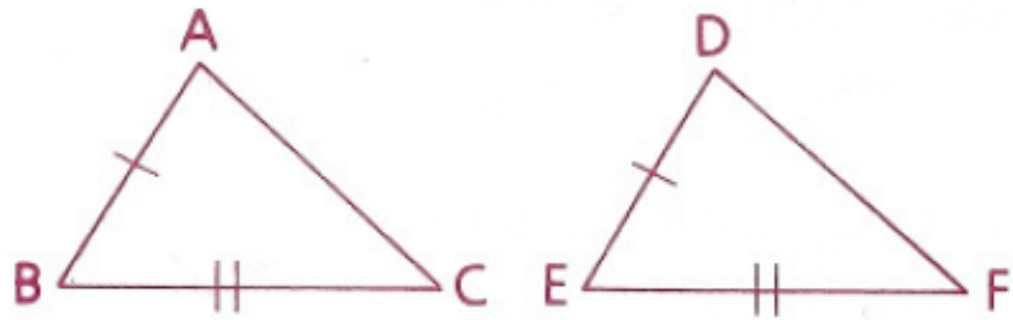
The ASA Postulate

If there exists a correspondence between the vertices of two triangles such that two angles and the included side of one triangle are congruent to the corresponding parts of the other triangle, the two triangles are congruent (ASA).

In problems 1–3 and 5, you are given the congruent angles and sides shown by the tick marks. Name the additional congruent sides or angles needed to prove that the triangles are congruent by each specified method.

Problem 1 **a** *SSS*

b *SAS*

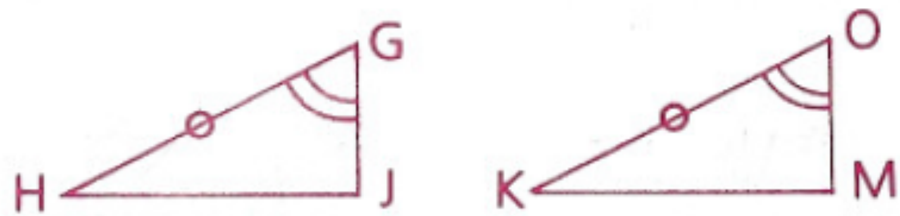


Answers **a** $\overline{AC} \cong \overline{DF}$

b $\angle B \cong \angle E$

Problem 2 **a** *SAS*

b *ASA*



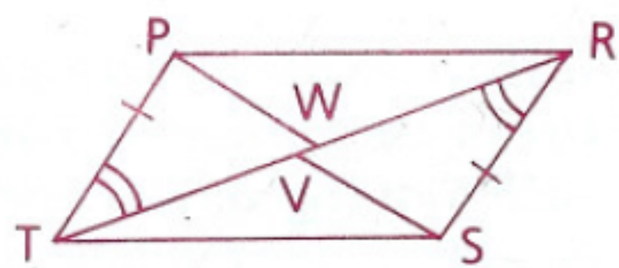
Answers **a** $\overline{GJ} \cong \overline{OM}$

b $\angle H \cong \angle K$

Problem 3 *Prove:* $\triangle PWT \cong \triangle SVR$

a *SAS*

b *ASA*

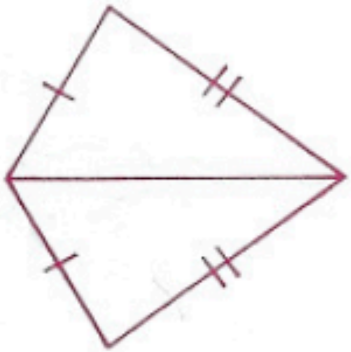


Answers **a** $\overline{TW} \cong \overline{RV}$

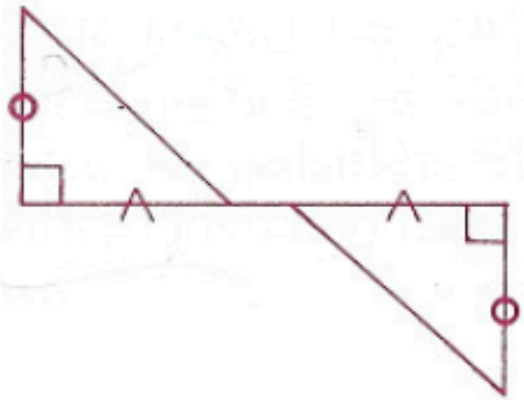
b $\angle TPW \cong \angle RSV$

Using the tick marks for each pair of triangles, name the method (SSS, SAS, or ASA), if any, that can be used to prove the triangles congruent.

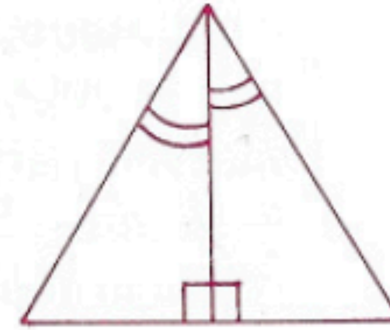
a



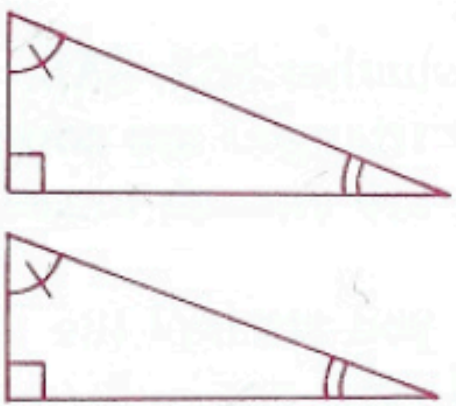
c



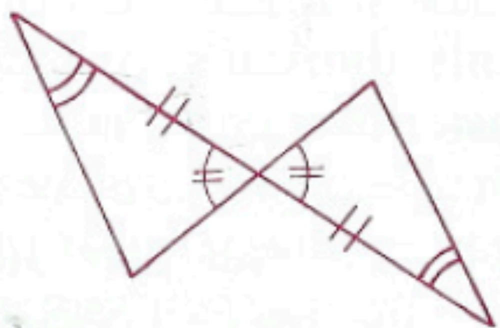
e



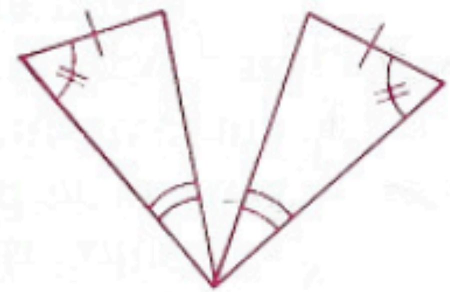
b



d

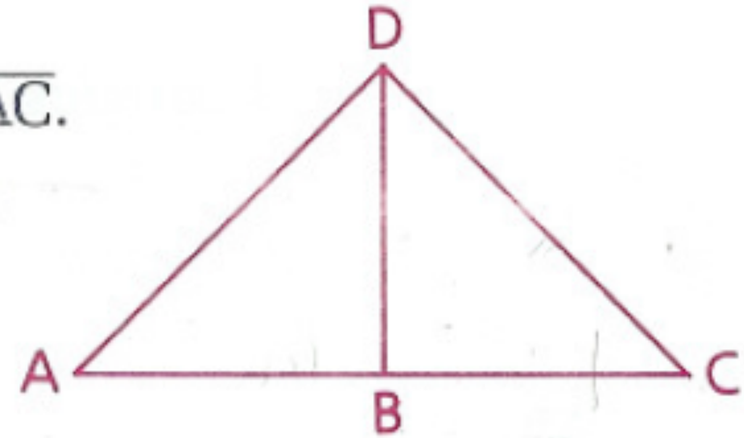


f



a SSS
b None
c SAS
d ASA
e ASA
f AAS

Given: $\overline{AD} \cong \overline{CD}$;
B is the midpoint of $\overline{AC}$.
Conclusion: $\triangle ABD \cong \triangle CBD$

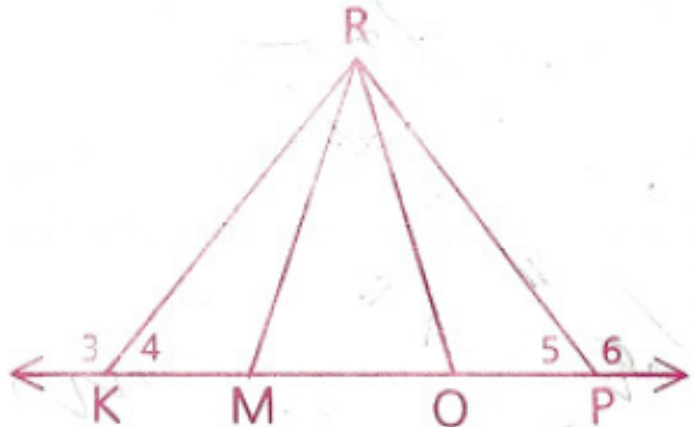


Statements	Reasons
1 $\overline{AD} \cong \overline{CD}$	1 Given
2 B is the midpt. of $\overline{AC}$.	2 Given
3 $\overline{AB} \cong \overline{CB}$	3 If a point is the midpoint of a segment, it divides the segment into two $\cong$ segments.
4 $\overline{BD} \cong \overline{BD}$	4 Reflexive Property
5 $\triangle ABD \cong \triangle CBD$	5 *SSS* (1, 3, 4)

Note After *SSS*, *SAS*, or *ASA* we shall identify the numbers of the statements in which the pairs of congruent parts were found.

Given: $\angle 3 \cong \angle 6$,
$\overline{KR} \cong \overline{PR}$,
$\angle KRO \cong \angle PRM$

Prove: $\triangle KRM \cong \triangle PRO$



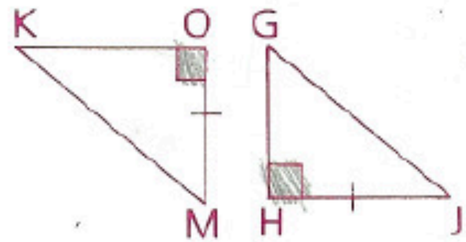
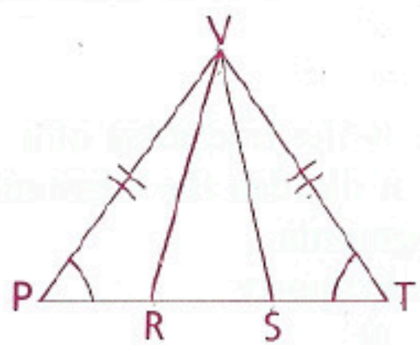
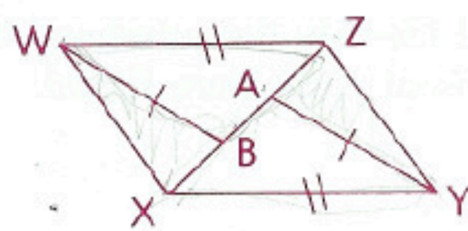
Statements	Reasons
1 $\angle 3 \cong \angle 6$	1 Given
2 $\angle 3$ is supp. to $\angle 4$.	2 If two $\angle$s form a straight $\angle$ (assumed from diagram), they are supp.
3 $\angle 5$ is supp. to $\angle 6$.	3 Same as 2
4 $\angle 4 \cong \angle 5$	4 Angles supp. to $\cong$ $\angle$s are $\cong$.
5 $\overline{KR} \cong \overline{PR}$	5 Given
6 $\angle KRO \cong \angle PRM$	6 Given
7 $\angle KRM \cong \angle PRO$	7 Subtraction Property
8 $\triangle KRM \cong \triangle PRO$	8 ASA (4, 5, 7)

Note The assumption of straight angles and the fact that two angles that form a straight angle are supplementary may now be combined in one step (as in step 2 above).

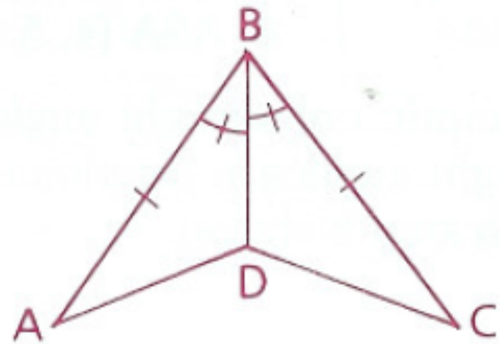
Homework

p. 120: 1, 3, 4, 6, 15

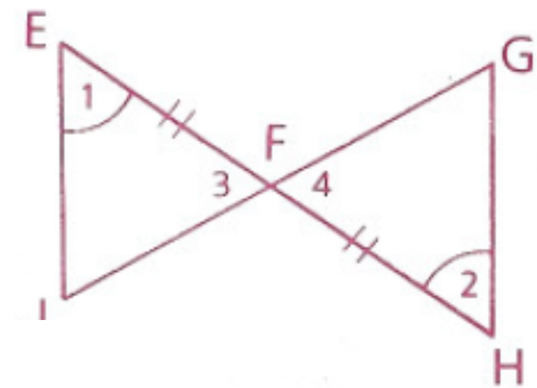
1 Study the congruent sides and angles shown by the tick marks, then identify the additional information needed to support the specified method of proving that the indicated triangles are congruent.

		Triangles	Method	Needed Information
a		△HGJ and △OKM	SAS ASA	? ?
b		△PSV and △TRV	SAS ASA	? ?
c		△WBZ and △YAX	SSS SAS	? ?

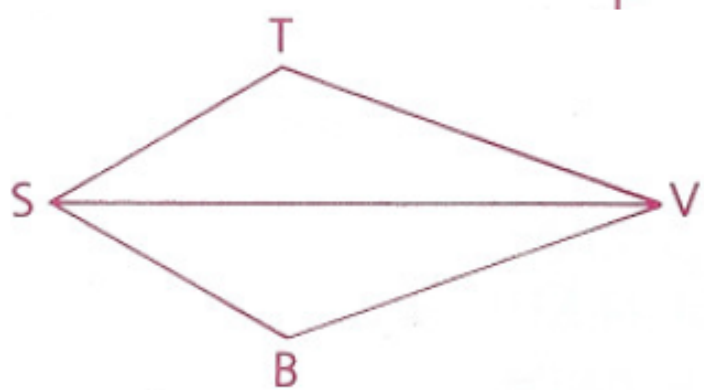
3 Given: $\overline{AB} \cong \overline{CB}$,
$\angle ABD \cong \angle CBD$
Prove: $\triangle ABD \cong \triangle CBD$



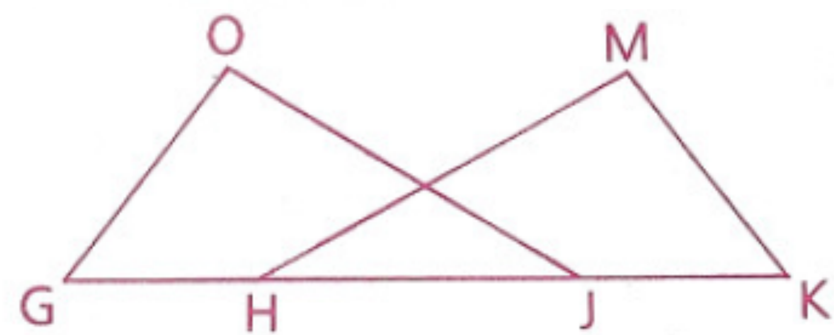
4 Given: $\angle 1 \cong \angle 2$,
$\overline{EF} \cong \overline{HF}$
Prove: $\triangle EFJ \cong \triangle HFG$



6 Given: $\overrightarrow{SV}$ bisects $\angle TSB$.
$\overrightarrow{VS}$ bisects $\angle TVB$.
Prove: $\triangle TSV \cong \triangle BSV$



15 Given: $\overline{GH} \cong \overline{KJ}$,
$\overline{HM} \cong \overline{JO}$,
$\overline{GO} \cong \overline{KM}$
Prove: $\triangle GOJ \cong \triangle KMH$

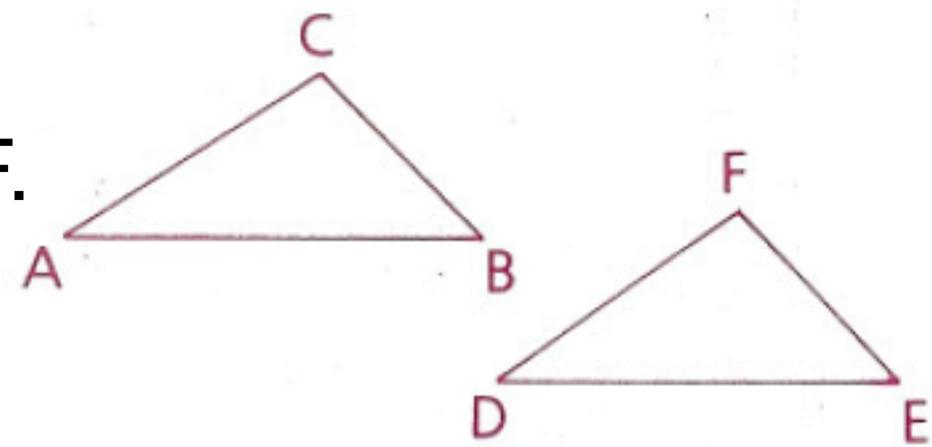


Objective

Students will be able to apply the principle of CPCTC and recognize some basic principles of circles.

CPCTC

Suppose that in the figure $\triangle ABC \cong \triangle DEF$. Is it therefore that $\angle B \cong \angle E$?



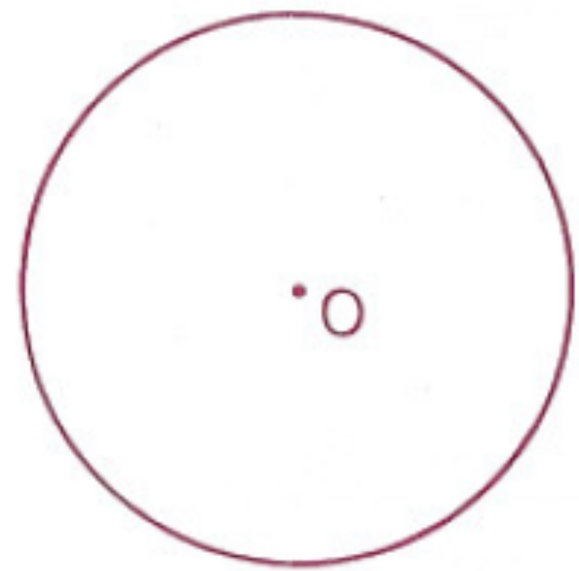
Yes, because of the definition of congruent triangles

We will often draw such conclusions after knowing that some triangles are congruent and CPCTC is the reason ("Corresponding Parts of Congruent Triangles are Congruent")

By corresponding parts, we mean only the matching angles and sides of the respective triangles.

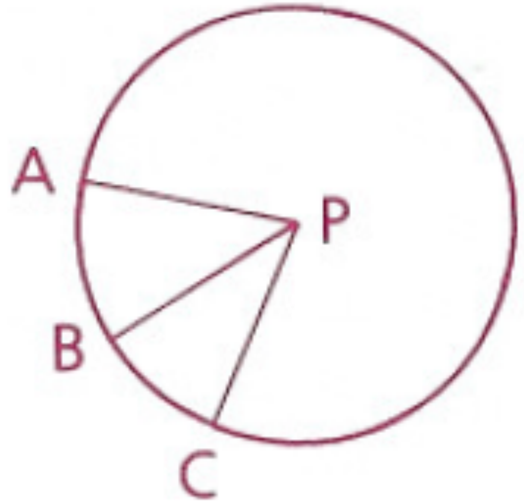
Introduction of Circles

Point O is the center of the circle shown at the right. By definition, every point of the circle is the same distance from the center. A circle is named by its center.



circle O (or ⊙O)

Points A, B, and C lie on circle P (⊙P).
$\overline{PA}$ is called a *radius*.
$\overline{PA}$, $\overline{PB}$, and $\overline{PC}$ are called *radii*.



Formula for area of a circle:

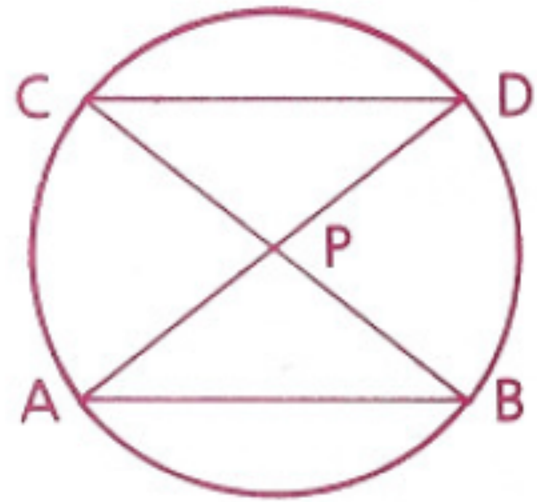
$$A = \pi r^2$$

Formula for circumference of a circle:

$$C = 2\pi r$$

Given: $\odot P$

Conclusion: $\overline{AB} \cong \overline{CD}$



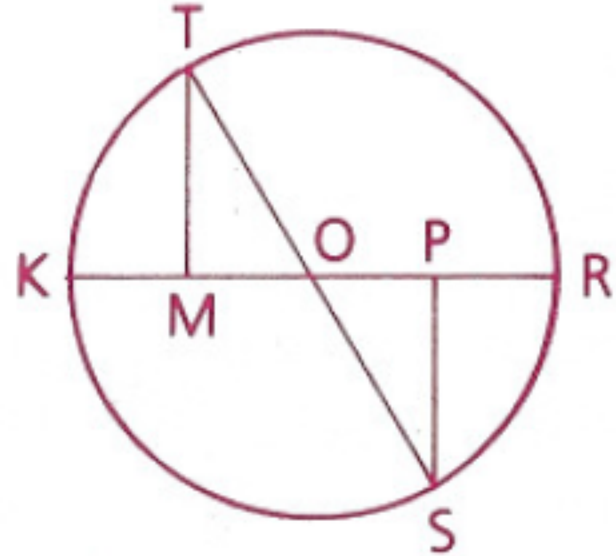
Statements	Reasons
1 $\odot P$	1 Given
2 $\overline{PA} \cong \overline{PB} \cong \overline{PC} \cong \overline{PD}$	2 All radii of a circle are $\cong$.
3 $\angle CPD \cong \angle APB$	3 Vertical angles are $\cong$.
4 $\triangle CPD \cong \triangle APB$	4 SAS (2, 3, 2)
5 $\overline{AB} \cong \overline{CD}$	5 CPCTC (Corresponding parts of congruent triangles are congruent.)

Given: $\odot O$;

$\angle T$ *is comp. to* $\angle MOT$.

$\angle S$ *is comp. to* $\angle POS$.

Prove: $\overline{MO} \cong \overline{PO}$

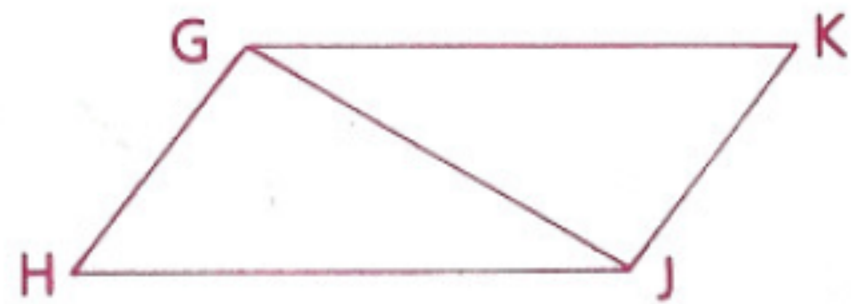


Statements	Reasons
1 $\odot O$	1 Given
2 $\overline{OT} \cong \overline{OS}$	2 All radii of a circle are $\cong$.
3 $\angle T$ is comp. to $\angle MOT$.	3 Given
4 $\angle S$ is comp. to $\angle POS$.	4 Given
5 $\angle MOT \cong \angle POS$	5 Vertical angles are $\cong$.
6 $\angle T \cong \angle S$	6 Complements of $\cong$ $\angle$s are $\cong$.
7 $\triangle MOT \cong \triangle POS$ (Watch the correspondence.)	7 ASA (5, 2, 6)
8 $\overline{MO} \cong \overline{PO}$	8 CPCTC

Homework p. 127: 2, 3, 6, 14, 15

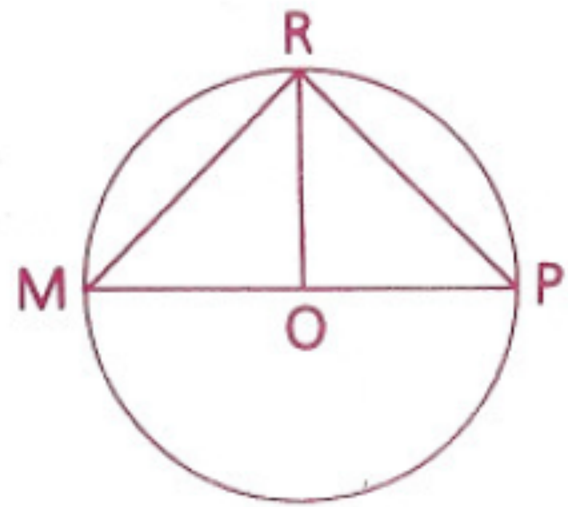
2 Given: $\angle HGJ \cong \angle KJG$,
$\angle KGJ \cong \angle HJG$

Conclusion: $\overline{HG} \cong \overline{KJ}$

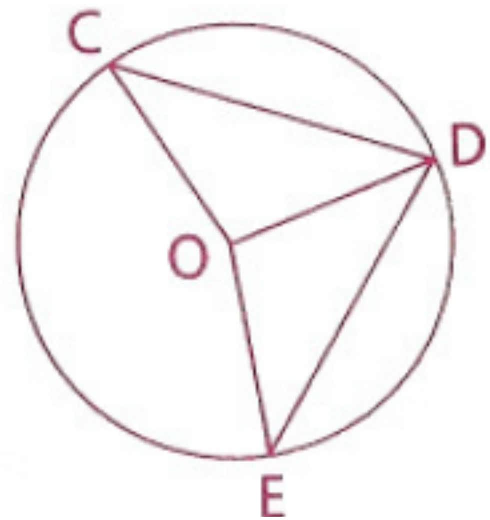


3 Given: $\odot O$,
$\overline{RO} \perp \overline{MP}$

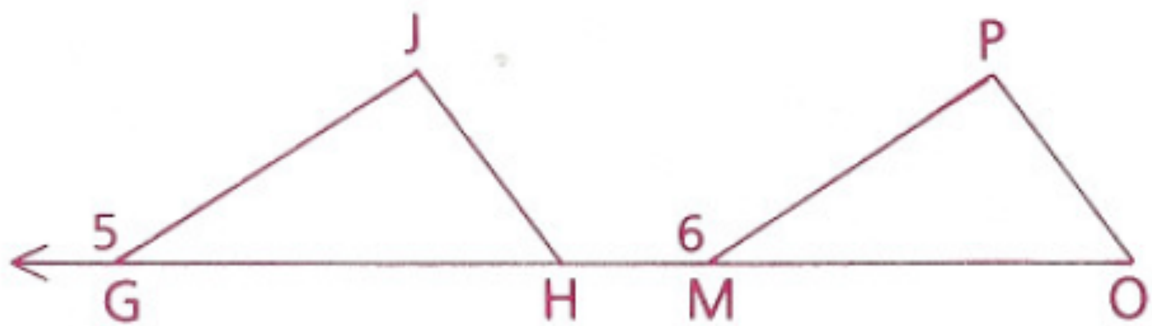
Prove: $\overline{MR} \cong \overline{PR}$



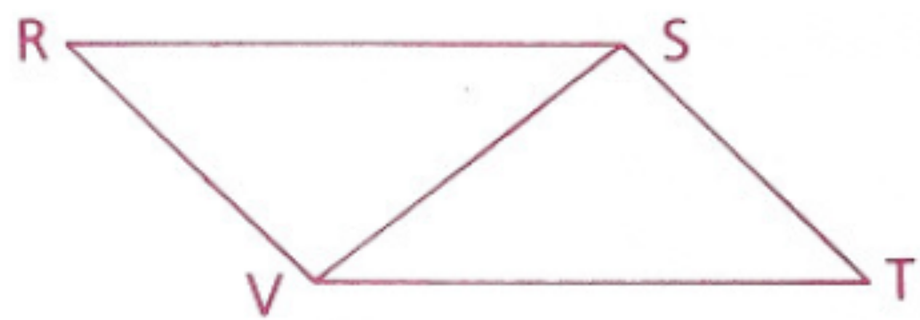
6 Given: $\odot O$,
$\overline{CD} \cong \overline{DE}$
Prove: $\angle COD \cong \angle DOE$



14 Given: $\angle 5 \cong \angle 6$,
$\angle JHG \cong \angle O$,
$\overline{GH} \cong \overline{MO}$
Conclusion: $\angle J \cong \angle P$



15 Given: $\angle RST \cong \angle RVT$,
$\angle RVS \cong \angle TSV$
Conclusion: $\overline{RS} \cong \overline{VT}$



Objective

Students will be able to use the HL postulate to prove right triangles congruent and apply the No-Choice Theorem and the AAS theorem.

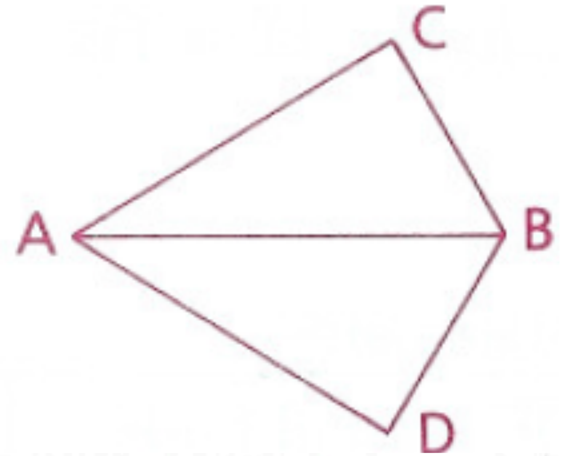
The HL Postulate

If there exists a correspondence between the vertices of two right triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the other triangle, the two right triangles are congruent (HL).

$\triangle ABC \cong \triangle DEF$ by HL

Given: $\overline{BC} \perp \overline{AC}$,
$\overline{BD} \perp \overline{AD}$,
$\overline{AC} \cong \overline{AD}$

Prove: $\overrightarrow{AB}$ *bisects* $\angle CAD$.



Statements	Reasons
1 $\overline{BC} \perp \overline{AC}$	1 Given
2 $\angle ACB$ is a right $\angle$.	2 If two segments are $\perp$, they form right $\angle$s.
3 $\overline{BD} \perp \overline{AD}$	3 Given
4 $\angle BDA$ is a right $\angle$.	4 Same as 2
5 $\overline{AC} \cong \overline{AD}$	5 Given
6 $\overline{AB} \cong \overline{AB}$	6 Reflexive Property
7 $\triangle ACB \cong \triangle ADB$	7 HL (2, 4, 6, 5)
8 $\angle CAB \cong \angle DAB$	8 CPCTC
9 $\overrightarrow{AB}$ bisects $\angle CAD$.	9 A ray that divides an $\angle$ into two $\cong$ $\angle$s bisects the $\angle$.

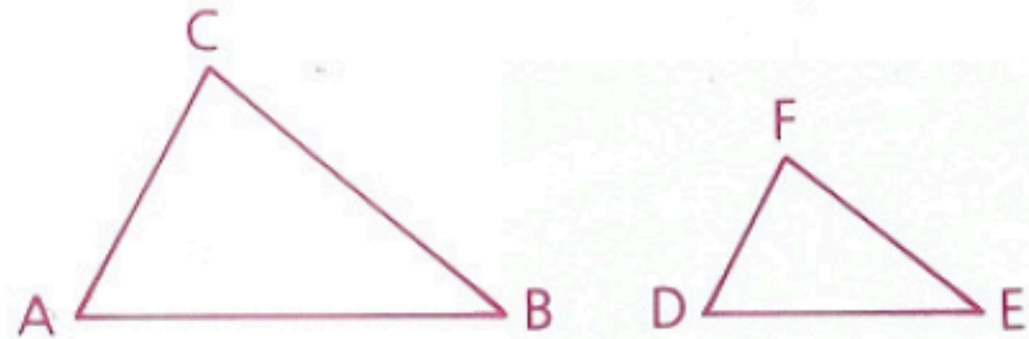
No-Choice Theorem (Third Angle Theorem)

We refer to the following theorem as the No-Choice Theorem since it shows that two angles "have no choice" but to be congruent.

Theorem 53: If two angles of one triangle are congruent to two angles of a second triangle, then the third angles are congruent. (No-Choice Theorem)

Given: $\angle A \cong \angle D$,
$\angle B \cong \angle E$

Conclusion: $\angle C \cong \angle F$



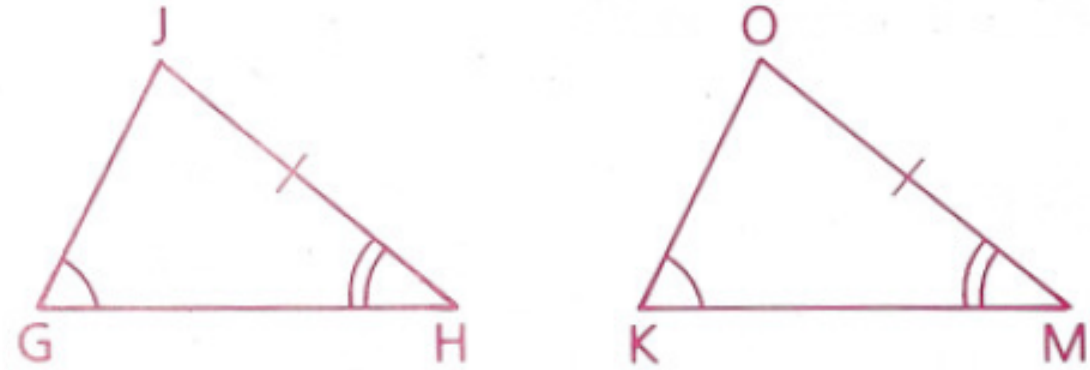
Proof: Since the sum of the angles in each triangle is 180°, the sums may be set equal. If we then apply the Subtraction Property, we see that $\angle C$ and $\angle F$ must be congruent.

Note The two triangles need not be congruent for us to apply the No-Choice Theorem.

Theorem 54: If there exists a correspondence between the vertices of two triangles such that two angles and a nonincluded side of one triangle are congruent to the corresponding parts of the other triangle, the two triangles are congruent (AAS).

Given: $\angle G \cong \angle K$,
$\angle H \cong \angle M$,
$\overline{JH} \cong \overline{OM}$

Prove: $\triangle GHJ \cong \triangle KMO$



Proof:

1 $\angle G \cong \angle K$	1 Given
2 $\angle H \cong \angle M$	2 Given
3 $\angle J \cong \angle O$	3 No-Choice Theorem
4 $\overline{JH} \cong \overline{OM}$	4 Given
5 $\triangle GHJ \cong \triangle KMO$	5 ASA (2, 4, 3)

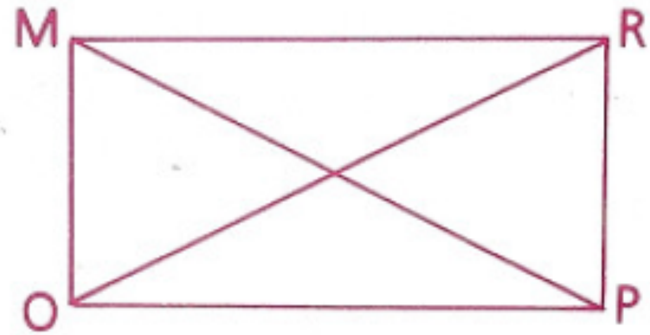
Homework

p. 158: 2, 4, 12
p. 304: 1, 2, 4, 5

p. 158: 2, 4, 12

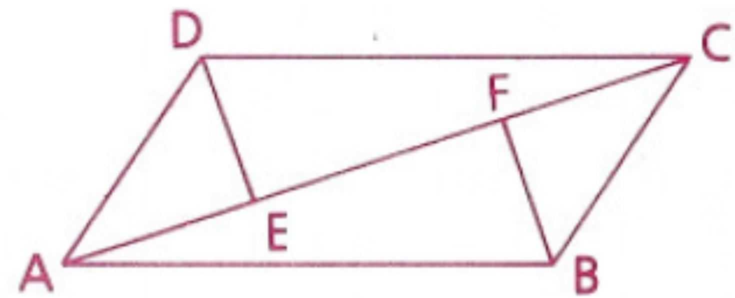
2 Given: $\overline{MO} \perp \overline{OP}$,
$\overline{RP} \perp \overline{OP}$,
$\overline{MP} \cong \overline{RO}$

Prove: $\triangle MOP \cong \triangle RPO$



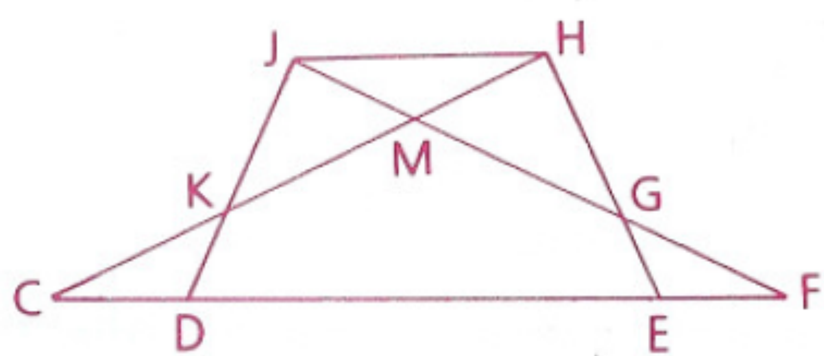
4 Given: $\overline{AE} \cong \overline{CF}$,
$\overline{AB} \cong \overline{CD}$;
$\angle BFA$ is a right angle.
$\angle DEC$ is a right angle.

Prove: $\angle CDE \cong \angle ABF$



12 Given: $\overline{CD} \cong \overline{EF}$,
$\overline{JF} \perp \overline{JD}$,
$\overline{CH} \perp \overline{HE}$,
$\overline{CH} \cong \overline{JF}$

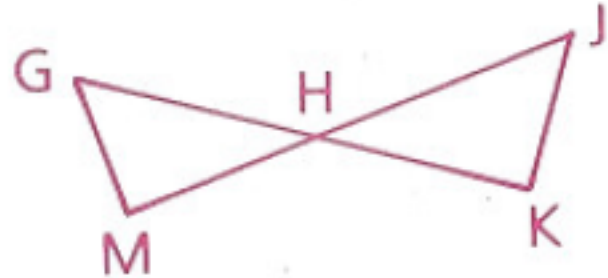
Prove: $\overline{JD} \cong \overline{HE}$



p. 304: 1, 2, 4, 5

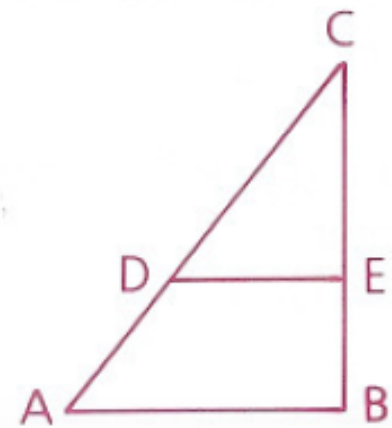
1 Given: $\overline{JM} \perp \overline{GM}$,
$\overline{GK} \perp \overline{KJ}$

Conclusion: $\angle G \cong \angle J$



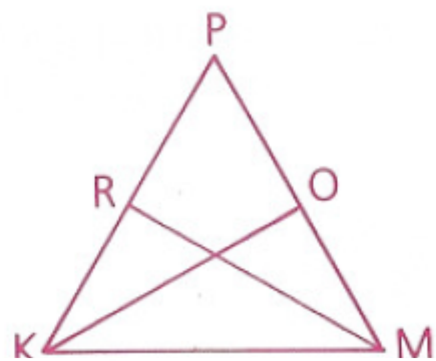
2 Given: $\overline{CB} \perp \overline{AB}$,
$\overleftrightarrow{DE} \parallel \overleftrightarrow{AB}$,
$\angle CDE = 40°$

Find: m∠A, m∠C, and m∠CED



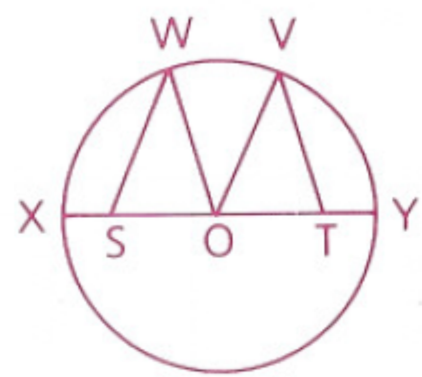
4 Given: $\overline{MR} \perp \overline{KP}$,
$\overline{KO} \perp \overline{PM}$,
$\angle RKM \cong \angle OMK$

Prove: $\triangle RKM \cong \triangle OMK$



5 Given: ⊙O,
$\angle SOV \cong \angle TOW$,
$\angle WSO \cong \angle VTO$

Prove: $\overline{SO} \cong \overline{TO}$



Objective

Students will be able to identify altitudes and medians of triangles and understand why auxiliary lines are used in some proofs.

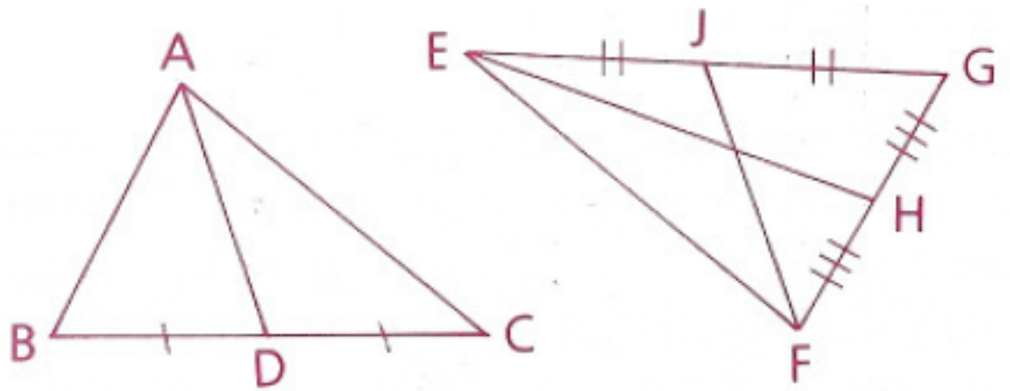
Triangle Congruency Quest on Tuesday! ☺

Medians of Triangles

Three ***medians*** are shown:

$\overline{AD}$ is a median of $\triangle ABC$.
$\overline{EH}$ is a median of $\triangle EFG$.
$\overline{FJ}$ is a median of $\triangle EFG$.

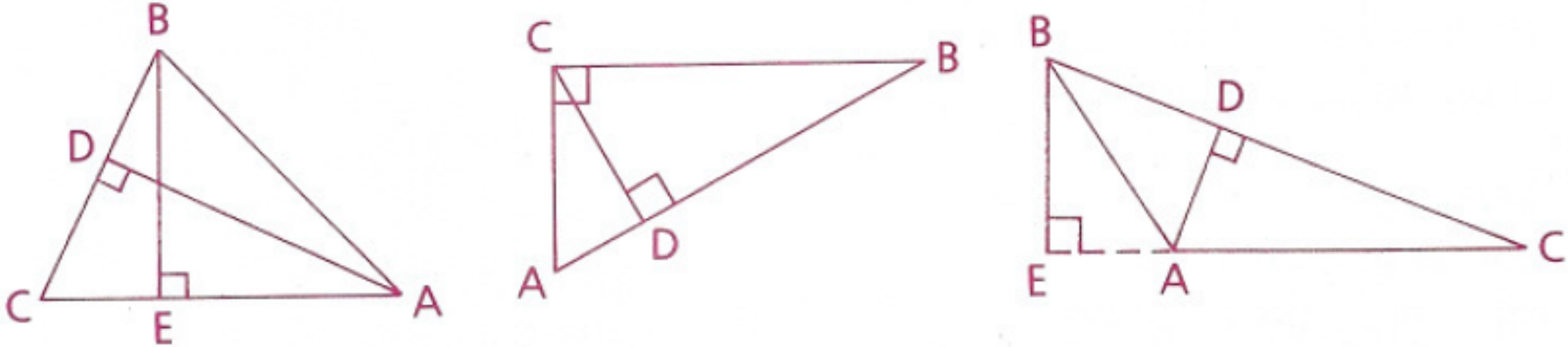
Every triangle has three medians.



A <u>median</u> of a triangle is a line segment drawn from any vertex of the triangle to the midpoint of the opposite side. (A median divides into two congruent segments, or bisects the side to which it is drawn.)

Altitudes of Triangles

In the first diagram below, $\overline{AD}$ and $\overline{BE}$ are altitudes of $\triangle ABC$.



Notice in the third diagram, BE falls outside of the triangle.

Every triangle has three altitudes

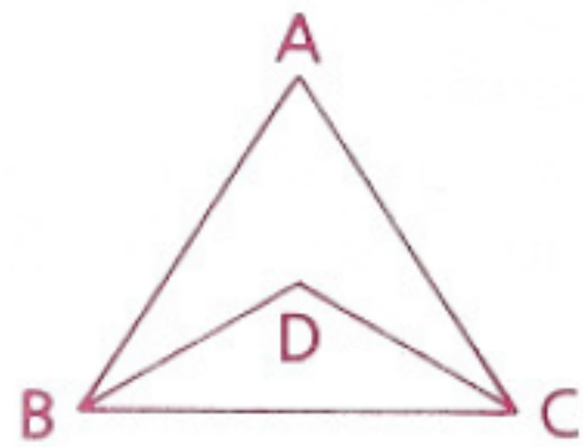
An <u>altitude</u> of a triangle is a line segment drawn from any vertex of the triangle to the opposite side, extended if necessary, and perpendicular to that side. (An altitude of a triangle forms right (90^o) angles with one of the sides.)

Could an altitude of a triangle be a median as well?

Consider the following problem.

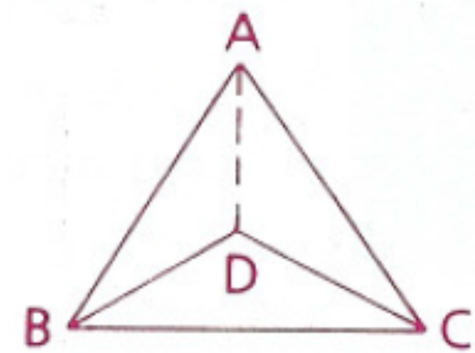
Given: $\overline{AB} \cong \overline{AC}$,
$\overline{BD} \cong \overline{CD}$

Conclusion: $\angle ABD \cong \angle ACD$



Is there a way to make this proof easier?

We could draw a line segment from A to D and then prove $\triangle ABD \cong \triangle ACD$ (by SSS) and that $\angle ABD \cong \angle ACD$ (by CPCTC)



Auxiliary Lines

You will find that many proofs involve lines, rays, or segments that do not appear in the original figure. These additions to diagrams are called auxiliary lines. Most auxiliary lines connect two points already in the diagram.

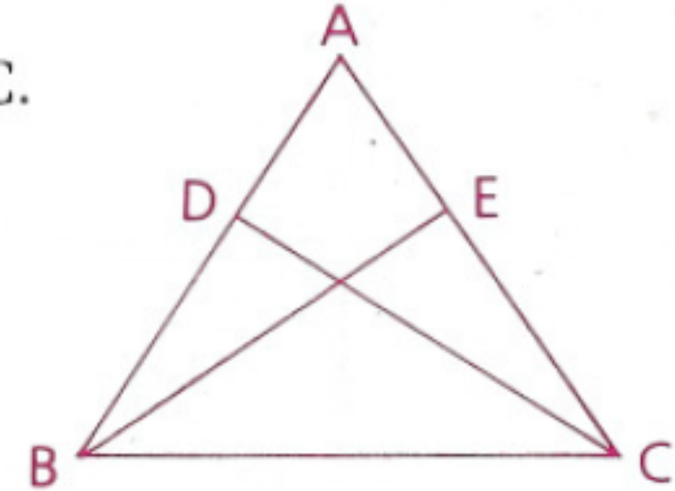
Postulate: Two points determine a line (or ray or segment)

The word determines indicates that there is a line through the given points and there is *no more than one* such line.

**In proof, S: Draw segment R: Two points determine a segment

Given: $\overline{CD}$ *and* $\overline{BE}$ *are altitudes of* $\triangle ABC$.
$\overline{AD} \cong \overline{AE}$

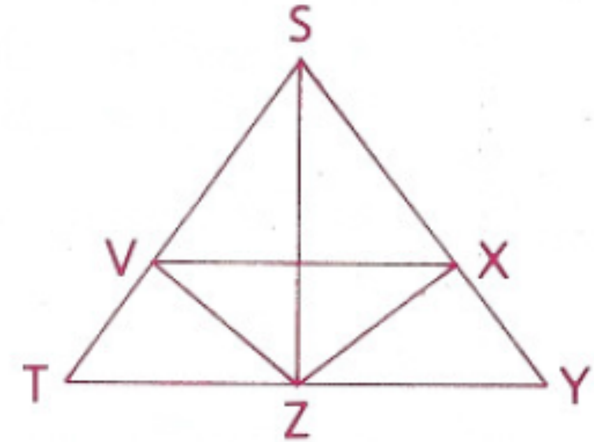
Prove: $\overline{DB} \cong \overline{EC}$



Statements	Reasons
1 $\overline{CD}$ and $\overline{BE}$ are altitudes of $\triangle ABC$.	1 Given
2 $\angle ADC$ is a right $\angle$.	2 An altitude of a $\triangle$ forms right $\angle$s with the side to which it is drawn.
3 $\angle AEB$ is a right $\angle$.	3 Same as 2
4 $\angle ADC \cong \angle AEB$	4 If $\angle$s are right $\angle$s, they are $\cong$.
5 $\angle A \cong \angle A$	5 Reflexive Property
6 $\overline{AD} \cong \overline{AE}$	6 Given
7 $\triangle ADC \cong \triangle AEB$	7 ASA (4, 6, 5)
8 $\overline{AB} \cong \overline{AC}$	8 CPCTC
9 $\overline{DB} \cong \overline{EC}$	9 Subtraction Property (6 from 8)

Given: $\angle T \cong \angle Y$,
$\angle SVZ \cong \angle SXZ$,
$\overline{TV} \cong \overline{YX}$

Conclusion: $\overline{SZ}$ *is the median to* $\overline{TY}$.



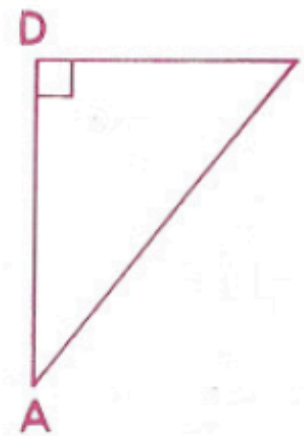
Statements	Reasons
1 $\angle T \cong \angle Y$	1 Given
2 $\angle SVZ \cong \angle SXZ$	2 Given
3 $\angle SVZ$ is supp. to $\angle TVZ$.	3 If two $\angle$s form a straight $\angle$, they are supplementary.
4 $\angle SXZ$ is supp. to $\angle YXZ$.	4 Same as 3
5 $\angle TVZ \cong \angle YXZ$	5 Supplements of $\cong$ $\angle$s are $\cong$.
6 $\overline{TV} \cong \overline{YX}$	6 Given
7 $\triangle TVZ \cong \triangle YXZ$	7 ASA (1, 6, 5)
8 $\overline{TZ} \cong \overline{YZ}$	8 CPCTC
9 $\overline{SZ}$ is the median to $\overline{TY}$.	9 If a segment from a vertex of a $\triangle$ divides the opposite side into two $\cong$ segments, it is a median.

Homework p. 135: 1, 4, 5, 6, 12

1 For the following figures, identify $\overline{AD}$ as a median, an altitude, neither, or both according to what can be proved.

a 

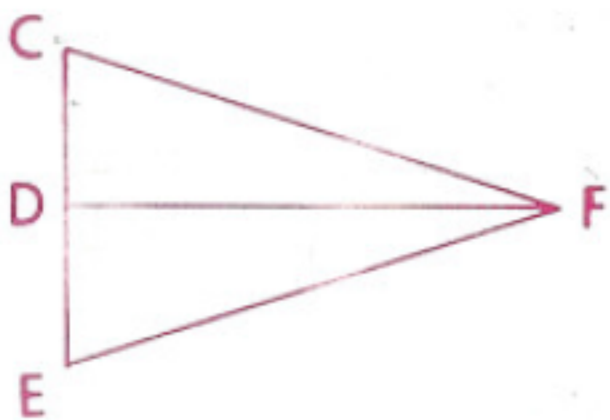
b 

c 

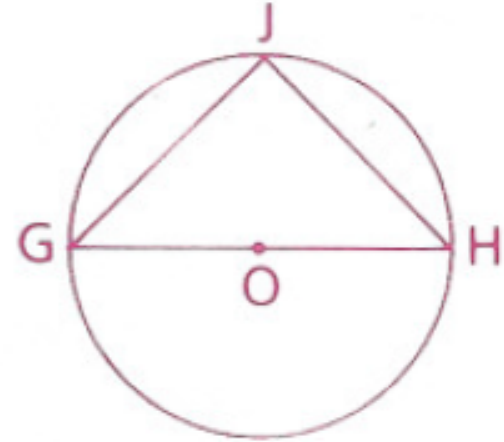
d 

4 Given: $\angle CFD \cong \angle EFD$;
$\overline{FD}$ is an altitude.

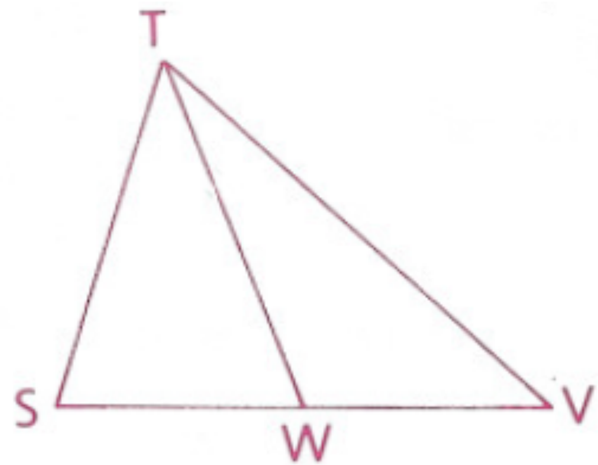
Prove: $\overline{FD}$ is a median.



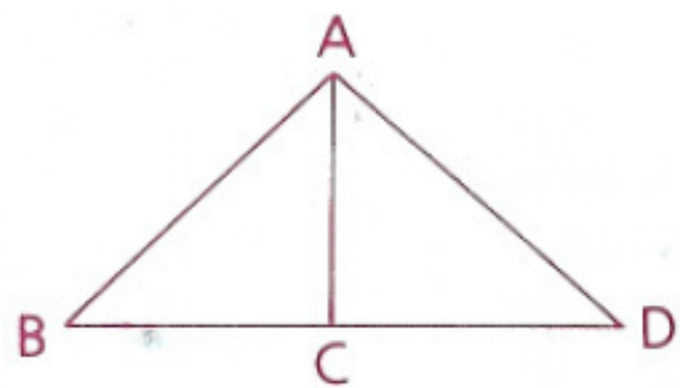
5 Given: $\odot O$,
$\overline{GJ} \cong \overline{HJ}$
Prove: $\angle G \cong \angle H$



6 Given: $\overline{TW}$ is a median.
$ST = x + 40$,
$SW = 2x + 30$,
$WV = 5x - 6$
Find: SW, WV, and ST



12 Given: $\overline{AC}$ is the altitude to $\overline{BD}$.
$\overline{AC}$ is a median.
$\angle BAC$ is comp. to $\angle D$.
Conclusion: $\angle DAC$ is comp. to $\angle B$.



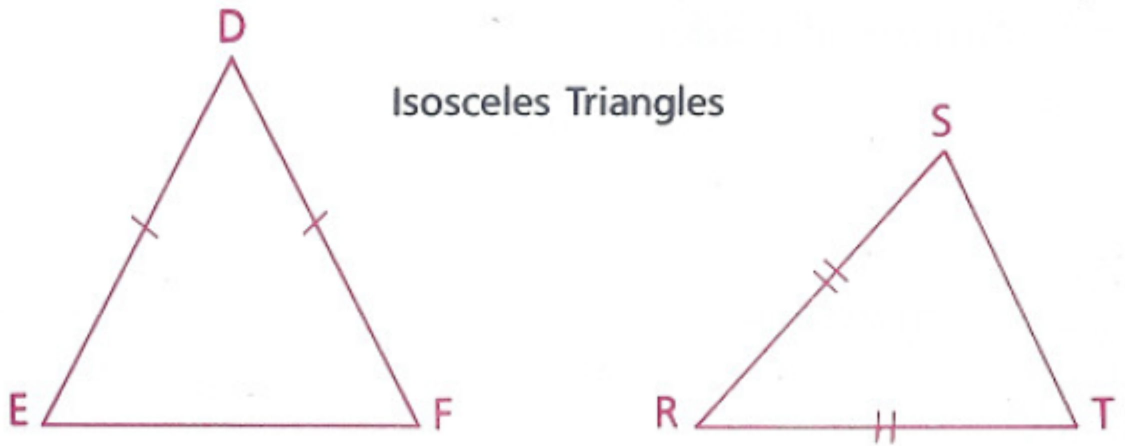
Objective

Students will be able to apply theorems relating the angle measures and side lengths of triangles.

Triangle Congruency Quest on Tuesday! ☺

Isosceles Triangle

An <u>isosceles triangle</u> is a triangle in which at least two sides are congruent.

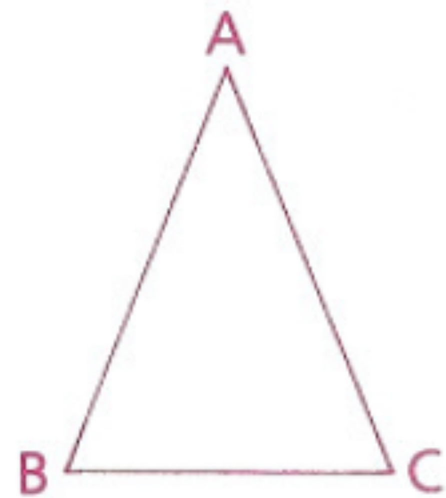


In $\triangle DEF$ above, $\overline{DE} \cong \overline{DF}$. $\overline{DE}$ and $\overline{DF}$ are called ***legs*** of the isosceles triangle, $\overline{EF}$ is called the ***base***, $\angle E$ and $\angle F$ are called ***base angles***, and $\angle D$ is called the ***vertex angle.*** Can you name these parts in $\triangle RST$?

Theorem 20: If two sides of a triangle are congruent, then the angles opposite the sides are congruent. *(If △, then △.)*

Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$



We proved that the triangle is congruent to itself (its mirror image)

Proof:

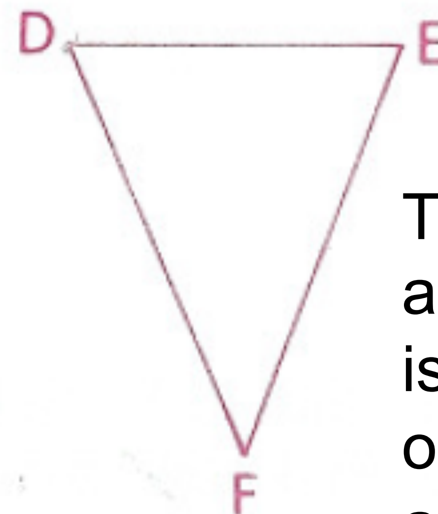
Statements	Reasons
1 $\overline{AB} \cong \overline{AC}$	1 Given
2 $\overline{BC} \cong \overline{BC}$	2 Reflexive Property
3 $\triangle ABC \cong \triangle ACB$	3 SSS (1, 2, 1)
4 $\angle B \cong \angle C$	4 CPCTC

Theorem 21: If two angles of a triangle are congruent, then the sides opposite the angles are congruent. (*If △, then △.*)

Converse of Theorem 20

Given: $\angle D \cong \angle E$

Conclusion: $\overline{DF} \cong \overline{EF}$



This tells us that a triangle is isosceles if two or more of its angles are congruent

Proof:

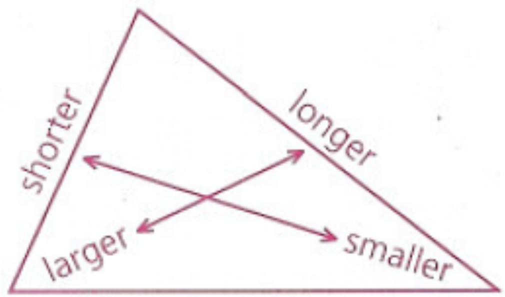
Statements	Reasons
1 $\angle D \cong \angle E$	1 Given
2 $\overline{DE} \cong \overline{DE}$	2 Reflexive Property
3 $\triangle DEF \cong \triangle EDF$	3 ASA (1, 2, 1)
4 $\overline{DF} \cong \overline{EF}$	4 CPCTC

Ways to Prove that a Triangle is Isosceles

1. If at least two sides of a triangle are congruent, then the triangle is isosceles.

2. If at least two angles of a triangle are congruent, then the triangle is isosceles.

It can be proved that the inequalities of sides and angles are related.



Inverses of Theorems 20 and 21

Theorem: If two sides of a triangle are not congruent, then the angles opposite them are not congruent, and the larger angle is opposite the longer side. *(If △, then △.)*

Theorem: If two angles of a triangle are not congruent, then the sides opposite them are not congruent, and the larger side is opposite the longer angle. *(If △, then △.)*

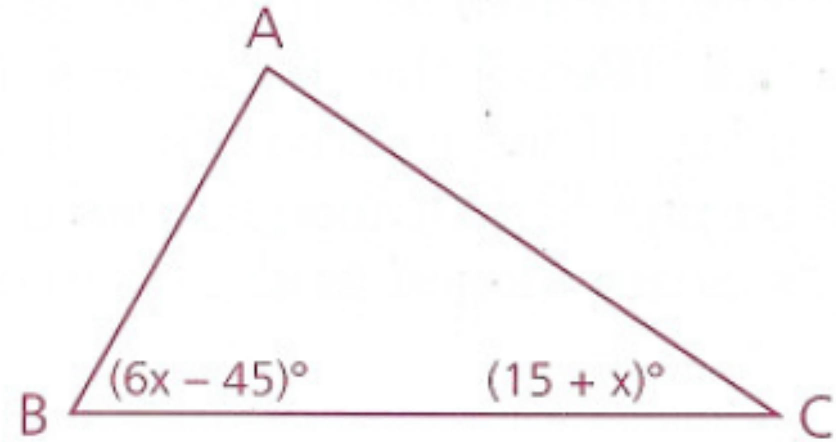
Given: $AC > AB$,

$m\angle B + m\angle C < 180$,

$m\angle B = 6x - 45$,

$m\angle C = 15 + x$

What are the restrictions on the value of x?



Since $AC > AB$, $m\angle B > m\angle C$.

$$6x - 45 > 15 + x$$
$$5x > 60$$
$$x > 12$$

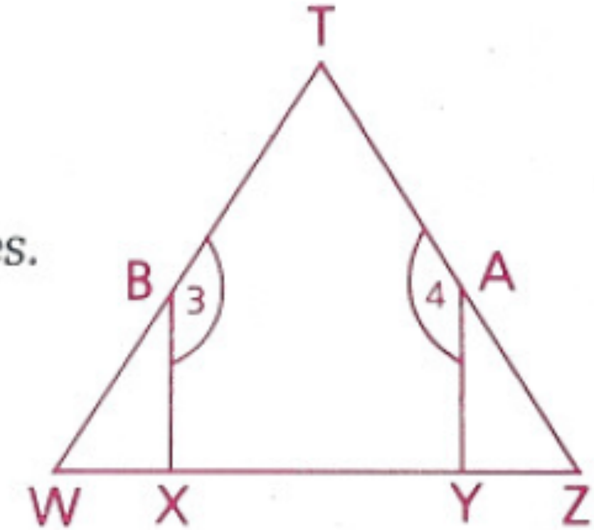
We also know that $m\angle B + m\angle C < 180$.

$$6x - 45 + 15 + x < 180$$
$$7x < 210$$
$$x < 30$$

Therefore, x must be between 12 and 30.

Given: $\angle 3 \cong \angle 4$,
$\overline{BX} \cong \overline{AY}$,
$\overline{BW} \cong \overline{AZ}$

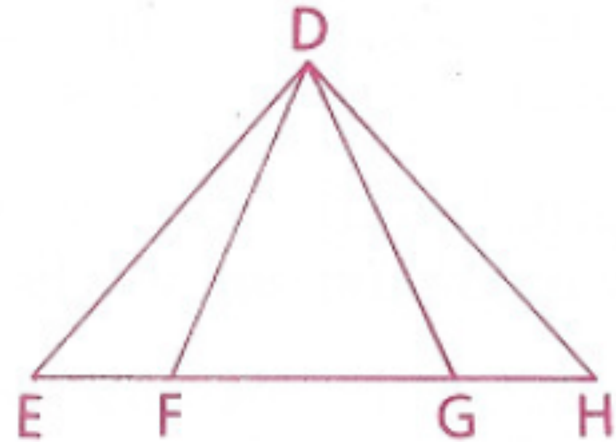
Conclusion: $\triangle WTZ$ *is isosceles.*



Statements	Reasons
1 $\angle 3 \cong \angle 4$	1 Given
2 $\angle 3$ is supp. to $\angle WBX$.	2 If two $\angle$s form a straight $\angle$, they are supplementary.
3 $\angle 4$ is supp. to $\angle YAZ$.	3 Same as 2
4 $\angle WBX \cong \angle YAZ$	4 $\angle$s supp. to $\cong$ $\angle$s, are $\cong$.
5 $\overline{BX} \cong \overline{AY}$	5 Given
6 $\overline{BW} \cong \overline{AZ}$	6 Given
7 $\triangle BWX \cong \triangle AZY$	7 SAS (5, 4, 6)
8 $\angle W \cong \angle Z$	8 CPCTC
9 $\triangle WTZ$ is isosceles.	9 If at least two $\angle$s of a $\triangle$ are $\cong$, the $\triangle$ is isosceles.

Given: $\angle E \cong \angle H$,
$\overline{EF} \cong \overline{GH}$

Conclusion: $\overline{DF} \cong \overline{DG}$



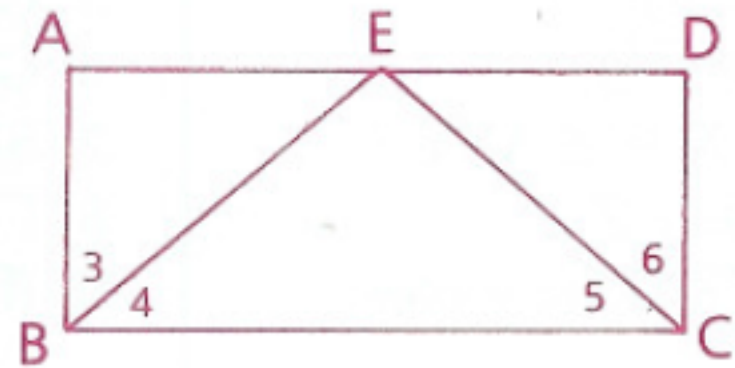
	Statements		Reasons
1	$\angle E \cong \angle H$	1	Given
2	$\overline{DE} \cong \overline{DH}$	2	If $\triangle$, then $\triangle$.
3	$\overline{EF} \cong \overline{GH}$	3	Given
4	$\triangle DEF \cong \triangle DHG$	4	SAS (2, 1, 3)
5	$\overline{DF} \cong \overline{DG}$	5	CPCTC

Homework

p.152: 4, 6, 11, 12, 16

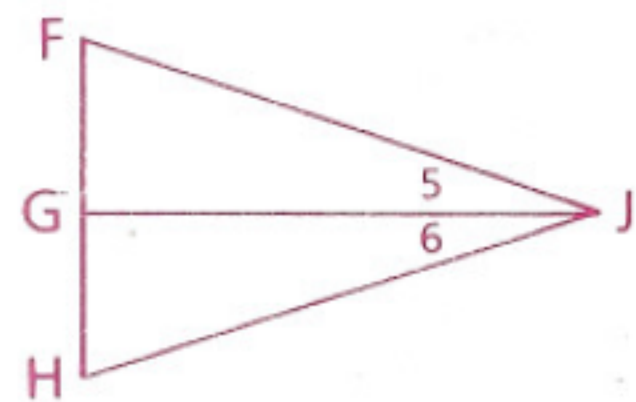
4 Given: $\angle 3 \cong \angle 6$;
$\angle 3$ is comp. to $\angle 4$.
$\angle 6$ is comp. to $\angle 5$.

Prove: $\triangle EBC$ is isosceles.



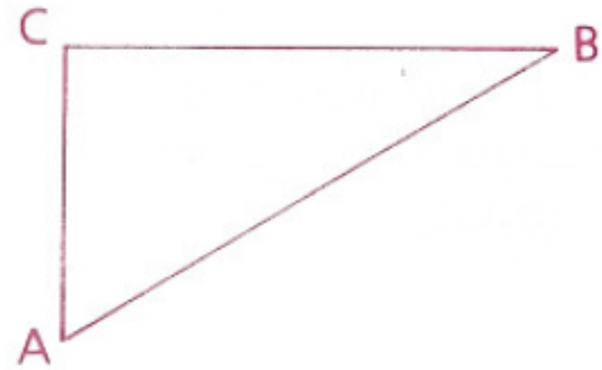
6 Given: $\angle 5 \cong \angle 6$;
$\overline{JG}$ is the altitude to $\overline{FH}$.

Prove: $\triangle FJH$ is isosceles.



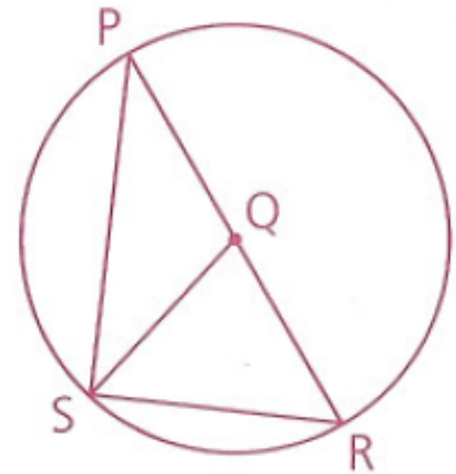
11 Given: $\overline{AC} \perp \overline{BC}$,
$\angle C = (3x)°$,
$BC = x + 20$,
$AC = 2x - 20$

Is $\triangle ABC$ isosceles?



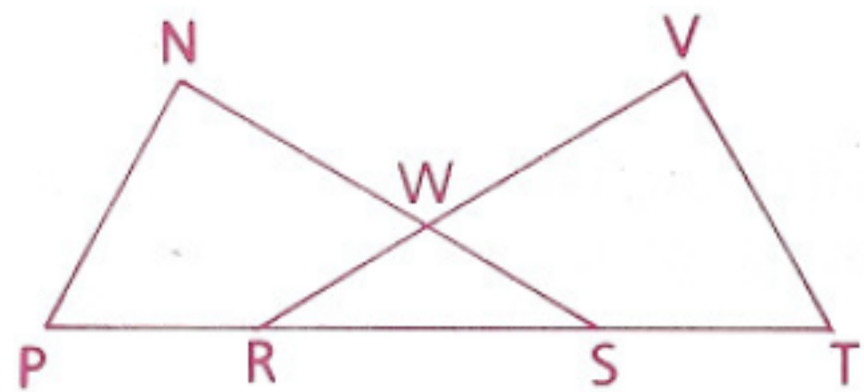
12 Given: $\odot Q$,
$\overline{PS} \perp \overline{SR}$,
$\angle P = 36°$

Find: **a** $\angle PSQ$
b $\angle R$



16 Given: $\overline{PR} \cong \overline{ST}$,
$\overline{NP} \cong \overline{VT}$,
$\angle P \cong \angle T$

Prove: $\triangle WRS$ is isosceles.



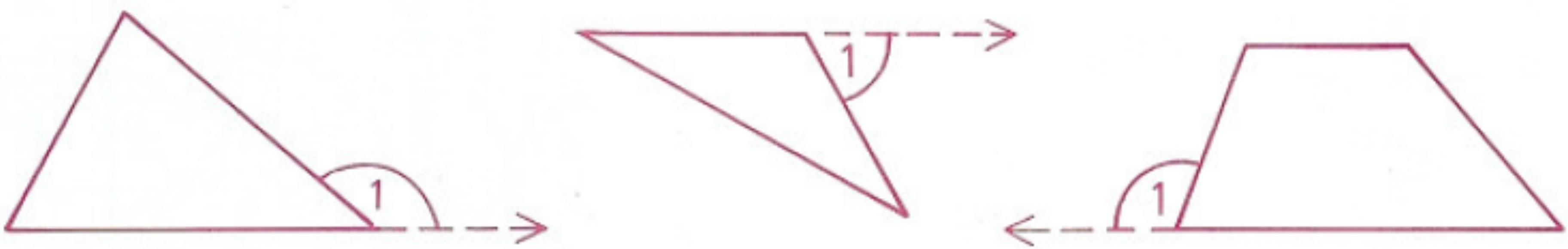
Objective

Students will be able to apply theorems about the exterior angles and the midsegments of triangles.

Triangle Congruency Quest on Tuesday! ☺

Exterior Angle

∠1 is an exterior angle of the polygons below. An exterior angle of a polygon is formed by extending one of the sides of the polygon.



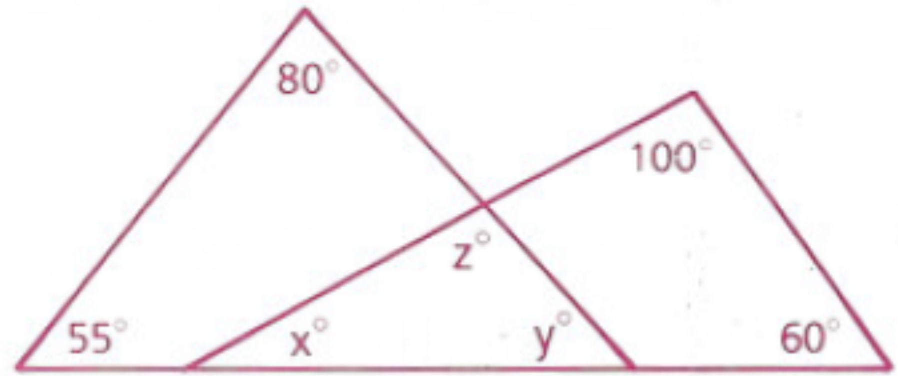
An <u>exterior angle</u> of a polygon is an angle that is adjacent to and supplementary to an interior angle of the remote interior angles.

Theorem 51: The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

Theorem 52: A segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is one-half the length of the third side. (Midsegment Theorem)

Given: Diagram as marked

Find: x, y, and z



Since the sum of the measures of the angles of a triangle is 180,

$$x + 100 + 60 = 180 \qquad 55 + 80 + y = 180 \qquad x + y + z = 180$$

$$x + 160 = 180 \qquad 135 + y = 180 \qquad 20 + 45 + z = 180$$

$$x = 20 \qquad y = 45 \qquad z = 115$$

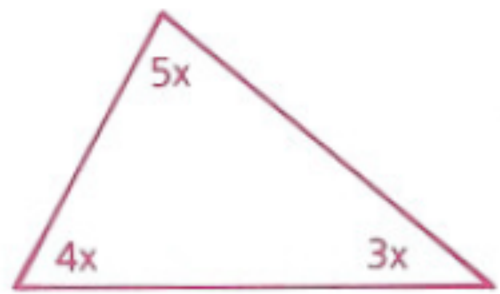
Substitution

Substitution

The measures of the three angles of a triangle are in the ratio 3:4:5. Find the measure of the largest angle.

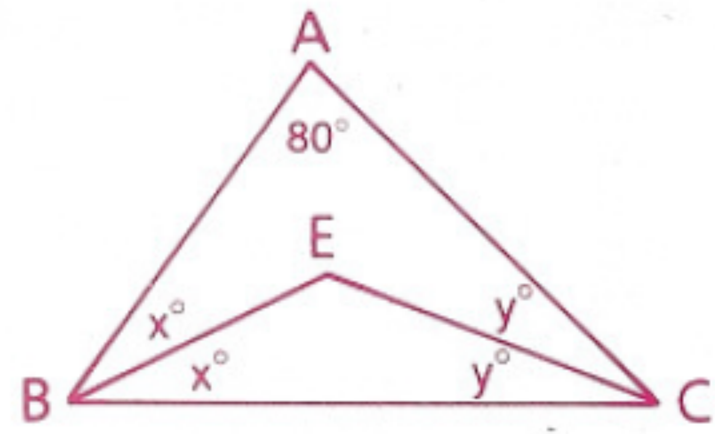
Let the measures of the three angles be 3x, 4x, and 5x. Since the sum of the measures of the three angles of a triangle is 180,

$$\begin{aligned} 3x + 4x + 5x &= 180 \\ 12x &= 180 \\ x &= 15 \end{aligned}$$



Therefore, the measure of the largest angle is 5(15), or 75.

If one of the angles of a triangle is 80°, find the measure of the angle formed by the bisectors of the other two angles.



The bisectors, $\overrightarrow{BE}$ and $\overrightarrow{CE}$, meet at E, so we want to find $m\angle E$. Let $\angle ABC = (2x)°$ and $\angle ACB = (2y)°$.

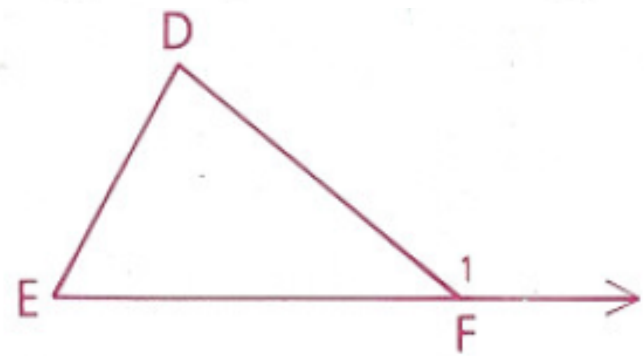
In $\triangle ABC$,

$$2x + 2y + 80 = 180$$
$$2x + 2y = 100$$
$$x + y = 50$$

In $\triangle EBC$,

$$x + y + m\angle E = 180$$
$$50 + m\angle E = 180 \quad \text{(Substitution)}$$
$$m\angle E = 130$$

$\angle 1 = 150°$, and the measure of $\angle D$ is twice that of $\angle E$. Find the measure of each angle of the triangle.



Let $\angle E = x°$ and $\angle D = (2x)°$.

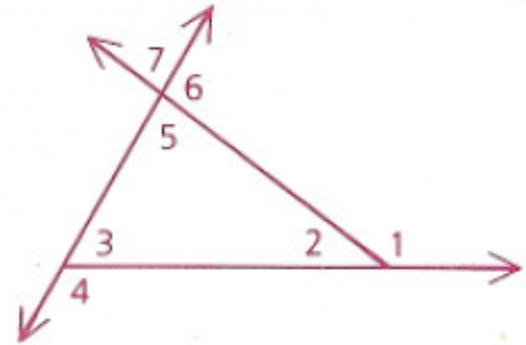
Since the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles,

$$150 = x + 2x$$
$$150 = 3x$$
$$50 = x$$

Hence, $\angle E = 50°$, $\angle D = 100°$, and $\angle DFE = 30°$.

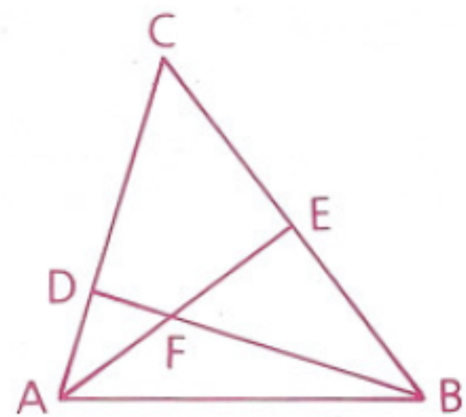
Homework

p. 298: 2, 3, 11, 16, 18



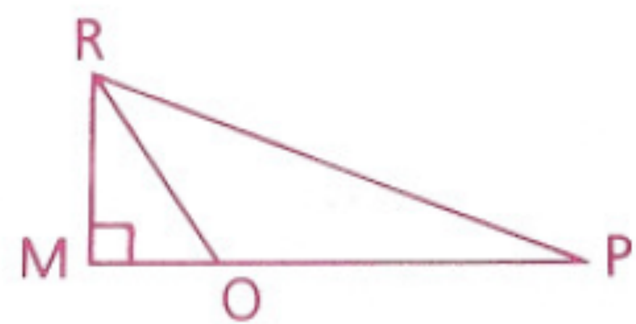
2 Given: $\angle 1 = 130°$,
$\angle 7 = 70°$

Find the measures of $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$.



3 Given: $\angle CAB = 80°$,
$\angle CBA = 60°$,
$\overline{AE}$ and $\overline{BD}$ are altitudes.

Find: $m\angle C$ and $m\angle AFB$

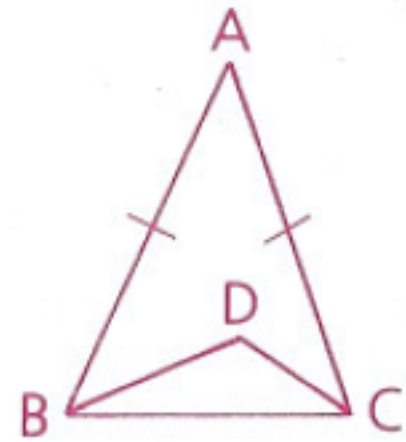


11 Given: $\angle P = 10°$;
$\overrightarrow{RO}$ bisects $\angle MRP$.

Find: $m\angle ORP$ and $m\angle MOR$

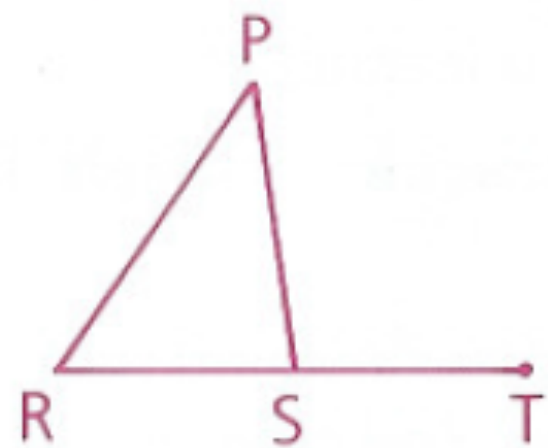
16 Given: $\angle A = 30°$, $\overline{AB} \cong \overline{AC}$;
$\overrightarrow{CD}$ bisects $\angle ACB$.
$\overrightarrow{BD}$ is one of the trisectors of $\angle ABC$.

Find: $m\angle D$



18 Given: $\angle PST = (x + 3y)°$,
$\angle P = 45°$, $\angle R = (2y)°$,
$\angle PSR = (5x)°$

Find: $m\angle PST$



Objective

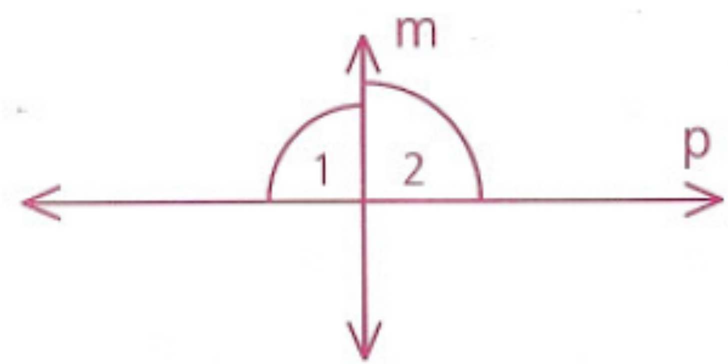
Students will be able to understand and apply that if two angles are both supplementary and congruent then they are right angles.

Right Angles

In order to prove that two lines are perpendicular, we need to prove that the angles formed are right angles.

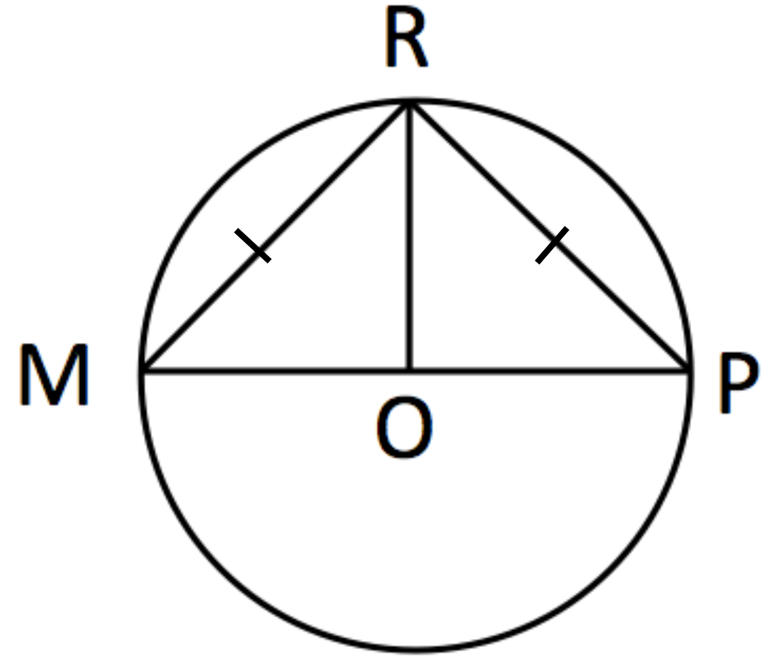
Theorem 23: If two angles are both supplementary and congruent, then they are right angles.

Given: $\angle 1 \cong \angle 2$
Prove: $\angle 1$ and $\angle 2$ are right angles.



Proof: Since $\angle 1$ and $\angle 2$ form a straight angle (line p), they are supplementary. Therefore, $m\angle 1 + m\angle 2 = 180$. Since $\angle 1 \cong \angle 2$, we can use substitution to get the equation $m\angle 1 + m\angle 1 = 180$, or $m\angle 1 = 90$. Thus, $\angle 1$ is a right angle, and so is $\angle 2$.

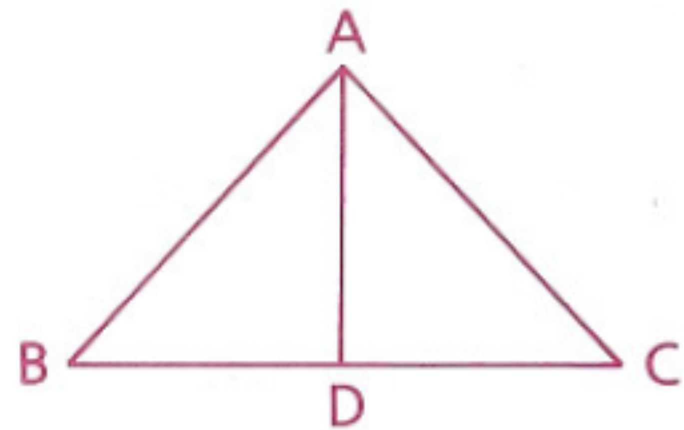
Given that O is the center of the circle, $\angle ROM = (5x - 185)^{o}$, $\angle ROP = (x + 35)^{o}$, and $\angle MRO = (9y)^{o}$, find x and y.



x = 55 and y = 5

Given: $\overline{AB} \cong \overline{AC}$,
$\overline{BD} \cong \overline{CD}$

Conclusion: $\overline{AD}$ *is an altitude.*

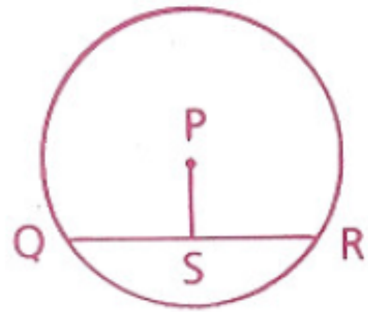


1 $\overline{AB} \cong \overline{AC}$	1 Given
2 $\overline{BD} \cong \overline{CD}$	2 Given
3 $\overline{AD} \cong \overline{AD}$	3 Reflexive Property
4 $\triangle ABD \cong \triangle ACD$	4 SSS (1, 2, 3)
5 $\angle ADB \cong \angle ADC$	5 CPCTC
6 $\angle ADB$ and $\angle ADC$ are right $\angle$s.	6 If two $\angle$s are both supp. and $\cong$, then they are right $\angle$s.
7 $\overline{AD}$ is an altitude	7 If a segment from a vertex of a $\triangle$ is $\perp$ to the opposite side, it is an altitude of the $\triangle$.
5b) $\angle ADB$ is supple. to $\angle ADC$	5b) If two $\angle$s form a straight $\angle$ (assumed from diagram), then they are supple.

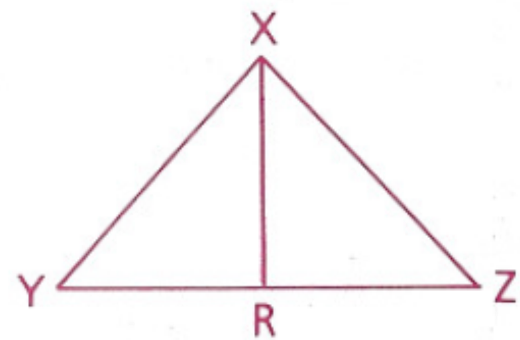
Homework

p. 182: 1, 4, 5, 9

1 Given: $\odot P$;
S is the midpt. of $\overline{QR}$.
Prove: $\overline{PS} \perp \overline{QR}$

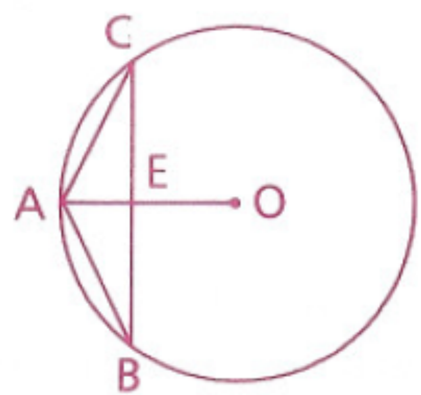


4 Given: $\overrightarrow{XR}$ bisects $\angle YXZ$.
$\angle Y \cong \angle Z$
Conclusion: $\overline{XR}$ is an altitude.



5 A diameter of a circle has endpoints with coordinates (2, 6) and (−4, 10). Find the coordinates of the center of the circle.

9 Given: $\odot O$,
$\angle B \cong \angle C$
Conclusion: $\overline{AO} \perp \overline{BC}$



Objective

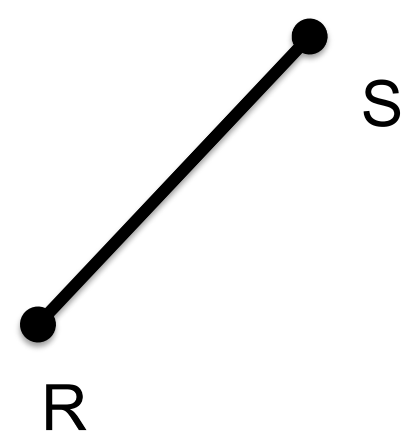
Students will be able to recognize the relationship between equidistance and perpendicular bisection.

Distance

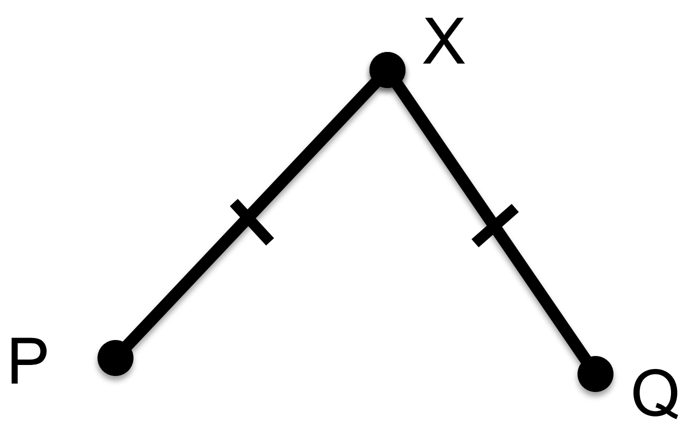
The <u>distance</u> between two objects is the length of the shortest path joining them.

A line segment is the shortest path between two points.

The distance between R and S is the length of $\overline{RS}$, or RS.



Equidistant



What can we conclude from this diagram?

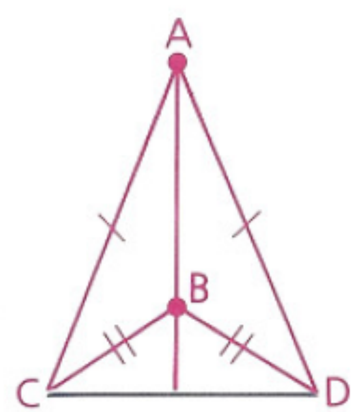
$$\overline{PX} \cong \overline{XQ}$$

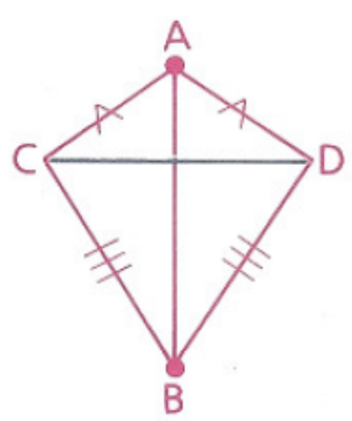
If two points P and Q are the same distance from a third point X, then X is said to be <u>equidistant</u> from P and Q.

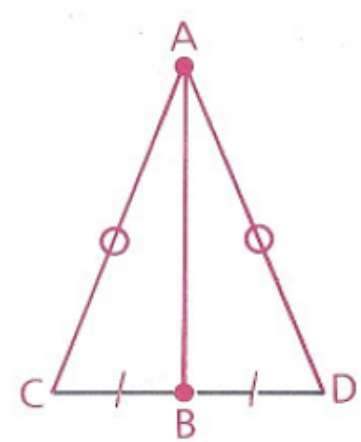
Perpendicular Bisector

Think back to when we did constructions and think about these words separately, what does it mean for something to be a perpendicular bisector?

The <u>perpendicular bisector</u> of a segment is the line that bisects and is perpendicular to the segment.



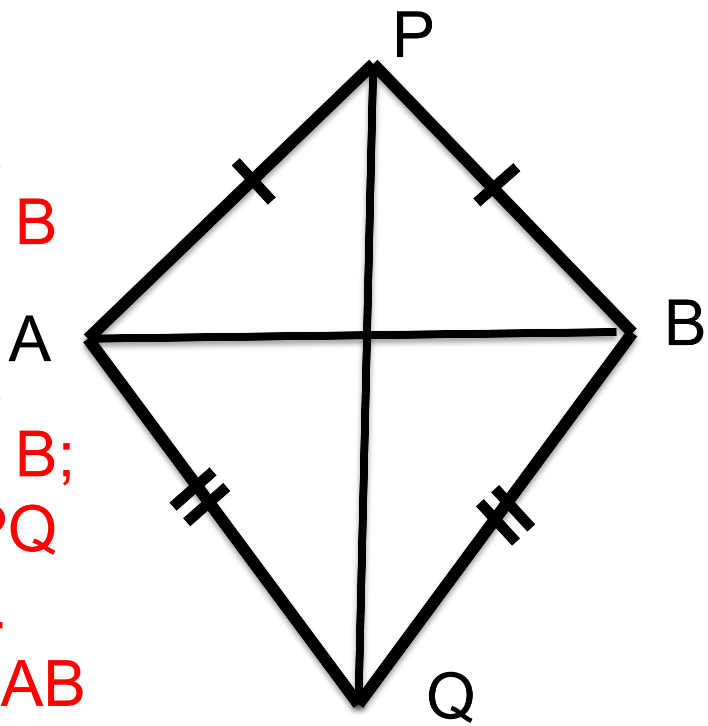




Perpendicular Bisector Theorem

Theorem 24: If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment.

P is equidistant from A and B & Q is equidistant from A and B; therefore PQ is the perp. bisector of AB



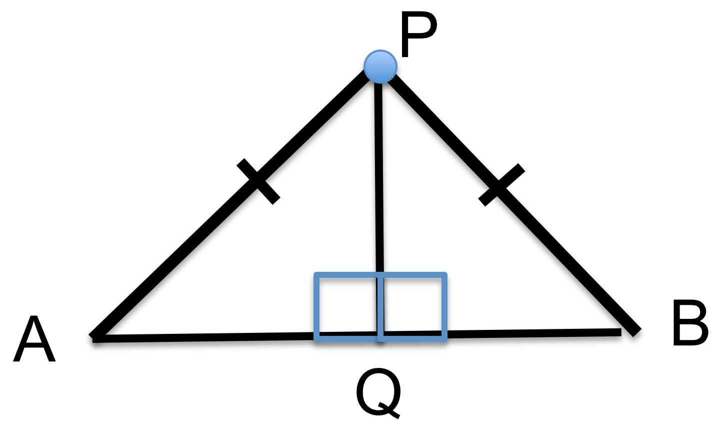
Given: $\overline{PA} \cong \overline{PB}$

$\overline{QA} \cong \overline{QB}$

Prove: $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$

Equidistant Theorem

Theorem 25: If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment. (converse of previous theorem)



Given: $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$

Prove: $\overline{PA} \cong \overline{PB}$

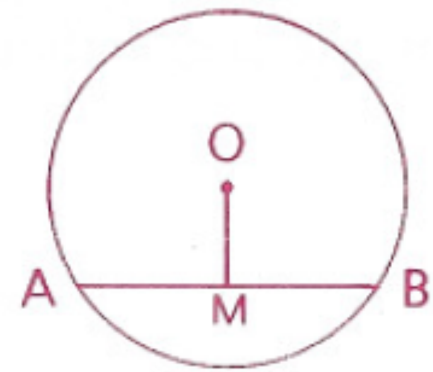
P is on PQ which is the perp. bisector of AB; therefore PA≅PB

Homework

p. 187: 1, 2, 4, 7, 8

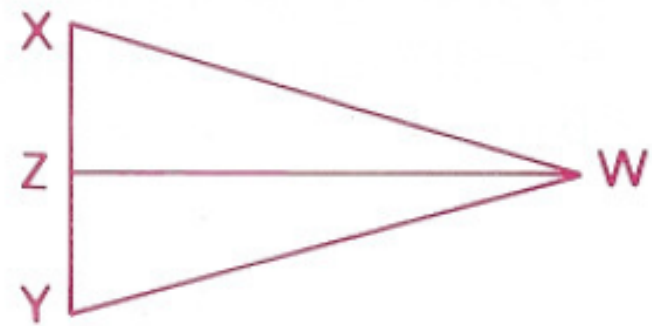
1 Given: ⊙O; M is the midpt. of $\overline{AB}$.

Conclusion: $\overline{OM} \perp \overline{AB}$ (Hint: Draw two auxiliary lines.)



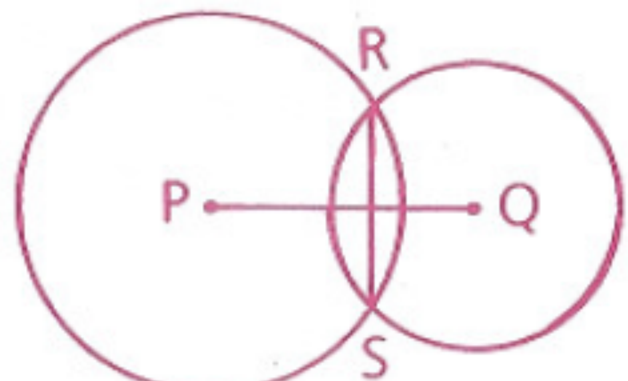
2 Given: $\overleftrightarrow{WZ} \perp$ bis. $\overline{XY}$

Prove: $\triangle WXY$ is isosceles. (Hint: This proof can be written in three steps by using Theorem 25.)



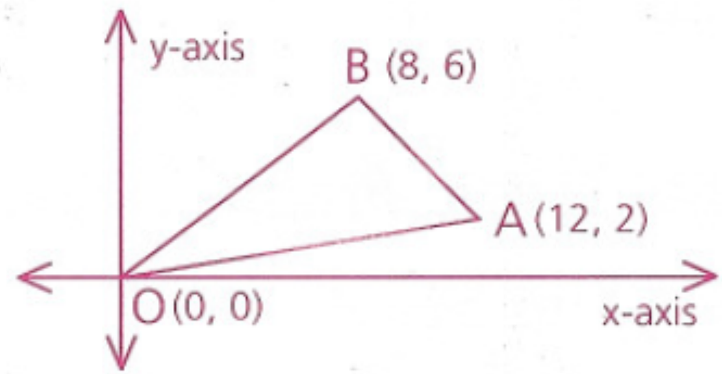
4 Given: Ⓢ P and Q

Prove: $\overleftrightarrow{PQ} \perp$ bis. $\overline{RS}$



s with a circle around it means "circles"

7 How much greater than the x-coordinate of the midpoint of $\overline{OA}$ is the x-coordinate of the midpoint of $\overline{AB}$?



8 In the graph, if a perpendicular is drawn from T to $\overleftrightarrow{PA}$, what will the coordinates of the point where the perpendicular intersects $\overleftrightarrow{PA}$ be?