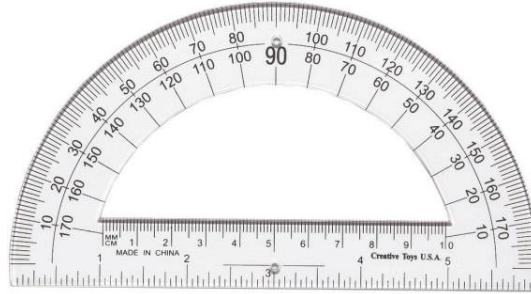


Objective

Students will be able to measure segments and angles, classify angles by size, and recognize congruent angles and segments.

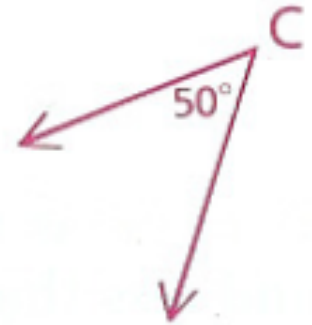
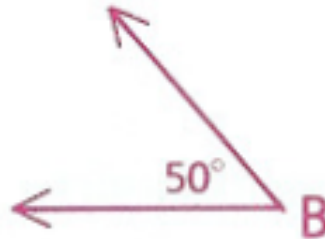
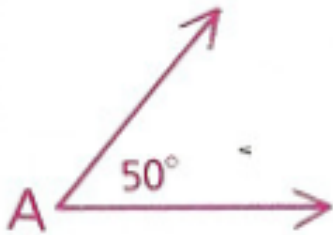


Classifying Angles Discovery



Congruent Angles

In the diagram below, \angle s A, B, and C are ***congruent***. We write $\angle A \cong \angle B \cong \angle C$.



Congruent (\cong) angles are angles that have the same measure (degree).

Congruent Segments

In a similar way, segments can be congruent.

Congruent (\cong) segments are segments that have the same length.

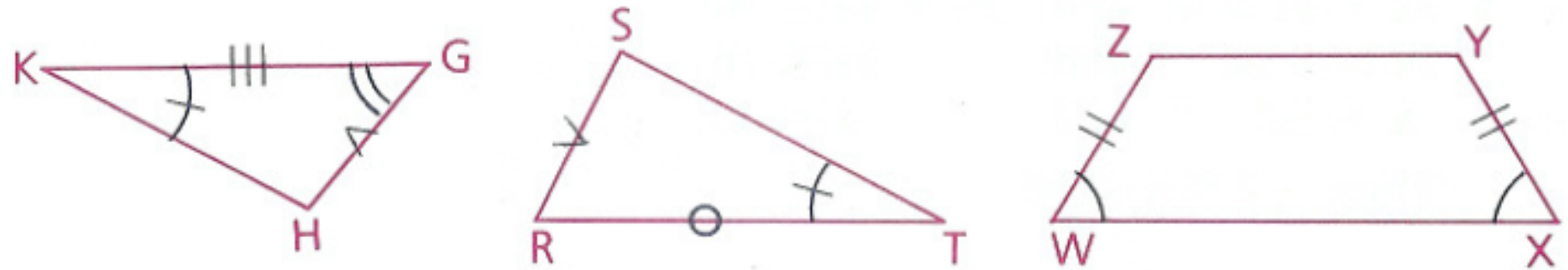


In the diagram above, segments \overline{AB} , \overline{CD} , and \overline{EF} are congruent. We write $\overline{AB} \cong \overline{CD} \cong \overline{EF}$.

Tick Marks

To show that angles and segments are congruent, use identical tick marks.

In the following diagram, the identical tick marks indicate that there are four pairs of congruent parts. Can you name them?

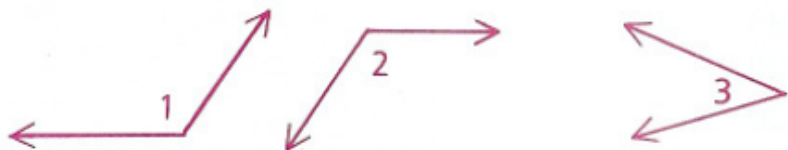


$$\overline{GH} \cong \overline{SR} \quad \overline{WZ} \cong \overline{XY} \quad \angle K \cong \angle T \quad \angle W \cong \angle X$$

Homework

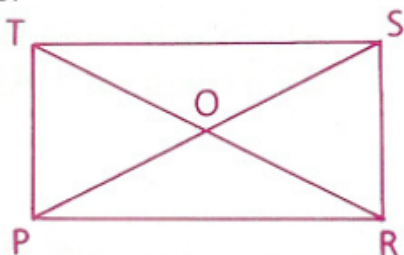
p. 14-16: 3, 6, 8, 13, 15,
17, 21

3 Which two of the angles below appear to be congruent?

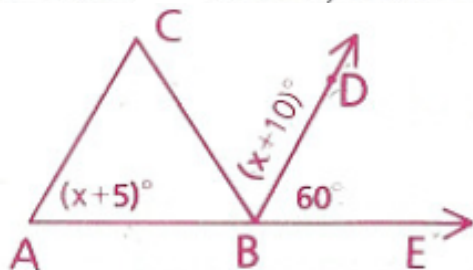


6 There is a right angle at each corner of PRST. (Later in the course you will learn that PRST is a rectangle.)

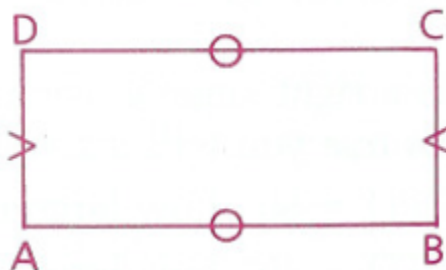
- a If $\angle TPO = 60^\circ$, how large is $\angle RPO$?
- b If $\angle PTO = 70^\circ$, how large is $\angle STO$?
- c If $\angle TOP = 50^\circ$, how large is $\angle POR$?
- d Classify $\angle TOS$ as acute, right, or obtuse.



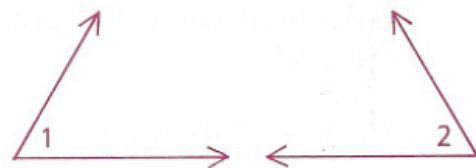
8 If $\angle CBD \cong \angle DBE$, find $m\angle A$.



13 The perimeter of (the distance around) ABCD is 66, and \overline{DC} is twice as long as \overline{CB} . How long is \overline{AB} ?



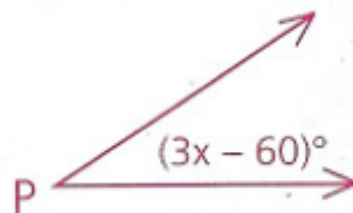
15 Given: $\angle 1 \cong \angle 2$,
 $m\angle 1 = x + 14$,
 $m\angle 2 = y - 3$



Solve for y in terms of x .

17 $\angle P$ is acute.

- a What are the restrictions on $m\angle P$?
- b What are the restrictions on x ?



21 Given: $\angle TRS$ is a straight angle.
 $\angle TRX$ is a right angle.
 $m\angle TRS = 2x + 5y$,
 $m\angle XRS = 3x + 3y$



Solve for x and y .

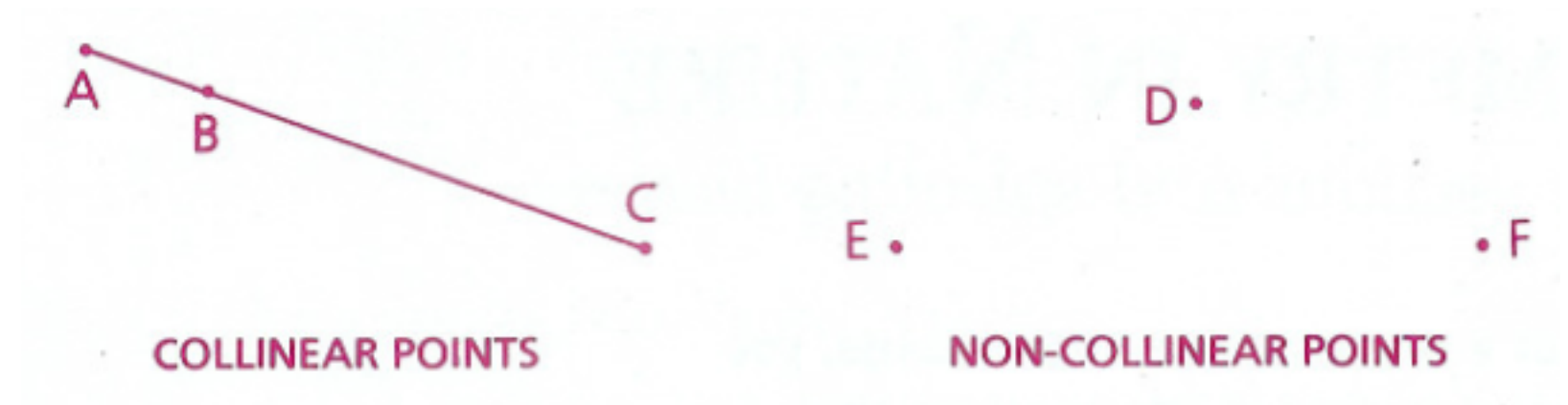
Objective

Students will be able to recognize collinear and non-collinear points, recognize when a point is between two other points, recognize triangle inequality, and correctly interpret geometric diagrams.

Quest over Introduction to Geometry on Thursday

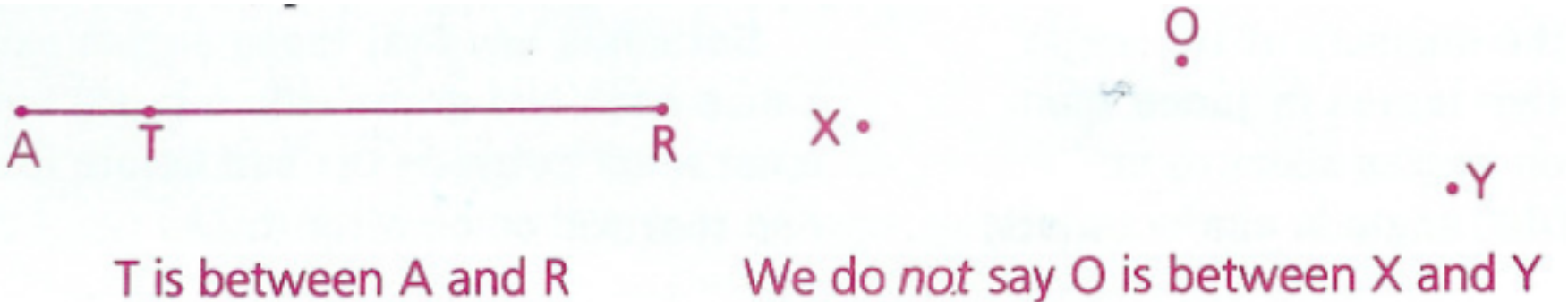
Collinearity

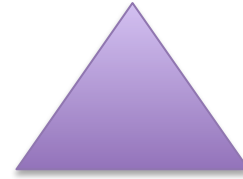
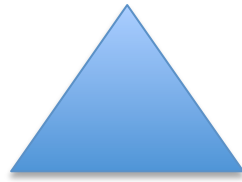
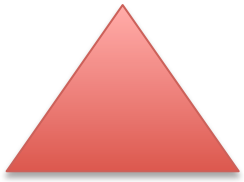
Points that lie on the same line are called collinear.
Points that do not lie on the same line are called noncollinear.



Betweenness of Points

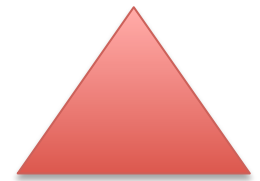
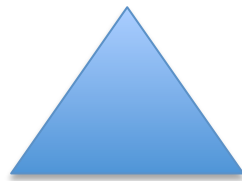
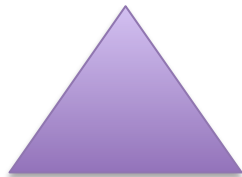
In order for us to say that a point is between two other points, all three of the points must be collinear.





Triangle Inequality Discovery

Triangle Inequality- The sum of the lengths of any two sides of a triangle is always greater than the length of the third.



Assumptions from Diagrams

We have to be very careful when we should and should not assume things.

How to Interpret a Diagram

You Should Assume

- Straight lines and angles
- Collinearity of points
- Betweenness of points
- Relative positions of points

You Should Not Assume

- Right angles
- Congruent segments
- Congruent angles
- Relative sizes of segments and angles

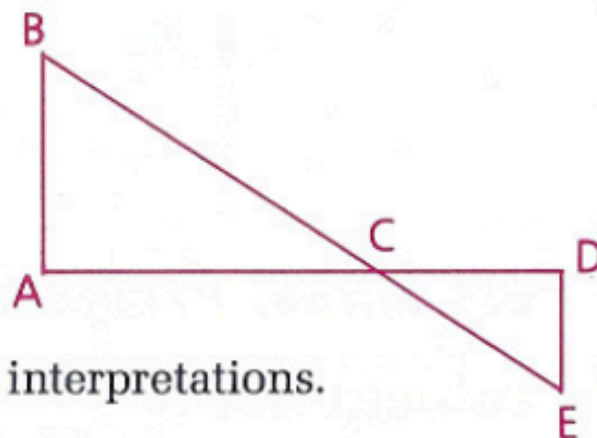
Example

The following example will help you understand what assumptions can be made.

Example

Given: Diagram as shown

Question: What should we assume?



The following are some of the many valid interpretations.

Do Assume

- \overleftrightarrow{ACD} and \overleftrightarrow{BCE} are straight lines.
- $\angle BCE$ is a straight angle.
- C, D, and E are noncollinear.
- C is between B and E.
- E is to the right of A.

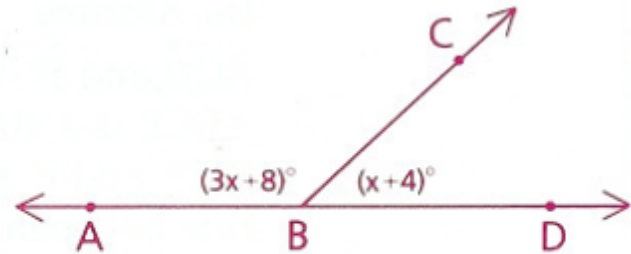
Do Not Assume

- $\angle BAC$ is a right \angle .
- $\overline{CD} \cong \overline{DE}$
- $\angle B \cong \angle E$
- $\angle CDE$ is an obtuse angle.
- \overline{BC} is longer than \overline{CE} .

Homework

p. 20-22: 1-7 (not 3 g, h), 10, 12, 13, 15

1 Find $m\angle ABC$ (the measure of $\angle ABC$).



2 Draw a diagram showing four points, no three of which are collinear.



- Should we assume that angles E, F, G, and H are right angles? Explain your answer.
- Should we assume that points E, F, and G are noncollinear? Explain your answer.

a Name all points collinear with E and F.

b Are G, E, and D collinear? Are F and C collinear?

c Which two segments do the tick marks indicate are congruent?

d Is $\angle A \cong \angle D$?

e Is $\angle F \cong \angle ABF$?

f Where do \overleftrightarrow{AC} and \overleftrightarrow{FE} intersect?

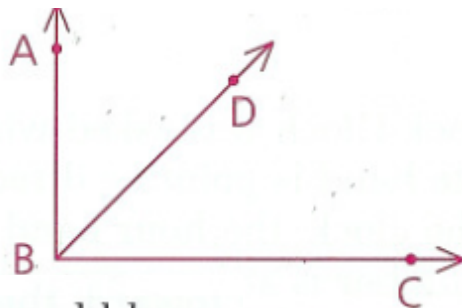
i B lies on a ray whose endpoint is E. Name this ray in all possible ways.

j Name all points between F and D.

5 Draw a number line and shade all points that are at or between -5 and 2 . Find the length of this shaded segment.

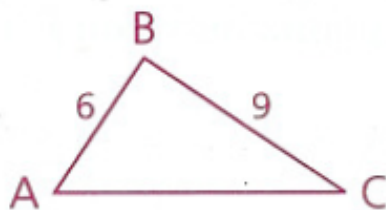
p. 20-22: 1-7 (not 3 g, h), 10, 12, 13, 15

- 6 $\angle ABC$ is a right angle. The ratio of the measures of $\angle ABD$ and $\angle DBC$ is 3 to 2. Find $m\angle ABD$. (Hint: Let $m\angle ABD = 3x$ and $m\angle DBC = 2x$.)

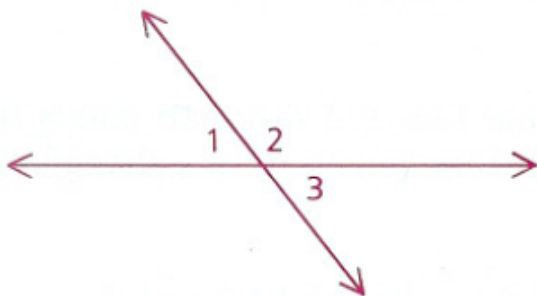


- 7 Explain how the sum of two acute angles could be
a Acute b Obtuse c Right
- 10 A, K, O, and Y are collinear points. K is between O and A, the length of \overline{AO} added to the length of \overline{AY} is equal to the length of \overline{OY} ($OA + AY = OY$), and A is to the right of O. Draw a diagram that correctly represents this information.
- 12 If $AB = 16$, $BC = 8$, and $AC = 24$, which point is between the other two?

- 13 a AC must be smaller than what number?
b AC must be larger than what number?



- 15 Given: $m\angle 1 = 2x + 40$,
 $m\angle 2 = 2y + 40$,
 $m\angle 3 = x + 2y$
Find: $m\angle 1$, $m\angle 2$, and $m\angle 3$



Objective

Students will be able to identify midpoints and bisectors of segments, trisection points and trisectors of segments, angle bisectors, and angle trisectors.

Quest over Introduction to Geometry on Thursday

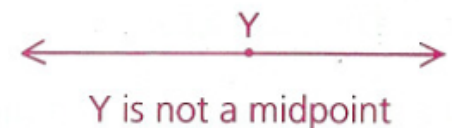
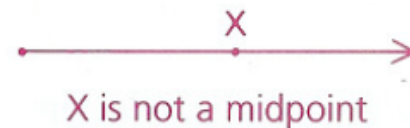
What do you think it means
to bisect something?
Trisect something?

Midpoints and Bisectors of Segments

A point (or segment, ray, or line) that divides a segment into two congruent segments bisects the segment. The bisection point is called the midpoint of the segment.

Can a ray have a midpoint?

How about a line?



Examples

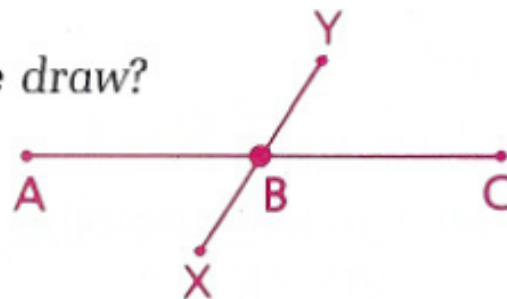
Example 1

If \overline{XY} bisects \overline{AC} at B, what conclusions can we draw?

Conclusions:

B is the midpoint of \overline{AC} .

$\overline{AB} \cong \overline{BC}$



Example 2

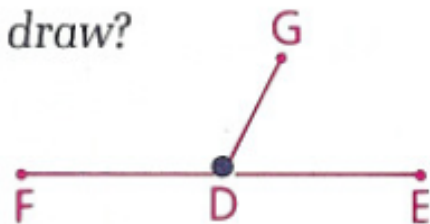
If D is the midpoint of \overline{FE} , what conclusions can we draw?

Conclusions:

$\overline{FD} \cong \overline{DE}$

Point D bisects \overline{FE} .

\overline{DG} bisects \overline{FE} .



Example 3

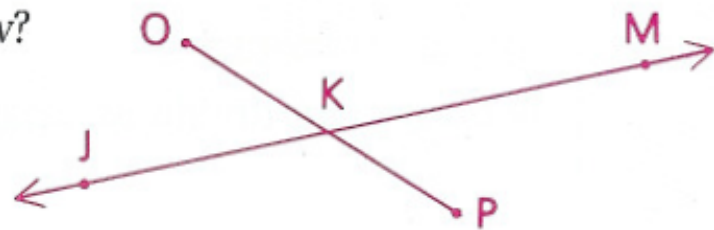
If $\overline{OK} \cong \overline{KP}$, what conclusions can we draw?

Conclusions:

K is the midpoint of \overline{OP} .

\overleftrightarrow{JM} is a bisector of \overline{OP} .

Point K bisects \overline{OP} .



Trisection Points and Trisecting a Segment

Two points (or segments, rays, or lines) that divide a segment into three congruent segments trisection the segment. The two points at which the segment is divided are called the trisection points of the segment.

A segment into three congruent parts is said to be trisected.

Rays and lines do not have trisected points.

Examples

Example 1

If $\overline{AR} \cong \overline{RS} \cong \overline{SC}$, what conclusions can we draw?

Conclusions:

R and S are trisection points of \overline{AC} .
 \overline{AC} is trisected by R and S.



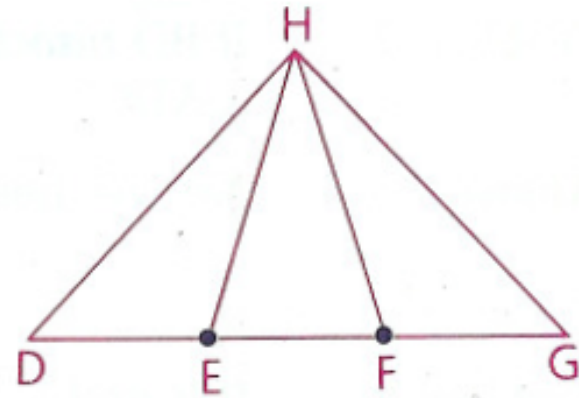
Example 2

If E and F are trisection points of \overline{DG} , what conclusions can we draw?

Conclusions:

$\overline{DE} \cong \overline{EF} \cong \overline{FG}$

\overline{HE} and \overline{HF} are trisectors of \overline{DG} .



Example Problem

\overline{EH} is divided by F and G in the ratio $5:3:2$ from left to right. If $EH = 30$, find FG



$$5x + 3x + 2x = 30$$

$$10x = 30$$

$$x = 3$$

$$FG = 3x = 3(3) = 9$$

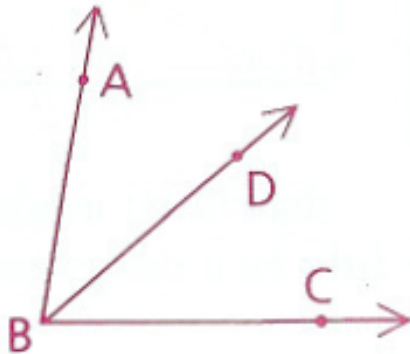
Is F the midpoint of EH ?

Since $EF = 5x = 5(3) = 15$ and $FH = 3x + 2x = 5x = 5(3) = 15$,
 F is the midpoint of EH .

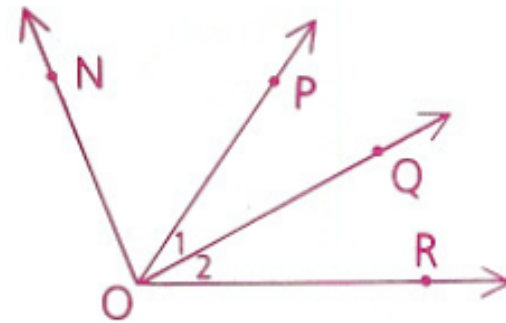
Angle Bisectors

A ray that divides an angle into two congruent angles bisects the angle. The dividing ray is called the bisector of the angle.

If $\angle ABD \cong \angle DBC$, then \overrightarrow{BD} (not \overrightarrow{DB}) is the bisector of $\angle ABC$.



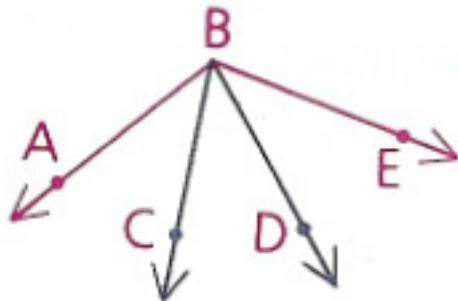
If $\angle NOP \cong \angle POR$ and \overrightarrow{OQ} bisects $\angle POR$, then \overrightarrow{OP} (not \overrightarrow{PO}) is the bisector of $\angle NOR$, and $\angle 1 \cong \angle 2$.



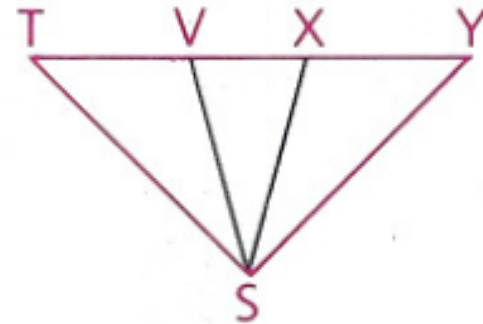
Angle Trisectors

Two rays that divide an angle into three congruent angles trisection the angle. The two dividing parts are called trisectors of the angle.

If $\angle ABC \cong \angle CBD \cong \angle DBE$,
then \overrightarrow{BC} and \overrightarrow{BD} trisect
 $\angle ABE$.



If \overrightarrow{SV} and \overrightarrow{SX} are trisectors
of $\angle TSY$, then $\angle TSV \cong$
 $\angle VSX \cong \angle XSY$.

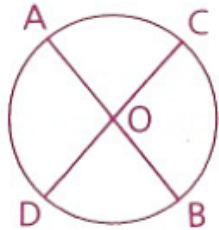


Homework

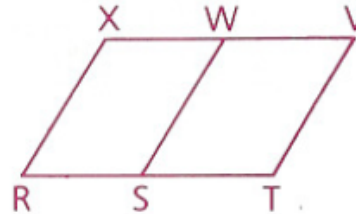
p. 32: 1-3, 5-9, 18, 19, 21, 22

1 Name the congruent segments.

a O is the midpoint of \overline{CD} .

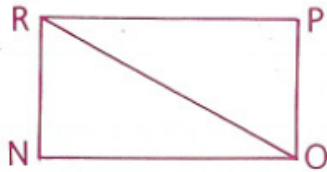


b \overline{SW} bisects \overline{XV} .

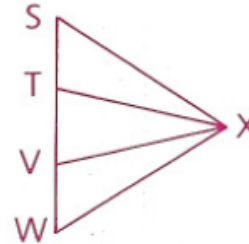


2 Name the congruent angles.

a \overrightarrow{RO} bisects $\angle NRP$.

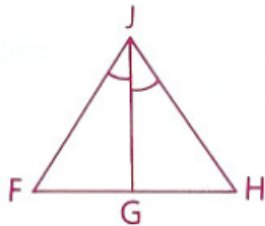


b \overrightarrow{XT} and \overrightarrow{XV} trisect $\angle SXW$.

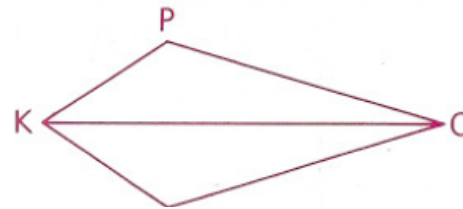


3 Name the angle bisector.

a



b $m\angle POK = m\angle MOK$



5 B and C trisect \overline{AD} .

a Find the coordinates of B and C.

b Find AC.



p. 32: 1-3, 5-9, 18, 19, 21, 22

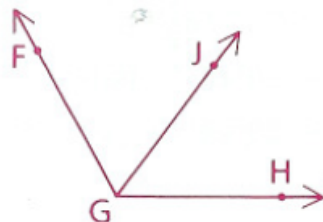
- 6 Given: $OM = x + 8$,
 $MP = 2x - 6$,
 $OP = 44$

Is M the midpoint of \overline{OP} ?



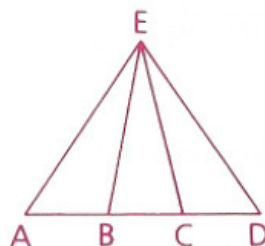
- 7 Given: $m\angle FGJ = 3x - 5$,
 $m\angle JGH = x + 27$;
 \overrightarrow{GJ} bisects $\angle FGH$.

Find: $m\angle FGJ$

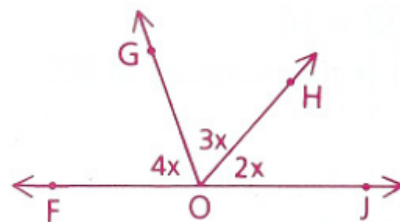


- 8 B and C are trisection points of \overline{AD} , and $\overline{AD} = 12$.

- a Find AB.
 b Find AC.
 c If $AB = x + 3$, solve for x.
 d If $AB = x + 3$ and $AE = 3x + 6$, find AE.
 e What segment is C the midpoint of?
 f Do \overrightarrow{EB} and \overrightarrow{EC} trisect $\angle AED$?

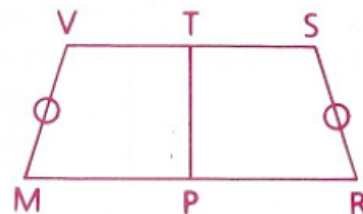


- 18 \overrightarrow{OG} and \overrightarrow{OH} divide straight angle FOJ into three angles whose measures are in the ratio 4:3:2. Find $m\angle FOG$.



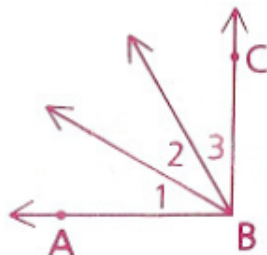
- 19 Given: \overleftrightarrow{TP} bisects \overline{VS} and \overline{MR} .
 $\overline{VM} \cong \overline{SR}$,
 $MP = 9$, $VT = 6$,
 perimeter of $MRSV = 62$

Find: VM

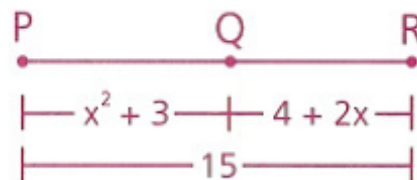


- 9 Given: $\angle ABC = 90^\circ$,
 $\angle 1 = (2x + 10)^\circ$,
 $\angle 2 = (x + 20)^\circ$,
 $\angle 3 = (3x)^\circ$

Has $\angle ABC$ been trisected.



- 21 a Find the value of x.
 b Is Q the midpoint of \overline{PR} ?



- 22 Given: \overrightarrow{OP} and \overrightarrow{OR} trisect $\angle NOS$.
 $m\angle NOP = 3x - 4y$,
 $m\angle POR = x - y$,
 $m\angle ROS = y - 10$

Find: $m\angle ROS$

