Objective

Students will be able to recognize polygons and identify special types of quadrilaterals.

Midterms are next week!

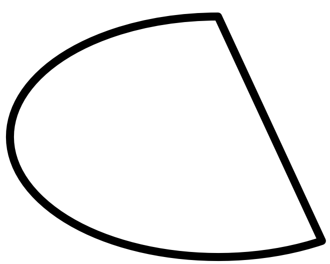
Polygons

What is a polygon?

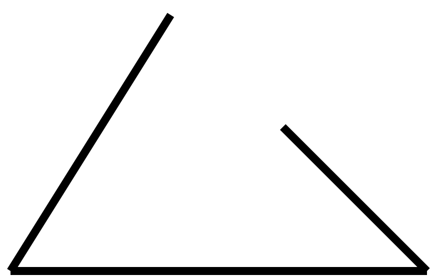
Polygons are closed plane figures.

- Contains all segments (no curves)
- Consecutive sides intersect only at endpoints
- Each vertex must belong to exactly two sides

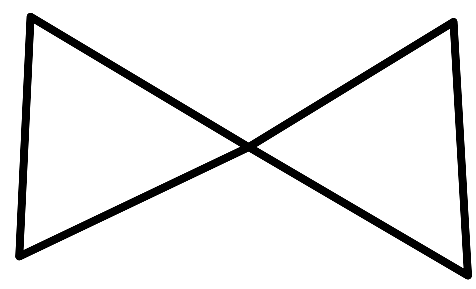
Polygon or not?



no; curved



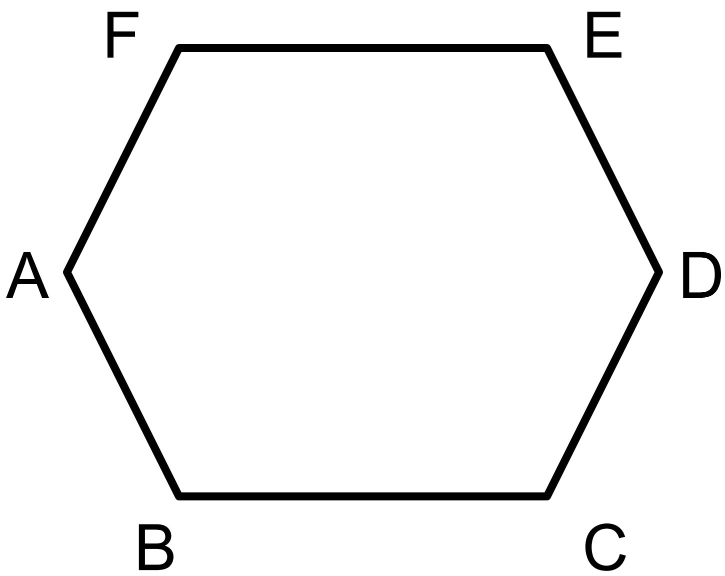
no; not closed



no; does not intersect at endpoints

Naming a Polygon

To name a polygon, you start at any vertex and you either go clockwise or counter-clockwise.

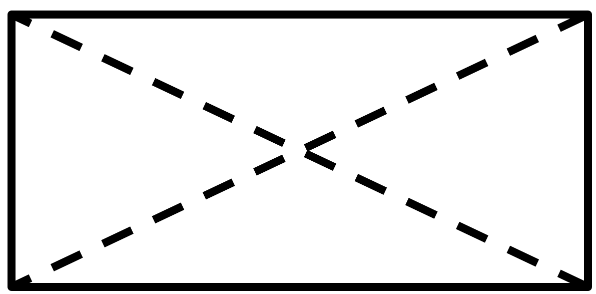


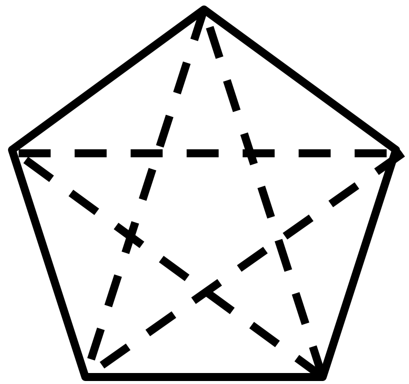
If we start at A, how can we name the polygon?

ABCDEF or AFEDCB

Diagonals

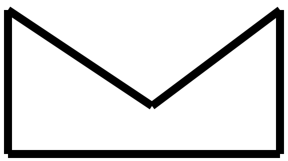
A diagonal of a polygon is any segment that connects two nonconsecutive (nonadjacent) vertices of the polygon.





A convex polygon is a polygon in which each interior angle has a measure less than 180.

A concave polygon is a polygon in which an interior angle has a measure greater than 180.



Quadrilateral Family Tree

Quadrilateral: four-sided polygon

1) Special Quadrilaterals:

Parallelogram: a quadrilateral with both pairs of opposite sides are parallel

Rectangle: a parallelogram with four right angles

Rhombus: a parallelogram with four congruent sides

Square: a parallelogram with four right angles and four congruent sides (a square is all of these!)

Quadrilateral Family Tree

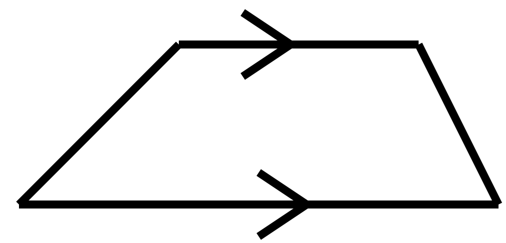
2) Special Quadrilaterals:

Kite: a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent

Quadrilateral Family Tree

3) Special Quadrilaterals:

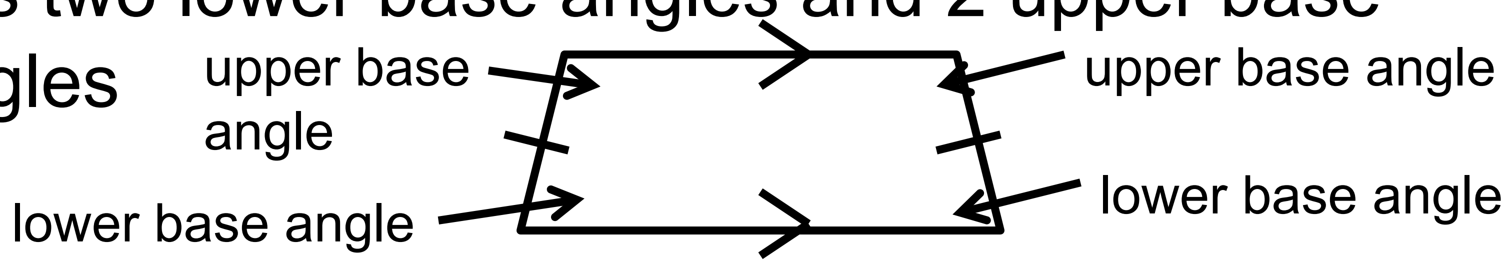
<u>Trapezoid</u>: a quadrilateral with exactly one pair of parallel sides



- parallel sides: bases
- nonparallel sides: legs
- base angles: two consecutive angles whose common side is a base

<u>Isosceles Trapezoid</u>: a trapezoid whose nonparallel sides (legs) are congruent

- has two lower base angles and 2 upper base angles

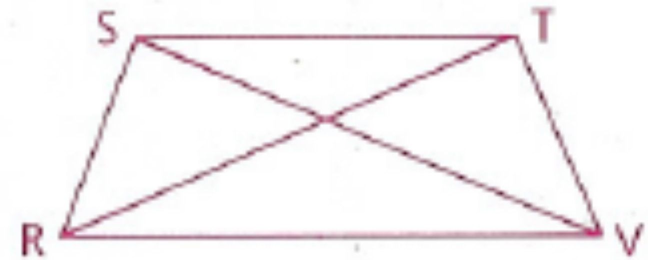


Homework

p. 237: 7-11, 13-16, 20

13, 14, 15
NOT TWO COLUMN PROOF

7 In the isosceles trapezoid shown, $\overline{ST} \parallel \overline{RV}$.



Name: **a** The bases
b The diagonals
c The legs
d The lower base angles
e The upper base angles
f All pairs of congruent alternate interior angles

8 Examine each statement below. If the statement is always true, write A; if sometimes true, write S; if never true, write N.

a A square is a rhombus.
b A rhombus is a square.
c A kite is a parallelogram.
d A rectangle is a polygon.
e A polygon has the same number of vertices as sides.
f A parallelogram has three diagonals.
g A trapezoid has three bases.

9 Why is a circle not a polygon?

10 Using the diagram, explain how the formula for the area of a parallelogram can be the same as that for the area of a rectangle.

11 If the sum of the measures of the angles of a triangle is 180, what is the sum of the measures of the angles in

a A quadrilateral?

b A pentagon (five-sided polygon)?

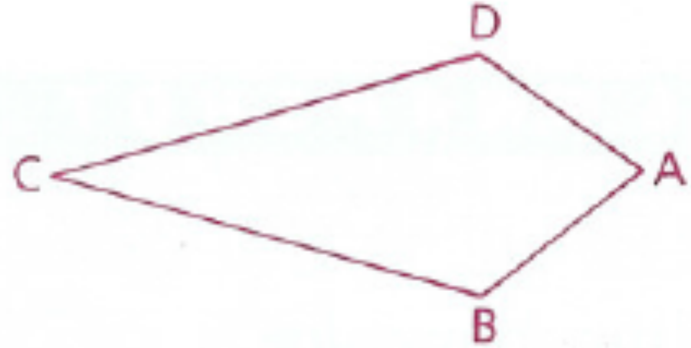
13 Prove that in a parallelogram each pair of consecutive angles are supplementary.

14 Prove that in a parallelogram each pair of opposite sides are congruent.

15 Prove that the diagonals of a rectangle are congruent.

16 Given: ABCD is a kite.

$AB = x + 3,$
$BC = x + 4,$
$CD = 2x - 1,$
$AD = 3x - y$



a Solve for x and y.
b What is the perimeter of the kite?
c Is it possible for $\overline{AC}$ to be 19 units long? Why or why not?

20 How many rectangles are shown in the figure at the right, in which all of the angles are right angles?

Objective

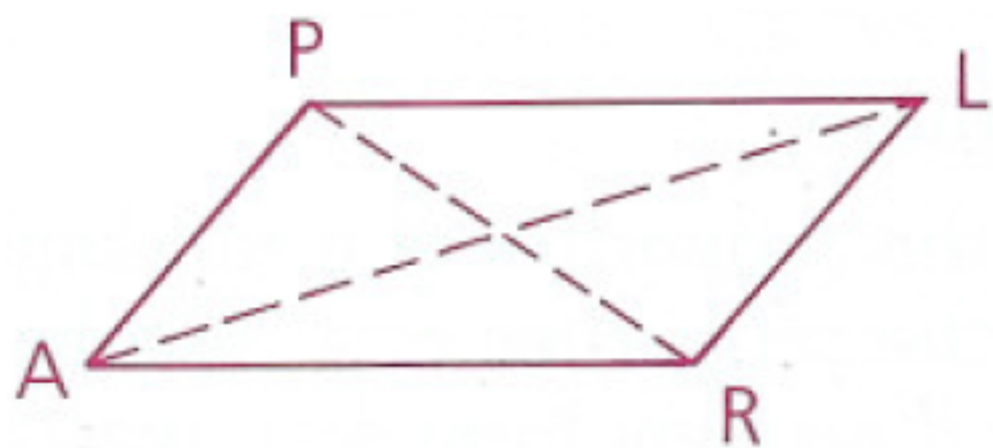
Students will be able to identify some properties of parallelograms.

Properties of Parallelograms

In a parallelogram,

1) The opposite sides are parallel by definition

$$\overline{PL} \parallel \overline{AR} \quad \overline{AP} \parallel \overline{RL}$$

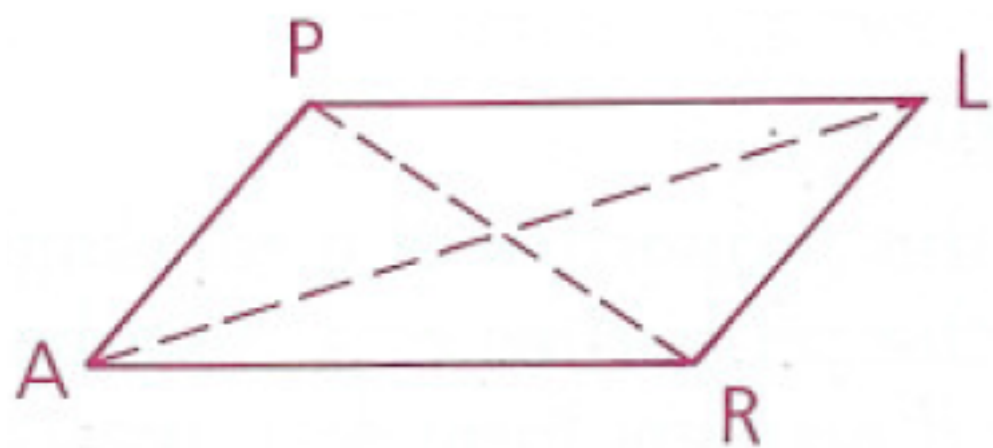


Properties of Parallelograms

In a parallelogram,

2) The opposite sides are congruent

$$\overline{PL} \cong \overline{AR} \qquad \overline{AP} \cong \overline{RL}$$

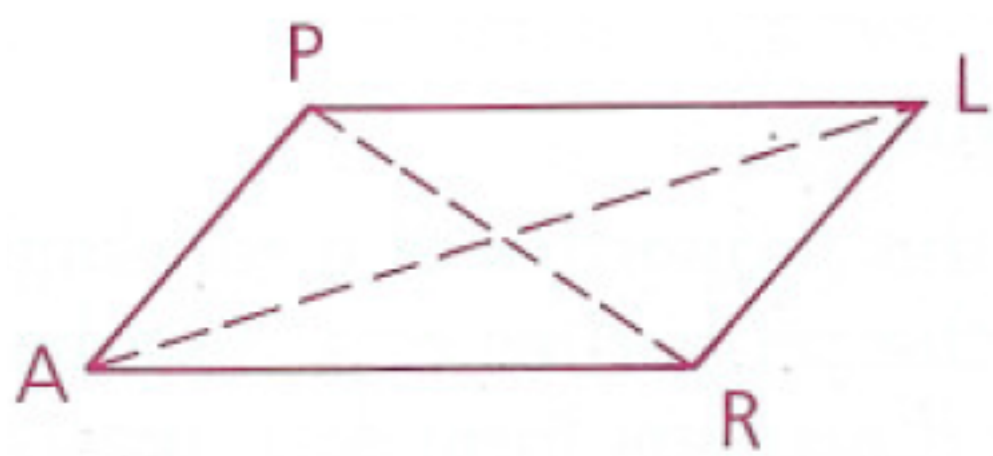


Properties of Parallelograms

In a parallelogram,

3) The opposite angles are congruent

$$\angle PAR \cong \angle PLR \qquad \angle ARL \cong \angle APL$$

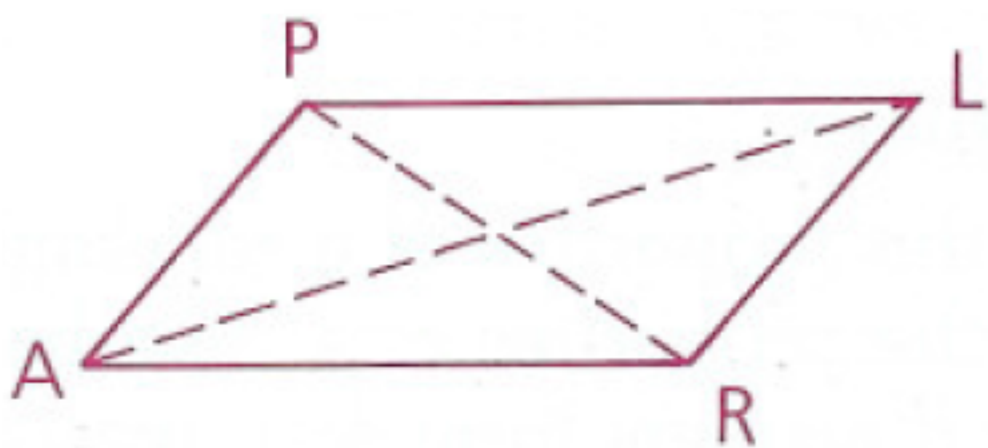


Properties of Parallelograms

In a parallelogram,

4) The diagonals bisect each other

$\overline{AL}$ bis. $\overline{PR}$ $\overline{PR}$ bis. $\overline{AL}$

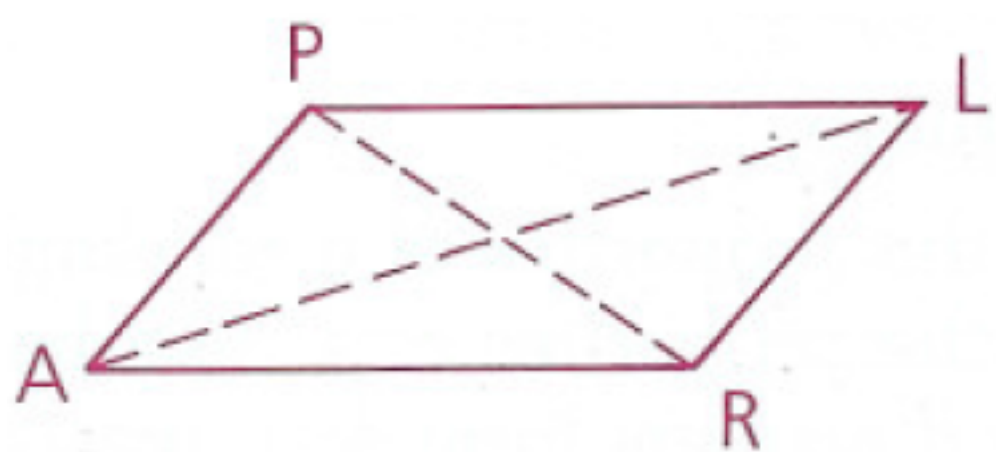


Properties of Parallelograms

In a parallelogram,

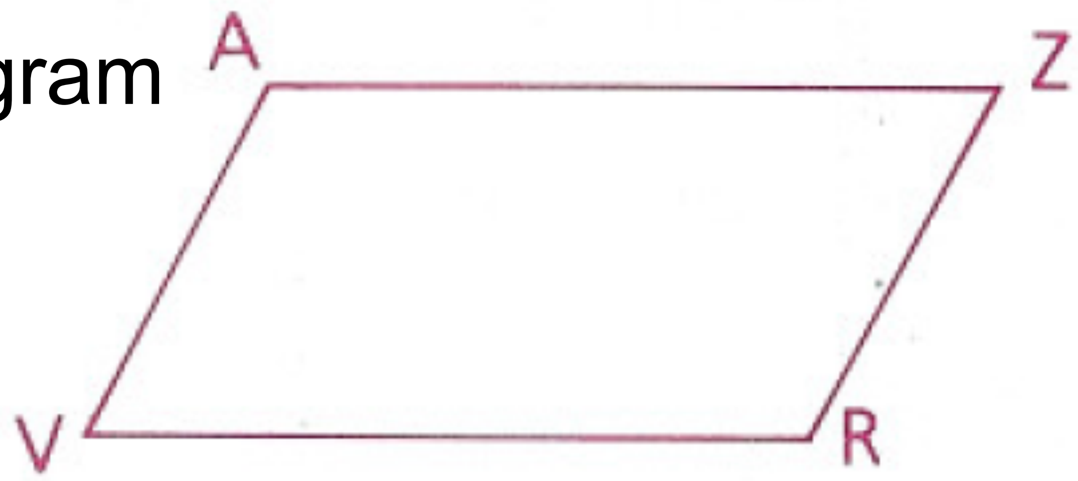
5) Any pair of consecutive angles are supplementary

$\angle PAR$ supp. $\angle ARL$



VRZA is a parallelogram

$AV = 2x - 4,$
$VR = 3y + 5,$
$RZ = \frac{1}{2}x + 8$
$ZA = y + 12$



Find the perimeter of *VRZA*

AV = 12 and RZ = 12

VR = 31/2 and ZA = 31/2

perimeter is 55

KMOP is a parallelogram

$\angle M = (x + 3y)^\circ$,
$\angle O = (x - 4)^\circ$,
$\angle P = (4y - 8)^\circ$

Find m∠K

$x + 3y = 4y - 8$

$x + 3y + x - 4 = 180$

$x - 4 + 4y - 8 = 180$

28

Objective

Students will be able to prove that a quadrilateral is a parallelogram.

Written Portion of Midterm is on Tuesday, March 28th

Multiple Choice Portion of Midterm is on Thursday, March 30th

Any one of the following methods might be used to prove that quadrilateral ABCD is a parallelogram.

1) If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram (reverse of the definition).

2) If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram (converse of the definition).

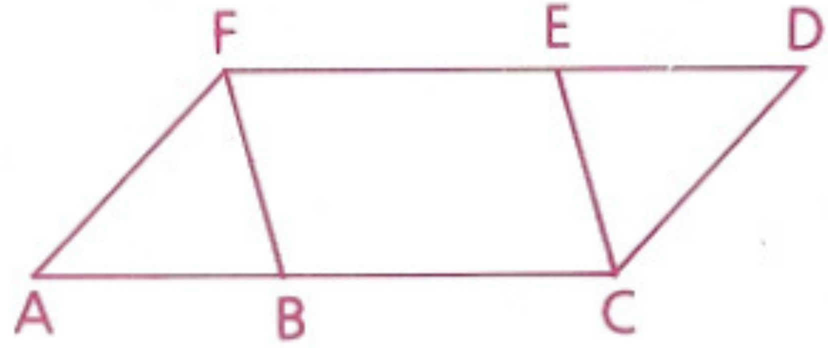
3) If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.

4) If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram (converse of a property).

5) If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram (converse of a property).

Given: ACDF is a ▱.
$\angle AFB \cong \angle ECD$

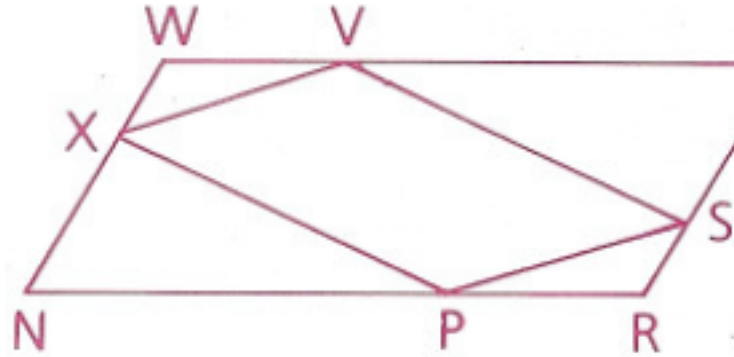
Prove: FBCE is a ▱.



1	ACDF is a ▱.	1	Given
2	$\angle A \cong \angle D$	2	Opposite $\angle$s of a ▱ are $\cong$.
3	$\overline{AF} \cong \overline{DC}$	3	Opposite sides of a ▱ are $\cong$.
4	$\angle AFB \cong \angle ECD$	4	Given
5	$\triangle AFB \cong \triangle DCE$	5	ASA (2, 3, 4)
6	$\overline{FB} \cong \overline{EC}$	6	CPCTC
7	$\overline{AB} \cong \overline{ED}$	7	CPCTC
8	$\overline{AC} \cong \overline{FD}$	8	Same as 3
9	$\overline{BC} \cong \overline{FE}$	9	Subtraction Property
10	FBCE is a ▱.	10	If both pairs of opposite sides of a quadrilateral are $\cong$, it is a ▱.

Given: NRTW *is a* ▱.
$\overline{NX} \cong \overline{TS}$,
$\overline{WV} \cong \overline{PR}$

Prove: XPSV *is a* ▱.



1 NRTW is a ▱.	1 Given
2 $\angle N \cong \angle T$	2 Opposite $\angle$s of a ▱ are $\cong$.
3 $\overline{NX} \cong \overline{TS}$	3 Given
4 $\overline{NR} \cong \overline{WT}$	4 Opposite sides of a ▱ are $\cong$.
5 $\overline{WV} \cong \overline{PR}$	5 Given
6 $\overline{NP} \cong \overline{VT}$	6 Subtraction Property
7 $\triangle NXP \cong \triangle TSV$	7 SAS (3, 2, 6)
8 $\overline{XP} \cong \overline{VS}$	8 CPCTC
9 In a similar manner, $\triangle WXV \cong \triangle RSP$ and $\overline{XV} \cong \overline{PS}$.	9 Steps 1–8
10 XPSV is a ▱.	10 If both pairs of opposite sides of a quadrilateral are $\cong$, it is a ▱.

Objective

Students will be able to identify some properties of rectangles, rhombuses, and squares.

Written Portion of Midterm is on Tuesday, March 28th
Multiple Choice Portion of Midterm is on Thursday, March 30th
Alpha Workshop today after school in room 125

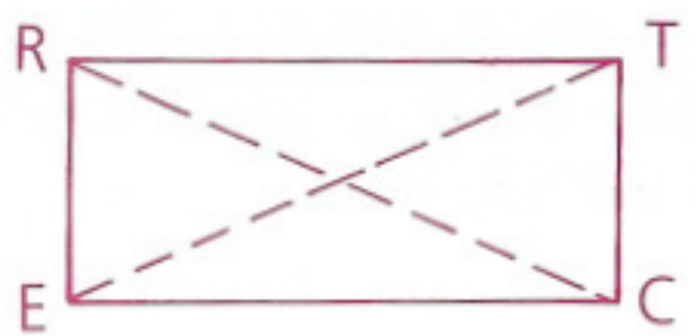
Properties of Rectangles

In a rectangle,

1) All the properties of a parallelogram apply by definition

2) All angles are right angles

3) The diagonals are congruent $\overline{ET} \cong \overline{RC}$

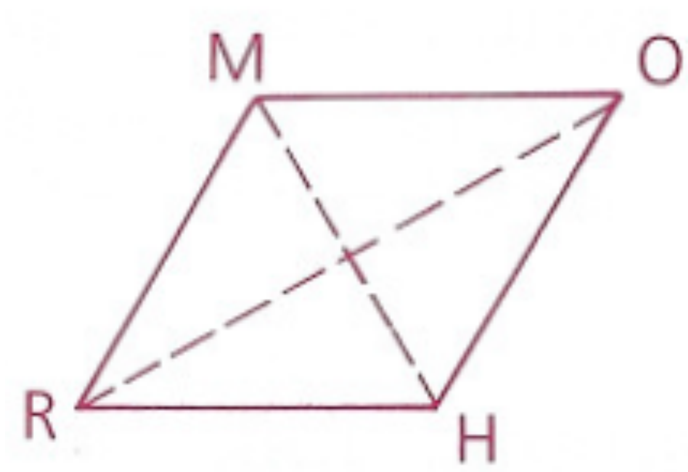


Properties of Rhombuses

In a rhombus,

1) All the properties of a parallelogram apply by definition

2) All sides are congruent– a rhombus is equilateral $\overline{RH} \cong \overline{HO} \cong \overline{OM} \cong \overline{MR}$



Properties of Rhombuses

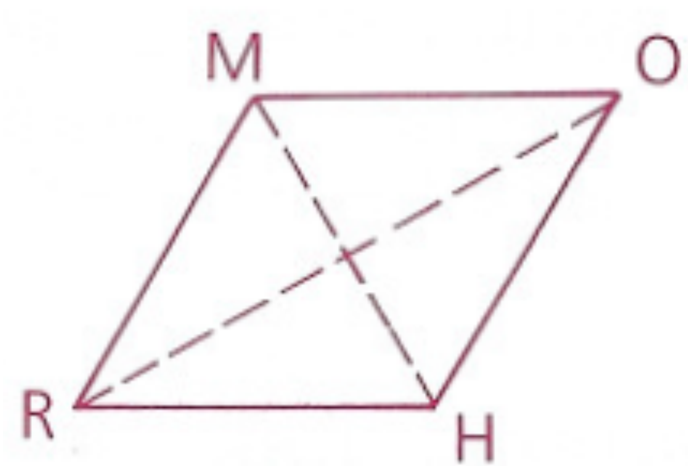
In a rhombus,

3) The diagonals bisect the angles

$\overrightarrow{RO}$ bis. $\angle MRH$ $\overrightarrow{RO}$ bis. $\angle MOH$

4) The diagonals are perpendicular bisectors of each other $\overline{RO} \perp$ bis. $\overline{MH}$

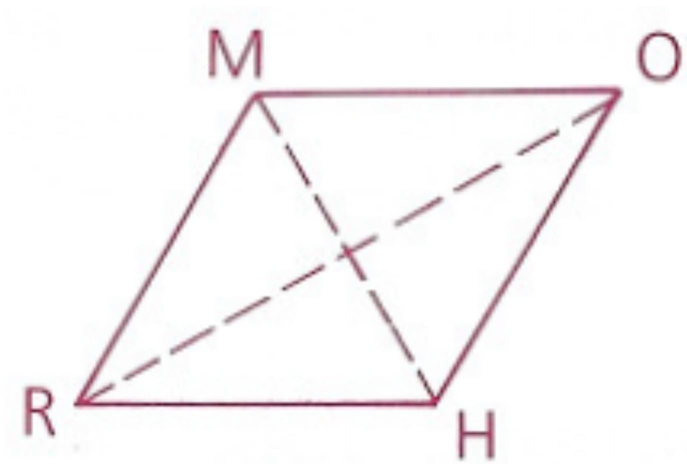
$\overline{MH} \perp$ bis. $\overline{RO}$



Properties of Rhombuses

In a rhombus,

5) The diagonals divide the rhombus into four congruent right triangles



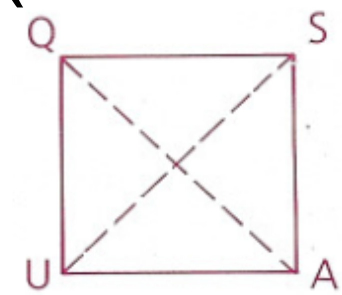
Properties of Squares

In a square,

1) All the properties of a rectangle apply by definition

2) All the properties of a rhombus apply by definition

3) The diagonals form four isosceles right triangles (45^o-45^o-90^o triangles)



Objective

Students will be able to identify some properties of kites and trapezoids.

Written Portion of Midterm is on Tuesday, March 28th
Multiple Choice Portion of Midterm is on Thursday, March 30th

Properties of Kites

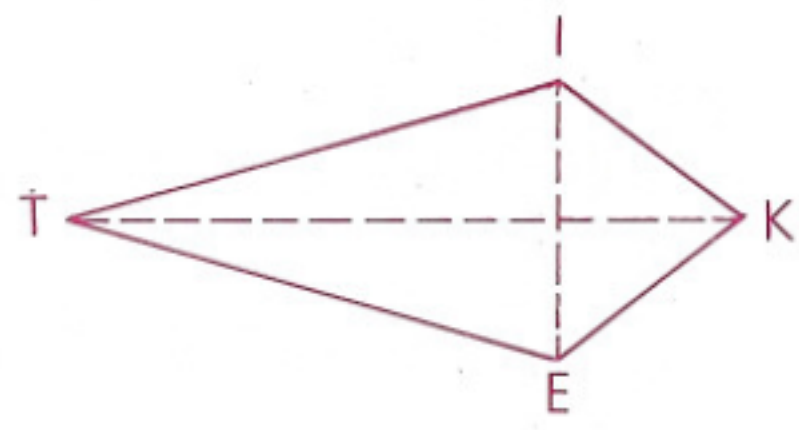
In a kite,

1) Two disjoint pairs of consecutive sides are congruent by definition

$\overline{IT} \cong \overline{ET}$ $\overline{IK} \cong \overline{EK}$

2) The diagonals are perpendicular

$\overline{TK} \perp \overline{IE}$

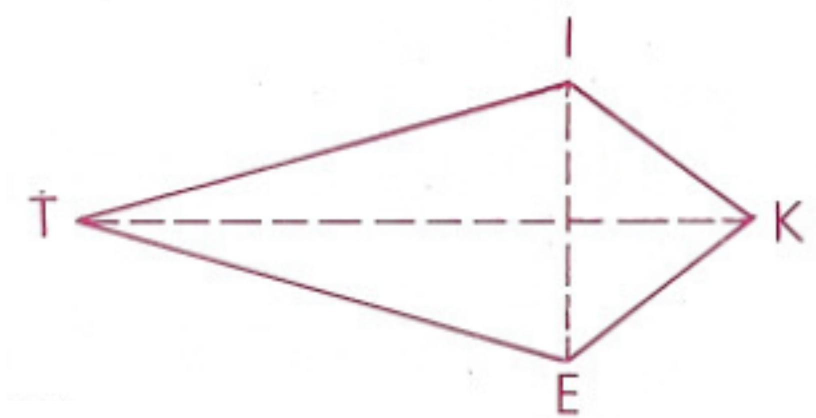


Properties of Kites

In a kite,

3) One diagonal is the perpendicular bisector of the other $\overline{TK} \perp$ bis. $\overline{IE}$

4) One of the diagonals bisects a pair of opposite angles



$\overrightarrow{TK}$ bis. $\angle IKE$

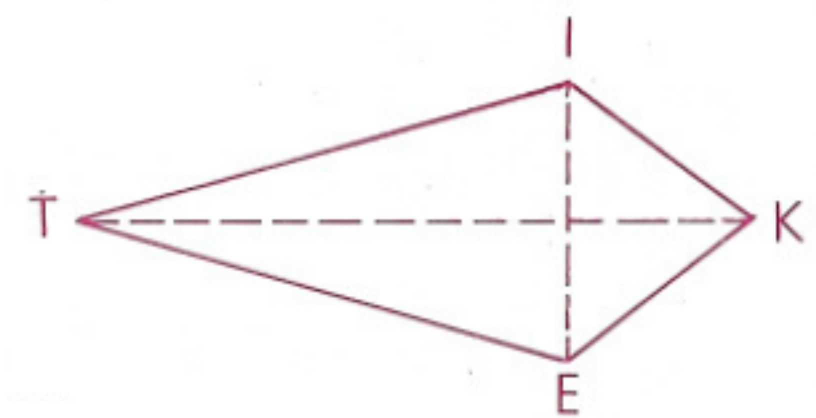
$\overrightarrow{TK}$ bis. $\angle ITE$

Properties of Kites

In a kite,

5) One pair of opposite angles are congruent $\angle TIK \cong \angle TEK$

Properties 3-5 are sometimes called the *half properties* of kites



Properties of Isosceles Trapezoids

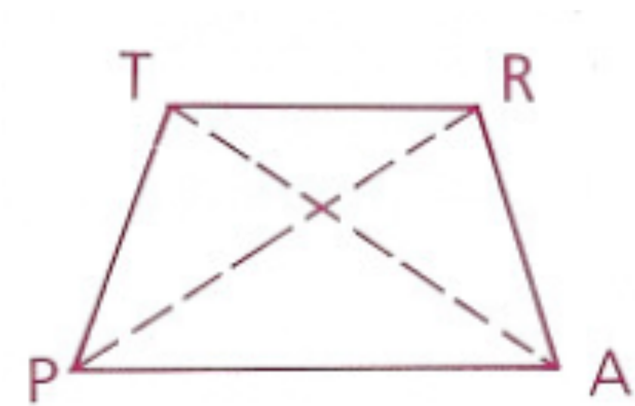
In an isosceles trapezoid,

1) The legs are congruent by definition

$$\overline{TP} \cong \overline{RA}$$

2) The bases are parallel (by definition of trapezoid)

$$\overline{TR} \parallel \overline{PA}$$



Properties of Isosceles Trapezoids

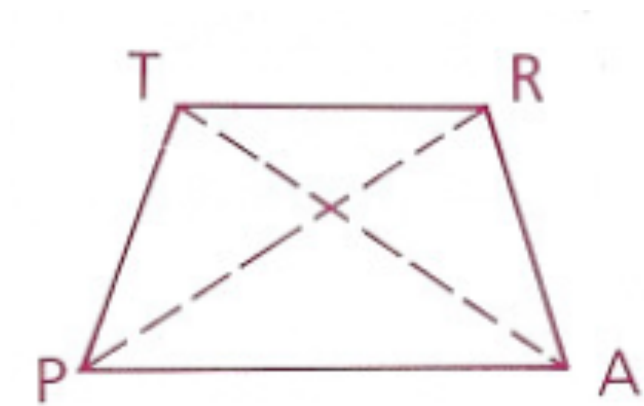
In an isosceles trapezoid,

3) The lower bases are congruent

$$\angle RAP \cong \angle TPA$$

4) The upper bases are congruent

$$\angle PTR \cong \angle ART$$



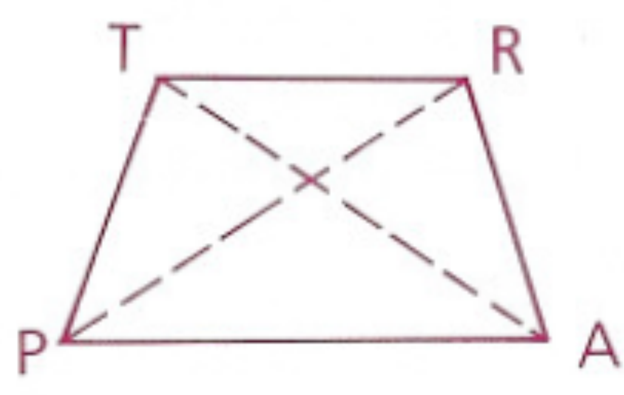
Properties of Isosceles Trapezoids

In an isosceles trapezoid,

5) The diagonals are congruent

$$\overline{PR} \cong \overline{AT}$$

6) Any lower base angle is supplementary to any upper base angle



$\angle PAR$ supp. $\angle PTR$

Objective

Students will be able to prove that a quadrilateral is a rectangle, kite, rhombus, square, and an isosceles trapezoid.

Rough Drafts of Data Due Today!!! I need your rubric and your Word document.

Quadrilateral Quiz on Thursday!

Proving that a Quadrilateral is a Rectangle

To prove that a quadrilateral is a rectangle, you must prove that it is a parallelogram first and then can use either of these methods:

1) If a parallelogram contains at least one right angle, then it is a rectangle (reverse of the definition)
2) If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle

Proving that a Quadrilateral is a Rectangle

You can also prove that a quadrilateral is a rectangle without first proving that it is a parallelogram:

3) If all four angles of a quadrilateral are right angles, then it is a rectangle

Proving that a Quadrilateral is a Kite

To prove a quadrilateral is a kite, either of the following methods can be used:

1) If two disjoint pairs of consecutive sides of a quadrilateral are congruent, then it is a kite (reverse of the definition)
2) If one of the diagonals of a quadrilateral is the perpendicular bisector of the other diagonal, then the quadrilateral is a kite

Proving that a Quadrilateral is a Rhombus

To prove that a quadrilateral is a rhombus, you must prove that it is a parallelogram first and then can use either of these methods:

1) If a parallelogram contains a pair of consecutive sides that are congruent, then it is a rhombus (reverse of definition)
2) If either diagonal of a parallelogram bisects two angles of the parallelogram, then it is a rhombus

Proving that a Quadrilateral is a Rhombus

You can also prove that a quadrilateral is a rhombus without first proving that it is a parallelogram:

3) If the diagonals of a quadrilateral are perpendicular bisectors of each other, then the quadrilateral is a rhombus

Proving that a Quadrilateral is a Square

To prove a quadrilateral is a square:

If a quadrilateral is both a rectangle and a rhombus, then it is a square (reverse of the definition)

Proving that a Trapezoid is Isosceles

Any one of the following methods can be used to prove that a trapezoid is isosceles:

1) If the nonparallel sides of a trapezoid are congruent, then it is isosceles (reverse of the definition)
2) If the lower or the upper base angles of a trapezoid are congruent, then it is isosceles
3) If the diagonals of a trapezoid are congruent, then it is isosceles

What is the most descriptive name for quadrilateral ABCD with vertices A = (-3, -7), B = (-9, 1), C = (3, 9), and D = (9, 1)?

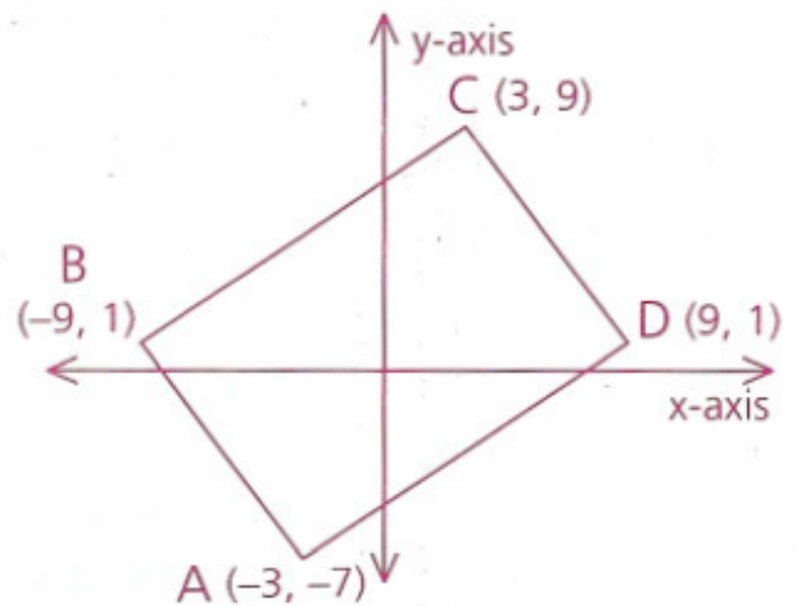
We must check every detail to see if sides are parallel or perpendicular, and we must also check diagonals. We must be careful to identify what we are finding with each calculation. A graph may prove helpful in directing our work.

Slope of AB $= \dfrac{1-(-7)}{-9-(-3)} = \dfrac{8}{-6} = -\dfrac{4}{3}$

Slope of BC $= \dfrac{9-1}{3-(-9)} = \dfrac{8}{12} = \dfrac{2}{3}$

Slope of CD $= \dfrac{1-9}{9-3} = \dfrac{-8}{6} = -\dfrac{4}{3}$

Slope of AD $= \dfrac{1-(-7)}{9-(-3)} = \dfrac{8}{12} = \dfrac{2}{3}$



What is the most descriptive name for quadrilateral ABCD with vertices A = (-3, -7), B = (-9, 1), C = (3, 9), and D = (9, 1)?

Slope of AB = -4/3　　Slope of CD = -4/3
Slope of BC = 2/3　　Slope of AD = 2/3

Since the slopes of AB and CD are equal, AB ll CD. Similarly, slopes of BC and AD are equal, so BC ll AD. Thus, ABCD is at least a parallelogram.

Is it a rectangle or rhombus?

Since the slopes of AB and BC are not opposite reciprocals of each other (they are not perpendicular), ∠ABC is not a right angle. ABCD is not a rectangle.

What is the most descriptive name for quadrilateral ABCD with vertices A = (-3, -7), B = (-9, 1), C = (3, 9), and D = (9, 1)?

For the figure to be a rhombus, the diagonals must be perpendicular.

$$\text{Slope of AC} = \frac{9-(-7)}{3-(-3)} = \frac{16}{6} = \frac{8}{3}$$

$$\text{Slope of BD} = \frac{1-1}{9-(-9)} = \frac{0}{18} = 0$$

The slopes are not opposite reciprocals, so AC is not perpendicular to BD. Thus, ABCD is not a rhombus.

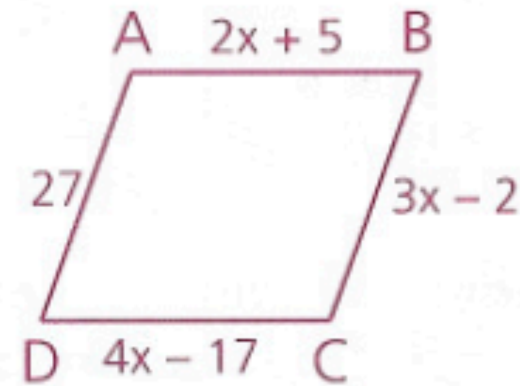
Therefore, ABCD is a parallelogram

Homework

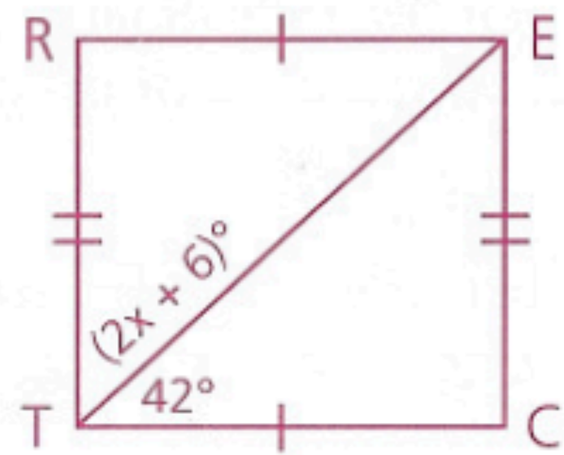
p. 258: 1, 2, 3, 14, 16, 19

1 Locate points Q = (2, 4), U = (2, 7), A = (10, 7), and D = (10, 4) on a graph. Then give the most descriptive name for QUAD.

2 If $\overline{AB} \cong \overline{DC}$, show that ABCD is not a rhombus.

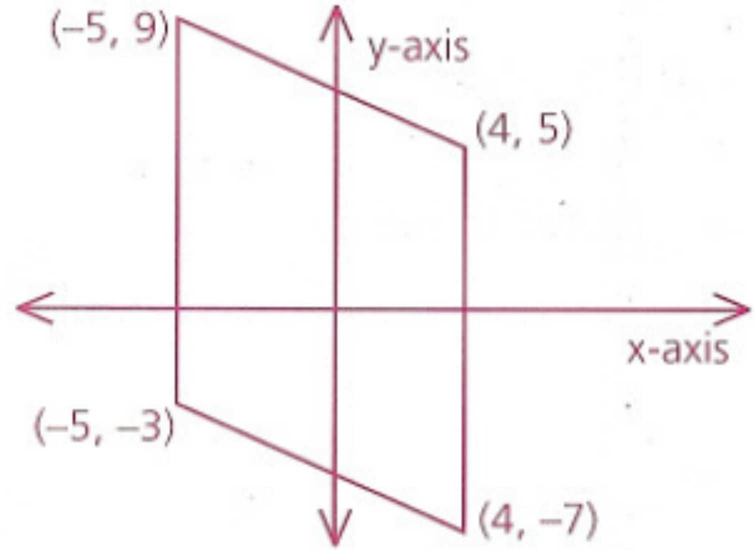


3 In order for RECT to be a rectangle, what must the value of x be?

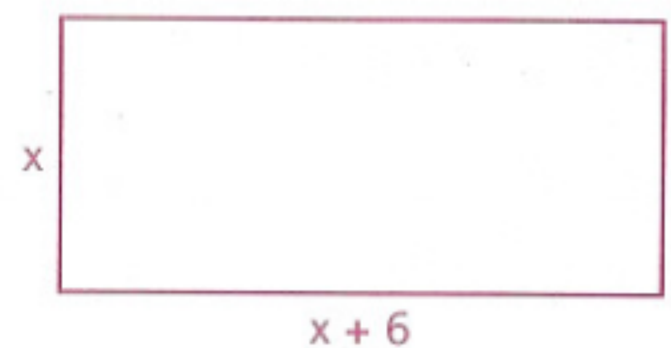


14 What is the most descriptive name for a quadrilateral with vertices $(-7, 2)$, $(2, 8)$, $(6, 2)$, and $(-3, -4)$? Justify your conclusion.

16 Find the area of the parallelogram. (Hint: Area = base · height.)



19 Write a quadratic equation to represent the area of the rectangle. If the area is 160 square meters, find the perimeter.



Objective

Students will be able to define, name, and identify various polygons and discover the sum of the interior angles of each.

Polygon Test on Friday, April 21st

Naming Polygons

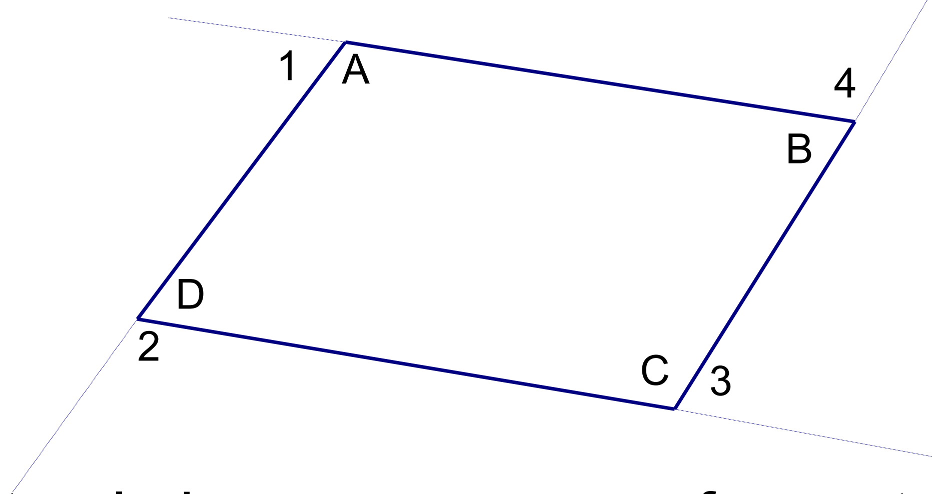
A polygon with three sides can be called a 3-gon. Similarly, a polygon with seven sides can be called a 7-gon. Most polygons we will discuss have special names.

# Sides/Vertices	Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
15	Pentadecagon
n	n-gon

Investigating the Sum of Polygon Angle Measures

The sum of the measures of the angles of any n-gon is

$$S = (n-2)180$$



In the quadrilateral above, we can see four exterior angles drawn.

We know that each interior angle is supplementary to one exterior angle.

In this case, $\angle A + \angle 1 = 180$; $\angle B + \angle 4 = 180$; $\angle C + \angle 3 = 180$; $\angle D + \angle 2 = 180$

The sum of all four of the interior angles and all four exterior angles is $4(180) = 720$

Since we have a quadrilateral, we know that the sum of the measures of the interior angles is 360^o.

Now, if we subtract the measures of the interior angles from the measures of all angles we get $720 – 360 = 360^o$

This is actually something that works with any n-gon.

The sum of the measures of the exterior angles, one taken at each vertex, is 360^{o}

Investigating the Number of Diagonals in a Polygon

The number of diagonals in any polygon is

$$d = \frac{n(n-3)}{2}$$

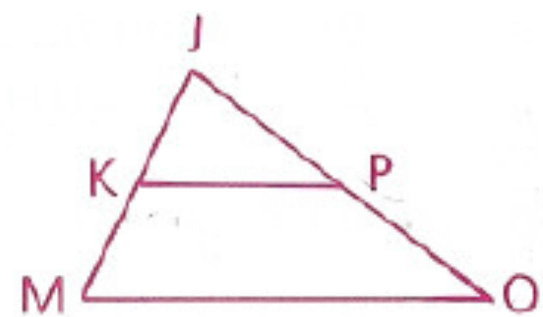
Homework p. 309: 1, 3, 5, 10, 13

1 Find the sum of the measures of the angles of

a A quadrilateral
b A heptagon
c An octagon
d A dodecagon
e A 93-gon

3 How many diagonals can be drawn in each figure below?

5 Given: K is a midpoint.
P is a midpoint.
$m\angle M = 70$,
$m\angle JKP = y + 15$,
$m\angle JPK = y - 10$



Find: **a** $m\angle JKP$ **b** $m\angle JPK$ **c** $m\angle J$

10 How many sides does a polygon have if the sum of the measures of its angles is

a 900?

b 1440?

c 2880?

d $180x - 720$?

e 436?

f Six right angles?

13 What are the names of the polygons that contain the following numbers of diagonals?

a 14

b 35

c 209