

Polynomials and Polynomial Functions (5.2 - 5.5, 5.8) Review
Advanced Algebra w/ Trig, Glawe

Name: Key
Date: _____ Period: _____

1) Find the sum and difference of the polynomials:

$$(x^2 + 6x + 2) + (5x^2 + 8x - 5) - (2x^2 + x - 4)$$

$$6x^2 + 14x - 3 - 2x^2 - x + 4 =$$

$$\boxed{4x^2 + 13x + 1}$$

3) Factor the polynomial completely:

$$x^4 - x^2 - 12 \quad \text{quadratic form}$$

$$(x^2 - 4)(x^2 + 3) \quad \begin{array}{c|c} \text{a.c} & b \\ -12 & 1 \\ \hline -4, 3 & \end{array}$$

difference of squares

$$\boxed{(x-2)(x+2)(x^2+3)}$$

5) Factor the polynomial completely:

$$4x^6 + 4x^3 - 8 \quad \text{GCF}$$

$$4(x^6 + x^3 - 2) \quad \text{quadratic form}$$

$$4(x^3 - 1)(x^3 + 2) \quad \begin{array}{c|c} -2 & 1 \\ \hline 2, -1 & \end{array}$$

difference of cubes

$$\boxed{4(x-1)(x^2+x+1)(x^3+2)}$$

7) Solve the polynomial:

$$3x^4 - 2x^2 - 8 = 0 \quad \text{quadratic form}$$

$$(3x^4 - 6x^2) + (4x^2 - 8) = 0 \quad \begin{array}{c|c} \text{a.c} & b \\ 3 \cdot -8 = -24 & -2 \\ \hline -6, 4 & \end{array}$$

$$3x^2(x^2 - 2) + 2(x^2 - 2) = 0$$

$$(3x^2 + 2)(x^2 - 2) = 0$$

$$3x^2 + 2 = 0$$

$$x^2 - 2 = 0$$

$$\frac{3x^2}{3} = \frac{-2}{3}$$

$$\sqrt{x^2 - 2} = \pm \sqrt{2}$$

$$x = \pm \sqrt{\frac{-2}{3}} = \pm i \sqrt{\frac{2}{3}} = \pm i \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm i \frac{\sqrt{6}}{3}$$

$$\boxed{x = \pm \sqrt{2}, \pm \frac{\sqrt{6}}{3}i}$$

2) Find the product of the two polynomials:

$$(2x^2 + 3x - 4)(x - 1)$$

$$2x^3 - 2x^2 + 3x^2 - 3x - 4x + 4 =$$

$$\boxed{2x^3 + x^2 - 7x + 4}$$

4) Factor the polynomial completely:

$$24x^4 - 375x \quad \text{GCF}$$

$$3x(8x^3 - 125) = 3x((2x)^3 - (5)^3) =$$

difference of cubes \rightarrow S.O.A.P

$$3x(2x-5)((2x)^2 + 2x(5) + 5^2) =$$

$$\boxed{3x(2x-5)(4x^2 + 10x + 25)}$$

6) Factor the polynomial completely:

$$x^4 - 81 \quad \text{difference of squares}$$

$$(x^2 - 9)(x^2 + 9)$$

difference of squares

$$\boxed{(x-3)(x+3)(x^2+9)}$$

8) Solve the polynomial:

$$27x^3 = 64$$

$$27x^3 - 64 = 0$$

difference of cubes

$$((3x)^3 - (4)^3) = 0$$

$$(3x-4)((3x)^2 + 3x(4) + (4)^2) = 0$$

$$(3x-4)(9x^2 + 12x + 16) = 0$$

$$3x-4=0$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$x = 4/3$$

$$\text{quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(9)(16)}}{2(9)}$$

$$\frac{-12 \pm \sqrt{144 - 576}}{18} = \frac{-12 \pm \sqrt{-432}}{18}$$

$$\frac{-12 \pm 12i\sqrt{3}}{18} = \frac{-2 \pm 2i\sqrt{3}}{3}$$

$$\boxed{x = 4/3, \frac{-2 \pm 2i\sqrt{3}}{3}}$$

$$\sqrt{-432} = \sqrt{-1 \cdot 144 \cdot 3} = i \sqrt{144 \cdot 3} = 12i\sqrt{3}$$

9) Divide using polynomial long division:

$$(x^3 + x^2 + 5x - 4) \div (x + 2)$$

$$\begin{array}{r} x^2 - x + 7 \\ x+2 \overline{) x^3 + x^2 + 5x - 4} \\ \underline{-(x^3 + 2x^2)} \\ -x^2 + 5x \\ \underline{-(-x^2 - 2x)} \\ 7x - 4 \\ \underline{-(7x + 14)} \\ -18 \end{array}$$

$$x^2 - x + 7 + \frac{-18}{x+2}$$

$$\frac{x^3}{x} = x^2$$

$$x^2(x+2) = x^3 + 2x^2$$

$$\frac{-x^2}{x} = -x$$

$$-x(x+2) = -x^2 - 2x$$

$$\frac{7x}{x} = 7$$

$$7(x+2) = 7x + 14$$

10) Given that $(x + 9)$ is a factor of the polynomial

$$f(x) = x^3 + 2x^2 - 51x + 108, \text{ factor completely.}$$

$$x+9=0$$

$$x=-9$$

$$\begin{array}{r|rrrr} -9 & 1 & 2 & -51 & 108 \\ \hline & \downarrow & -9 & 63 & -108 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$$f(x) = (x+9)(x^2 - 7x + 12)$$

$$f(x) = (x+9)(x-4)(x-3)$$

11) Divide the following polynomials: $(4x^4 - 5x^3 - 4) \div (x - 2)$

long division:

$$\begin{array}{r} 4x^3 + 8x^2 + 16x + 27 \\ x-2 \overline{) 4x^4 + 0x^3 + 0x^2 - 5x - 4} \\ \underline{-(4x^4 - 8x^3)} \\ 8x^3 + 0x^2 \\ \underline{-(8x^3 - 16x^2)} \\ 16x^2 - 5x \\ \underline{-(16x^2 - 32x)} \\ 27x - 4 \\ \underline{-(27x - 54)} \\ -54 \end{array}$$

$$\frac{4x^4}{x} = 4x^3$$

$$4x^3(x-2) = 4x^4 - 8x^3$$

$$\frac{8x^3}{x} = 8x^2$$

$$8x^2(x-2) = 8x^3 - 16x^2$$

$$\frac{16x^2}{x} = 16x$$

$$16x(x-2) = 16x^2 - 32x$$

$$\frac{27x}{x} = 27$$

$$27(x-2) = 27x - 54$$

synthetic division:

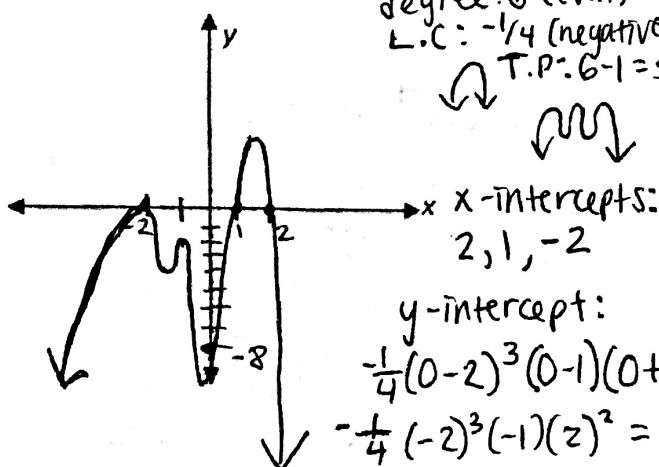
$$\begin{array}{r|rrrrr} 2 & 4 & 0 & 0 & -5 & -4 \\ \hline & \downarrow & 8 & 16 & 32 & 54 \\ \hline & 4 & 8 & 16 & 27 & -50 \end{array}$$

quotient: $4x^3 + 8x^2 + 16x + 27$
remainder: -50

Sketch the graph of the polynomial using end behavior, x-intercepts and y-intercept. Be sure to label your graphs.

$$12) f(x) = -\frac{1}{4}(x-2)^3(x-1)(x+2)^2$$

degree: 6 (even)
L.C.: $-\frac{1}{4}$ (negative)
T.P.: $6-1=5$



x-intercepts:
2, 1, -2

y-intercept:

$$-\frac{1}{4}(0-2)^3(0-1)(0+2)^2$$

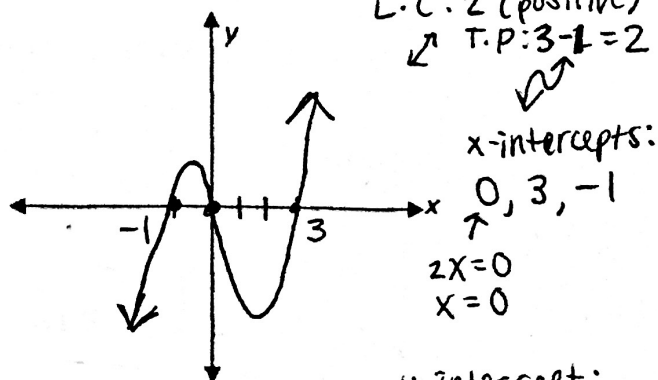
$$-\frac{1}{4}(-8)(-1)(4) = -8$$

$$-\frac{1}{4}(-8)(-1)(4) = -8$$

bounces at -2

$$13) f(x) = 2x(x-3)(x+1)$$

degree: 3 (odd)
L.C.: 2 (positive)
T.P.: $3-1=2$



x-intercepts:

0, 3, -1

$$2x=0$$

$$x=0$$

y-intercept:

$$2(0)(0-3)(0+1) = 0$$

$$0(-3)(1) = 0$$