Objective

Students will be able to recognize that geometry is based on a deductive structure and will be able to understand parts of a proof (undefined terms, postulates, definitions, and theorems).

ANNOUCEMENTS:

Logo Project due TOMORROW
Written Midterm: Thursday, October 27th
Multiple Choice Midterm: Monday, October 31st

Deductive Structure

Geometry is based on a <u>deductive structure</u>–
a system of thought in which
conclusions are justified by means of previously assumed or proved statements.

Contains four elements:

- Undefined terms
- Assumptions known as postulates
- Definitions
- Theorems and other conclusions

Undefined term

- Not defined but everyone has a clear idea of what they are
- Examples: a point and a line

Postulate

- Unproved assumption; we do not need to prove
- Example: any two points determine a line

Definition

- States the meaning of a term or idea
- Reversible
- Example: If an angle measure is 90 degrees, then it is a right angle

Theorem

- Statement you can prove
- Not always reversible
- Examples: From Warm Up

Conditional Statement (implication)

Definitions can be stated in the form "If p, then q" where p and q are declarative statements.

- $p \Rightarrow q$ ("p implies q")

Where have you heard this before?

What else does p and q mean?

The "if" part of the sentence is called the hypothesis. The "then" part of the sentence is called the conclusion.

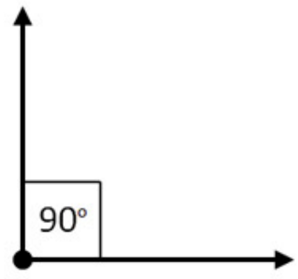
Since definitions are reversible, what do you think the reversible statement of a conditional statement is called and how would it be phrased/look?

Converse of $p \Rightarrow q$ is $q \Rightarrow p$.

- To write the converse of a conditional statement, you reverse parts p and q. The converse is “If q, then p”

Remember,

- Definitions are always reversible
- Theorems and postulates are not always reversible



Example

Conditional Statement: If an angle measures 90 degrees, then it is a right angle.

What is p and what is q?

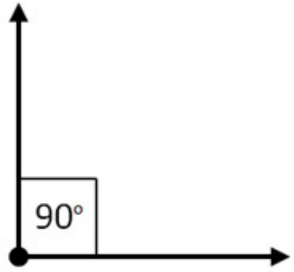
p is "an angle measures 90 degrees"

q is "it is a right angle"

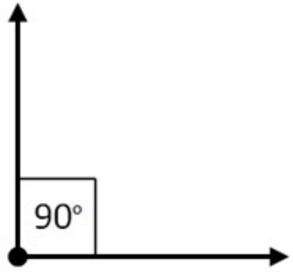
What is the converse of this statement?

If an angle is a right angle, then the angle measures 90 degrees.

Is this converse a true statement always? yes!



Example 2



Conditional Statement: If two angles are right angles, then they are congruent.

What is p and what is q?

p is "two angles are right angles"

q is "they are congruent"

What is the converse of this statement?

If two angles are congruent, then they are right angles.

Is this converse a true statement always? no!

More Examples

1) If Mary received a B on her history test, then she passed her test.

Is the converse always true? Why or why not? No!

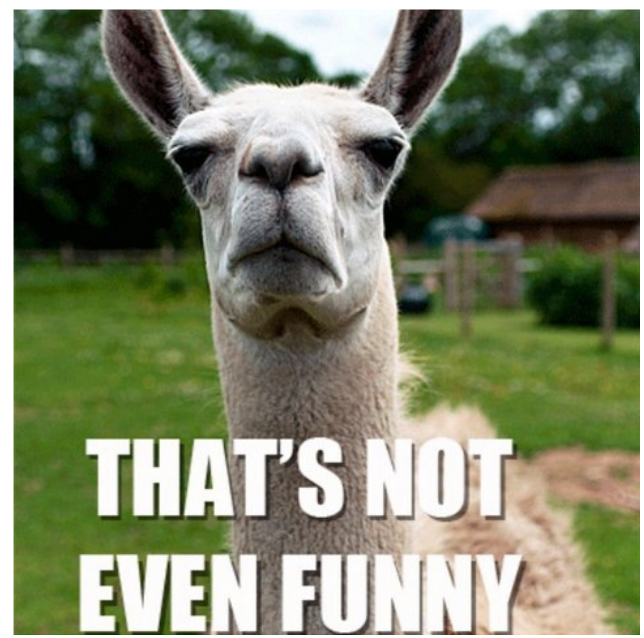
A C D

2) If an animal is a dog, then it has four legs.

Is the converse always true? Why or why not? No!

**These are examples of True, False Statements

With a partner, come up with a True True statement and a True False statement that you will then share with the rest of the class



Homework

p. 42-43: 1 - 5, 8 - 12, 14

1 What four elements are found in any deductive structure?

2 Which of the following kinds of statements are always reversible?

a Definitions **b** Theorems **c** Postulates

3 Answer each question Yes or No.

a Do we prove theorems? **b** Do we prove definitions?

4 Tell whether each of the following statements is a theorem or a definition.

a If two angles are right angles, then they are congruent.

b If a ray bisects an angle, it divides the angle into two congruent angles.

5 **a** Write the converse of each of the following statements.

- **i** If A, then B.
- **ii** Rain $\Rightarrow$ wet
- **iii** If an angle is a 45° angle, then it is acute.
- **iv** If a point is the midpoint of a segment, it divides the segment into two congruent segments.

b Discuss the truth of each of the converses in part **a**.

In problems 8–12, study each of the arguments and state whether or not the conclusion is deducible. If it is not, comment on the error in the reasoning.

8 If a student at Niles High has room 303 as his or her homeroom, the student is a freshman. Joe Jacobs is a student at Niles High and has room 303 as his homeroom. Therefore, Joe Jacobs is a freshman.

9 If the three angles of a triangle are acute, then the triangle is acute. In triangle ABC, angle A and angle B are acute. Therefore, triangle ABC is acute.

10 All school buses stop at railroad crossings. A vehicle stopped at the Santa Fe railroad crossing. Therefore, that vehicle is a school bus.

11 All cloudy days are depressing. Therefore, since I was depressed on Thursday, Thursday was cloudy.

12 If two angles of a triangle are congruent, then the sides opposite them are congruent. In $\triangle ABC$, $\angle A \cong \angle B$. Therefore, in $\triangle ABC$, $\overline{BC} \cong \overline{AC}$.

14 The Bronx Zoo has a green lizard, a red crocodile, and a purple monkey. They are the only animals of their kind in existence. One violently windy Saturday, their name tags blew off, and their keeper's journal was torn to shreds. Inasmuch as they were to appear on television at 7:30 Sunday morning, the night watchman had to replace their name tags. He managed to piece together the following information from the mangled journal.

1 Wendy cannot get along with the lizard.
2 Katie playfully took a bite out of the monkey's ear one month ago.
3 Wendy never casts a red reflection in the mirror.
4 Jody has the personality of a crocodile, but she isn't one.

Match the animals with their names.

Objective

Students will be able to recognize conditional statements, the negation, converse, and inverse of a statement.

ANNOUCEMENTS:

- Written Midterm: Thursday, October 27th
- Multiple Choice Midterm: Monday, October 31st

Five athletes are returning from a cross country race. Athlete C placed 3rd and Athlete E placed 2nd. How did the rest place?

Athlete A was not last. Athlete A came in after E. Athlete D was not 1st.

1st - ~~A~~ B ~~D~~

2nd - E

3rd - C

4th - A B D

5th - ~~A~~ B D

If I love math, then I am a student in Ms. Glawe’s class.

Negation

-The negation of any statement p is the statement “not p” or ~p

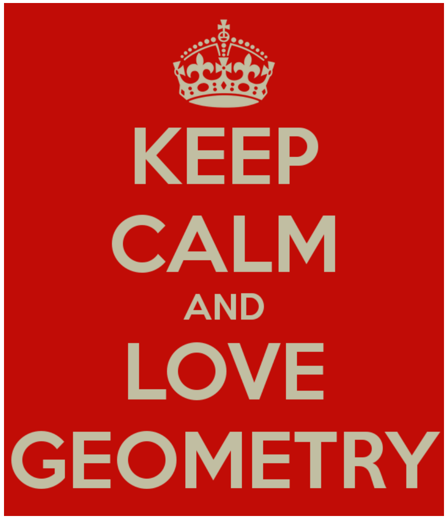
-The opposite

Ex: It is raining outside

Negation: It is not raining outside

Ex 2: I do not love geometry

Negation: I DO love geometry!



If p, then q…

- Converse
  - If q, then p

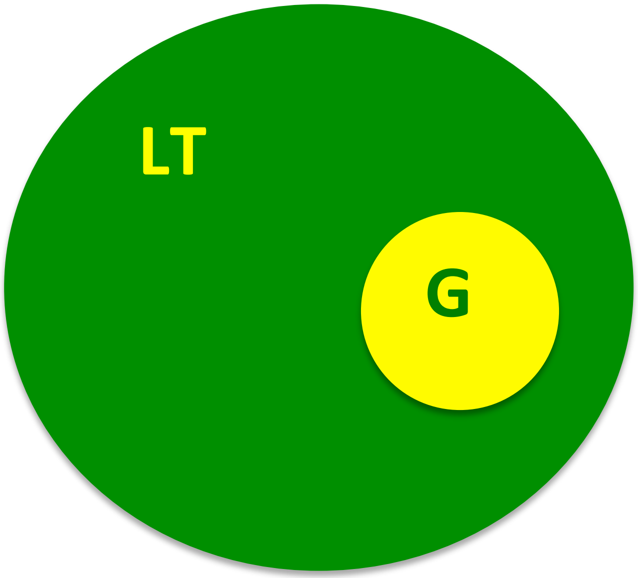
- Inverse
  - If ~p, then ~q

Example: If I have money, then I will go to the cubs game.

- Converse: If I go to the cubs game, then I have money.
- Inverse: If I do not have money, then I will not go to the cubs game.

If you are a student in Ms. Glawe's class, then you are a student at Lane Tech.

- Converse: If you are a student at Lane Tech, then you are in Ms. Glawe's class
- Inverse: If you are not in Ms. Glawe's class, then you are not a student at Lane Tech



Chain Rule: connection of conditional statements

$A \rightarrow B$
$B \rightarrow C$
$C \rightarrow D$

$A \rightarrow B \rightarrow C \rightarrow D$ then $A \rightarrow D$

Example

If a dog goes for a walk, then the dog will be tired.

If a dog takes a nap, then the dog will snore.

If a dog is tired, then the dog will take a nap.

$$\left.\begin{array}{l} W \rightarrow T \\ N \rightarrow S \\ T \rightarrow N \end{array}\right\} \quad W \rightarrow T \rightarrow N \rightarrow S \quad \text{then } W \rightarrow S$$

If a dog goes for a walk, then the dog will snore.

Homework

p. 47 – 48: 2, 4, 5a, 6, 8 (converse and inverse only)

2 Write the converse, the inverse, and the contrapositive of each statement. Determine the truth of each of the new statements.

a If each side of a triangle has a length of 10, then the triangle's perimeter is 30.

b If an angle is acute, then it has a measure greater than 0 and less than 90.

4 Draw a Venn diagram for the true conditional statement "If a person lives in Chicago, then the person lives in Illinois." Assuming that each of the following "Given . . ." statements is true, determine the truth of the conclusion.

a Given: Penny lives in Chicago.
Conclusion: Penny lives in Illinois.

b Given: Benny lives in Illinois.
Conclusion: Benny lives in Chicago.

c Given: Kenny does not live in Chicago.
Conclusion: Kenny must live in Illinois.

d Given: Denny does not live in Illinois.
Conclusion: Denny lives in Chicago.

5 Write a concluding statement for each of the following chains of reasoning.

a $a \Rightarrow b$
$d \Rightarrow \sim c$
$\sim c \Rightarrow a$
$b \Rightarrow f$

6 Write the converse, the inverse, and the contrapositive of "If M is the midpoint of $\overline{AB}$, then M, A, and B are collinear." Are these statements true or false?

8 Write the converse, the inverse, and the contrapositive of each statement.

a If a ray bisects an angle, it divides the angle into two congruent angles.

b If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Objective

Students will be able to write simple two-column proofs.

ANNOUCEMENTS:

Written Midterm: Thursday, October 27th

Multiple Choice Midterm: Monday, October 31st

Two-Column Proofs

-Theorem: A mathematical statement that can be proved

-Think of a proof as a puzzle: In order to complete the puzzle, you must have all of the pieces put together in order

- Needs to be written in a logical order
  - Think of pb&j steps

- The two-column proof helps us do that

Two-Column Proofs

Numbered statements and reasons that show the logical order of an argument

Given: We are always given some info

Conclusion/Prove: What we are trying to prove

Statements	Reasons
1.	1.
2.	2.
3.	3.

Statements Column: Holds statements that we need to prove true

Reasons Column: Holds reasons on why those mathematical statements are true

****Think of writing a paper: introduction (given), middle, and conclusion (prove)**

Algebraic Proofs

Example 1: Write a two-column proof.

Given: $3\left(x-\frac{5}{3}\right)=1$ Prove: $x=2$

Statements	Reasons
1. $3\left(x-\frac{5}{3}\right)=1$	1. Given
2. $3x-3\left(\frac{5}{3}\right)=1$	2. Distributive Property
3. $3x-5=1$	3. Simplify
4. $3x-5+5=1+5$	4. Addition Property
5. $3x=6$	5. Simplify
6. $\frac{3x}{3}=\frac{6}{3}$	6. Division Property
7. $x=2$	7. Simplify

Example 2: Write a two-column proof.

Given: $\frac{7}{2}-n=4-\frac{1}{2}n$ Prove: $n=-1$

Statements	Reasons
1. $\frac{7}{2}-n=4-\frac{1}{2}n$	1. Given
2. $\frac{7}{2}-n+\frac{1}{2}n=4-\frac{1}{2}n+\frac{1}{2}n$	2. Addition Property
3. $\frac{7}{2}-\frac{1}{2}n=4$	3. Simplify
4. $\frac{7}{2}-\frac{1}{2}n-\frac{7}{2}=4-\frac{7}{2}$	4. Subtraction Property
5. $-\frac{1}{2}n=\frac{1}{2}$	5. Simplify
6. $-2\left(-\frac{1}{2}n\right)=-2\left(\frac{1}{2}\right)$	6. Multiplication Property
7. $n=-1$	7. Simplify

Homework

PROOF **Write a two-column proof.**

24. Given: $\frac{3x+5}{2} = 7$

Prove: $x = 3$

25. Given: $2x - 7 = \frac{1}{3}x - 2$

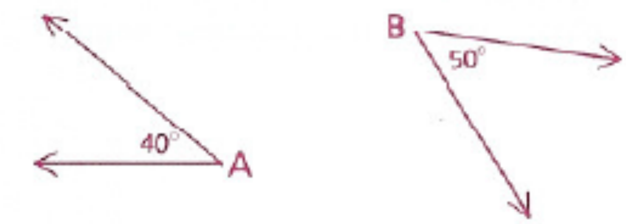
Prove: $x = 3$

26. If $4 - \frac{1}{2}a = \frac{7}{2} - a$, then $a = -1$.

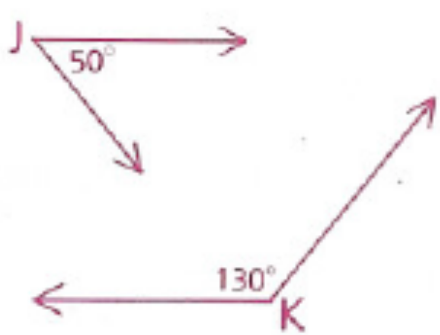
27. If $-2y + \frac{3}{2} = 8$, then $y = -\frac{13}{4}$.

Complementary vs. Supplementary Angles

Complementary angles are two angles whose sum is 90° (a right angle). Each of the two angles is called the complement of the other.



∠A and ∠B are complementary.



∠J and ∠K are supplementary.

Supplementary angles are two angles whose sum is 180° (a straight angle). Each of the two angles is called the supplement of the other.

Objective

Students will be able to write simple two-column proofs.

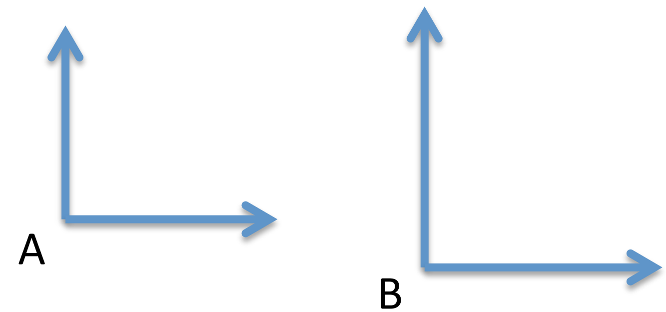
Geometric Proofs

We can use geometric proofs to prove different theorems.

When writing geometric proofs, you want to use if-then statements to write your reasons.

Theorem: If two angles are right angles, then they are congruent

Given: $\angle A$ is a right angle
$\angle B$ is a right angle
Prove: $\angle A \cong \angle B$

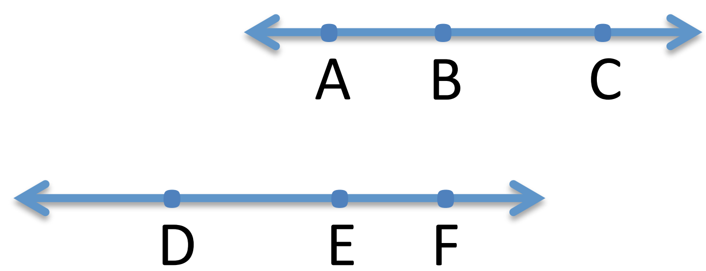


Statements	Reasons
1. $\angle A$ is a right angle	1. Given
2. $m\angle A = 90$	2. If an angle is a right angle, then its measure is 90 degrees.
3. $\angle B$ is a right angle	3. Given
4. $m\angle B = 90$	4. If an angle is a right angle, then its measure is 90 degrees.
5. $\angle A \cong \angle B$	5. If two angles have the same measure, then they are congruent (steps 2 and 4)

Theorem: If two angles are straight angles, then they are congruent

Given: $\angle ABC$ is a straight angle
$\angle DEF$ is a straight angle

Prove: $\angle ABC \cong \angle DEF$



Statements	Reasons
1. $\angle ABC$ is a straight angle	1. Given
2. $m\angle ABC = 180$	2. If an angle is a straight angle, then its measure is 180.
3. $\angle DEF$ is a straight angle	3. Given
4. $m\angle DEF = 180$	4. If an angle is a straight angle, then its measure is 180.
5. $\angle ABC \cong \angle DEF$	5. If two angles have the same measure, then they are congruent (steps 2 and 4)

I Love Theorems

-Use them as reasons to prove things (deductive structure)

-Use them in homework

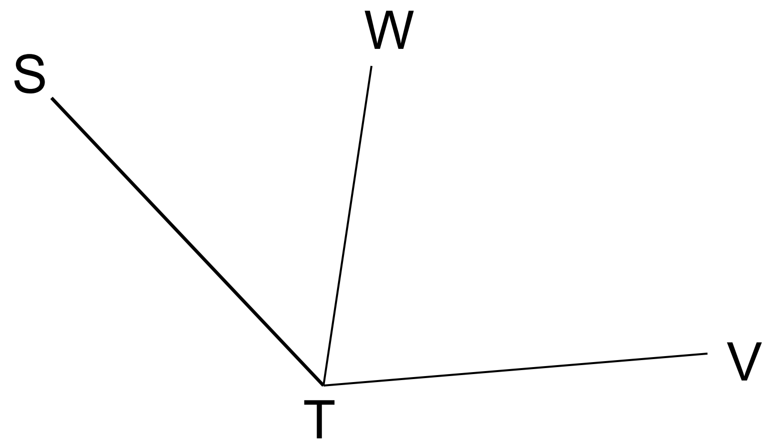
-They shorten your work

-Don’t fight the theorems, love them

-Read the three sample proofs on page 25

Example

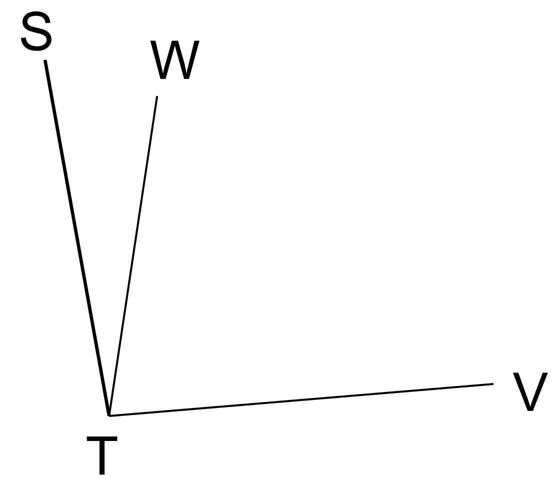
Given: ∠WTV = 80°
∠STW = 40°
Prove: ∠STV is obtuse



Statements	Reasons
1. ∠WTV = 80°	1. Given
2. ∠STW = 40°	2. Given
3. ∠STV = 120°	3. Addition (80° + 40° = 120°)
4. ∠STV is obtuse	4. If an ∠ measures more than 90° and less than 180°, then it's obtuse.

Example

Given: ∠STV is a right ∠
∠STW = 20°
Prove: ∠WTV is acute



Statements	Reasons
1. ∠STV is a right ∠	1. Given
2. ∠STV = 90°	2. If an ∠ is a right ∠, then its measure is 90°.
3. ∠STW = 20°	3. Given
4. ∠WTV = 70°	4. Subtraction (90° - 20° = 70°)
5. ∠WTV is acute	5. If an ∠ measures more than 0° and less than 90°, then it's acute.

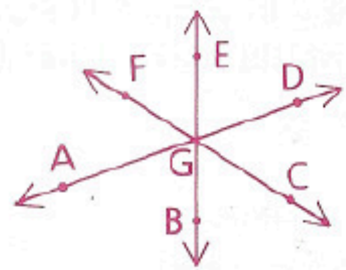
Homework

p. 26 – 27: 2, 3, 5, 6, 7, 15

In problems 1 and 2, copy the figure and the incomplete proof. Then complete the proof by filling in the missing reasons.

2 Given: Diagram as shown

Prove: $\angle AGD \cong \angle EGB$



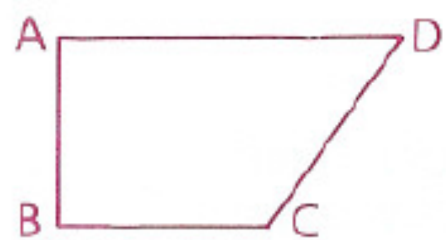
Statements	Reasons
1 Diagram as shown	1 ____________
2 $\angle AGD$ is a straight angle.	2 ____________
3 $\angle EGB$ is a straight angle.	3 ____________
4 $\angle AGD \cong \angle EGB$	4 ____________

In problems 3–7, use the two-column form of proof.

3 Given: $\angle A$ is a right angle.

$\angle B$ is a right angle.

Prove: $\angle A \cong \angle B$

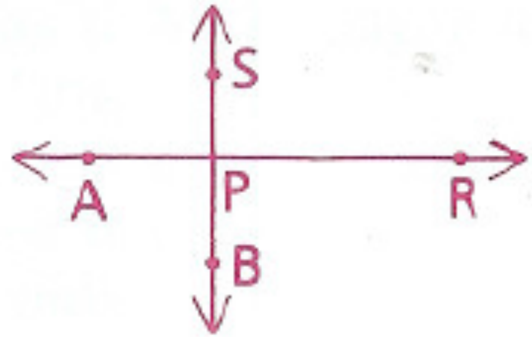


5 Given: JK = 2.5 cm, NO = 2.5 cm

Conclusion: $\overline{JK} \cong \overline{NO}$

6 Given: Diagram as shown

Prove: $\angle APR \cong \angle SPB$

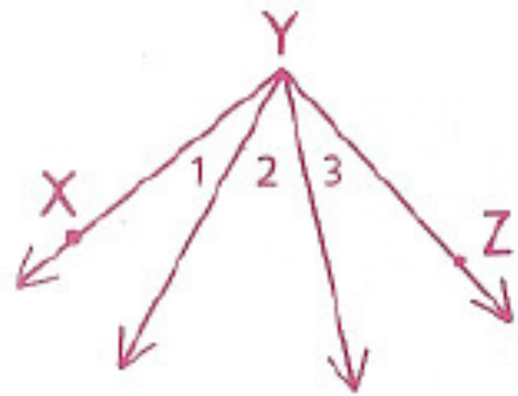


7 Given: $\angle 1 = 20^\circ$,

$\angle 2 = 40^\circ$,

$\angle 3 = 30^\circ$

Prove: $\angle XYZ$ is a right angle.



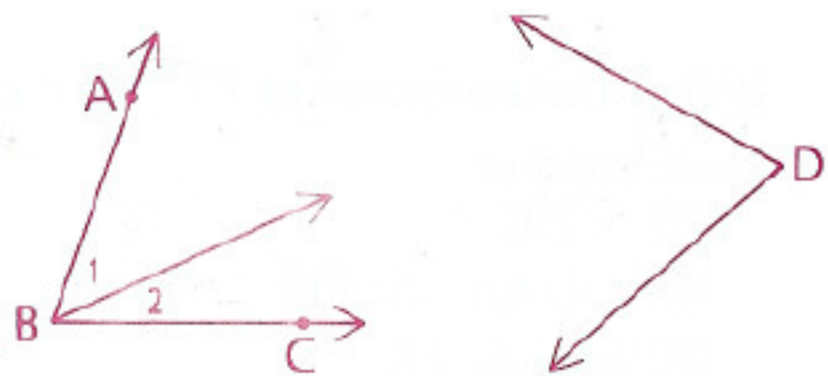
15 Given: $\angle 1 = (x + 7)^\circ$,

$\angle 2 = (2x - 3)^\circ$,

$\angle ABC = (x^2)^\circ$,

$\angle D = (5x - 4)^\circ$

Show that $\angle ABC \cong \angle D$.

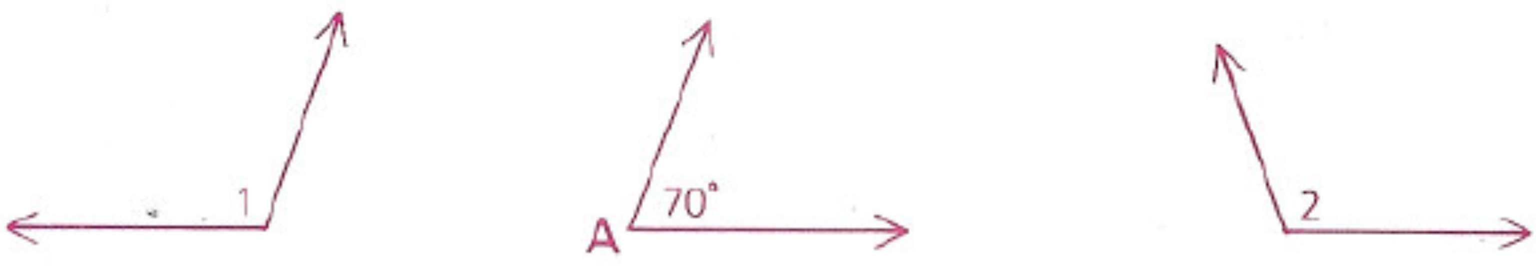


**Not a proof

Objective

Students will be able to understand that complements and supplements of congruent (or the same angle) are congruent and will be able to use theorems to justify it through two-column proofs.

In the diagram, $\angle 1$ is supplementary to $\angle A$, and $\angle 2$ is also supplementary to $\angle A$.



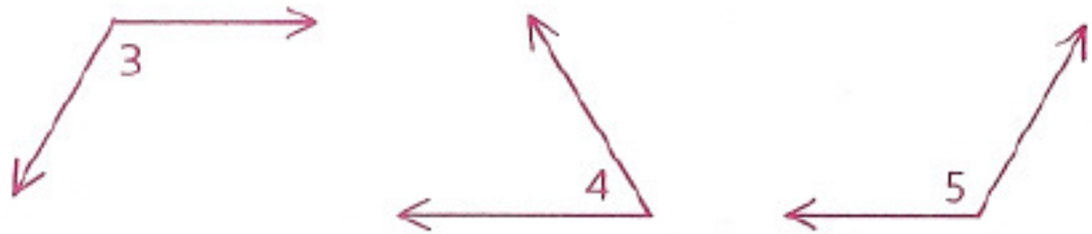
What is the $m\angle 1$? What is the $m\angle 2$? How does $\angle 1$ compare with $\angle 2$?

$\angle 1 \cong \angle 2$

Theorem 4: If angles are supplementary to the same angle, then they are congruent

Given: $\angle 3$ is supp. to $\angle 4$.
$\angle 5$ is supp. to $\angle 4$.

Prove: $\angle 3 \cong \angle 5$

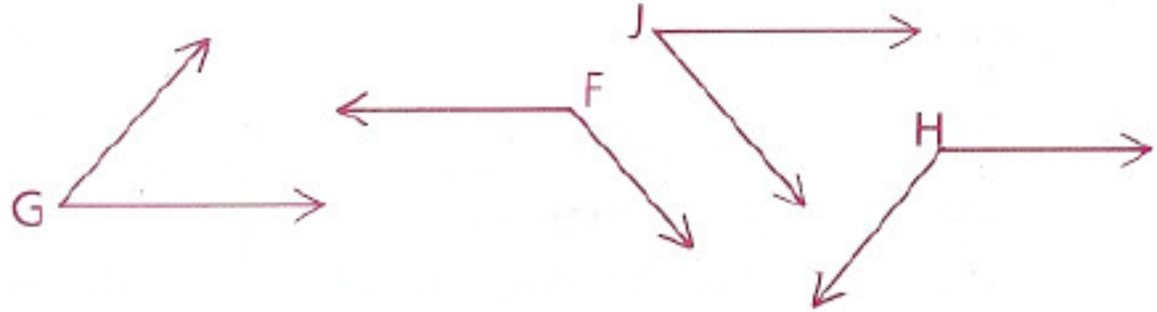


Proof: $\angle 3$ is supp. to $\angle 4$, so $m\angle 3 + m\angle 4 = 180$.
Therefore, $m\angle 3 = 180 - m\angle 4$.
$\angle 5$ is supp. to $\angle 4$, so $m\angle 5 + m\angle 4 = 180$.
Therefore, $m\angle 5 = 180 - m\angle 4$.
Since $\angle 3$ and $\angle 5$ have the same measure, $\angle 3 \cong \angle 5$.

Theorem 5: If angles are supplementary to congruent angles, then they are congruent

Given: $\angle F$ is supp. to $\angle G$.
$\angle H$ is supp. to $\angle J$.
$\angle G \cong \angle J$

Conclusion: $\angle F \cong \angle H$



The proof is very similar to the proof for Theorem 4.

Two similar theorems apply to complementary angles.

Theorem 6: If angles are complementary to the same angle, then they are congruent

Theorem 7: If angles are complementary to congruent angles, then they are congruent

Problem 1 *Given:* $\angle 1$ is supp. to $\angle 2$.
$\angle 3$ is supp. to $\angle 4$.
$\angle 1 \cong \angle 4$

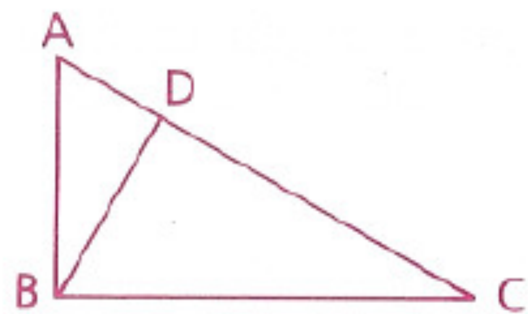
Conclusion: $\angle 2 \cong \angle 3$

Proof

Statements	Reasons
1 $\angle 1$ is supp. to $\angle 2$.	1 Given
2 $\angle 3$ is supp. to $\angle 4$.	2 Given
3 $\angle 1 \cong \angle 4$	3 Given
4 $\angle 2 \cong \angle 3$	4 If angles are supplementary to $\cong$ angles, they are $\cong$. (Short form: Supplements of $\cong$ $\angle$s are $\cong$.)

Problem 2 *Given:* $\angle A$ is comp. to $\angle C$.
$\angle DBC$ is comp. to $\angle C$.

Conclusion: ___?___



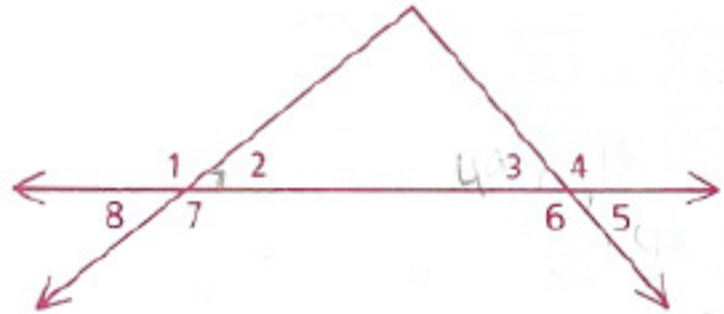
Proof

Statements	Reasons
1 $\angle A$ is comp. to $\angle C$.	1 Given
2 $\angle DBC$ is comp. to $\angle C$.	2 Given
3 $\angle A \cong \angle DBC$	3 If angles are complementary to the same angle, they are $\cong$. (Short form: Complements of the same $\angle$ are $\cong$.)

Homework

HW: p. 79-80: 1, 2, 4, 5, 12, 19

1 Given: $\angle 2$ is comp. to $\angle 3$.
$\angle 4 = 131°$



Find the measure of each of the following angles.

a $\angle 3$

b $\angle 6$

c $\angle 5$

d $\angle 2$

e $\angle 1$

f $\angle 8$

g $\angle 7$

2 Given: $\angle 1$ is supp. to $\angle 3$.
$\angle 2$ is supp. to $\angle 3$.

Prove: $\angle 1 \cong \angle 2$

4 One of two supplementary angles is four times the other. Find the larger angle.

5 One of two complementary angles is 20° larger than the other. Find the measure of each.

12 The measure of the supp. of an $\angle$ exceeds 3 times the measure of the comp. of the $\angle$ by 10. Find the measure of the comp.

19 Given: $\angle PQR$ supp. $\angle QRS$, $\angle QRS$ supp. $\angle TWX$,
$\angle PQR = (5x - 48)^\circ$, $\angle TWX = (2x + 30)^\circ$

Find: $m\angle QRS$

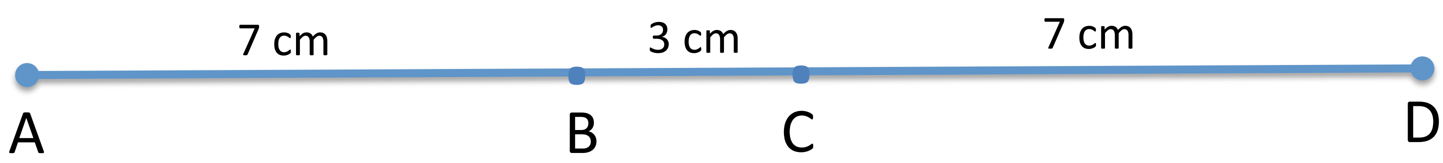
Objective

Students will be able to understand and apply addition and subtraction properties of segments and angles.

Data for Exercise Lab is due on Monday, 21st November

Addition Properties

In the diagram below, AB = CD. Do you think that AC = BD? If BC is 3 cm, would AC = CD? If AB = CD, does the length of BC have any effect on whether AC = BD?



AC = BD in every case and the length of BC does not effect that equality.

Addition Property of Equality (AB + BC = CD + BC)

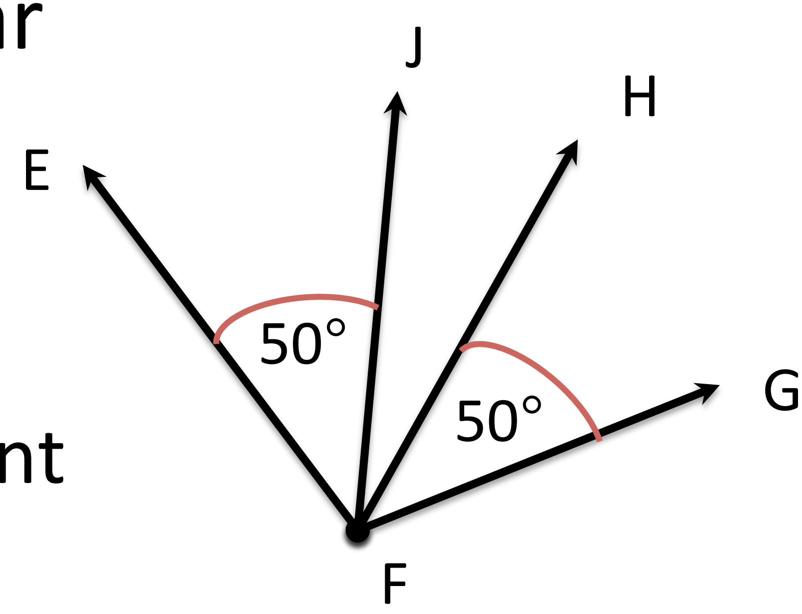
Theorem 8: If a segment is added to two congruent segments, then the sums are congruent. (Addition Property)

Given: $\overline{PQ} \cong \overline{RS}$

Conclusion: $\overline{PR} \cong \overline{QS}$

Proof: $\overline{PQ} \cong \overline{RS}$, so by definition of congruent segments, $PQ = RS$. Now, the Addition Property of Equality says that we may add QR to both sides, so $PQ + QR = RS + QR$. Substituting, we get $PR = QS$. Therefore, $\overline{PR} \cong \overline{QS}$ by the definition of *congruent segments* (reversed).

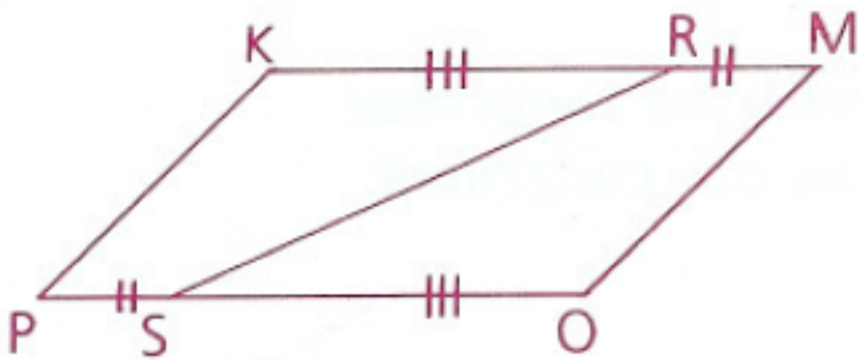
Do you think that a similar relationship holds for angles?

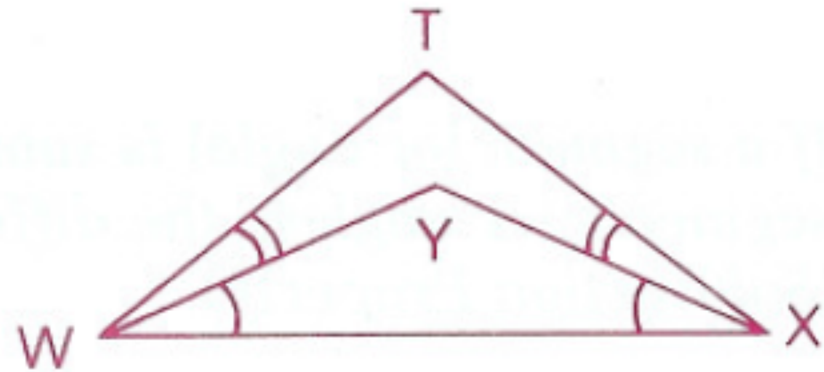


Is ∠EFH necessarily congruent to ∠JFG?

Theorem 9: If an angle is added to two congruent angles, then the sums are congruent. (Addition Property)

In the figures below, identical tick marks indicate congruent parts.





Do you think $\overline{KM}$ is necessarily congruent to $\overline{PO}$?

Do you think $\angle TWX$ is necessarily congruent to $\angle TXW$?

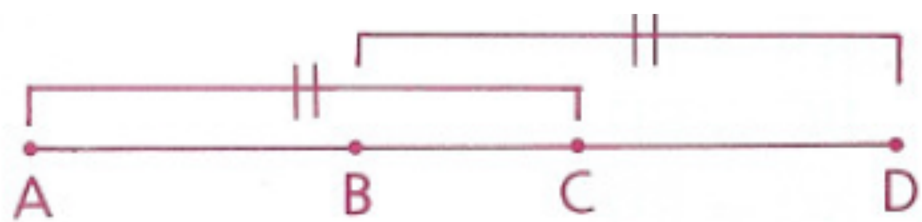
Theorem 10: If congruent segments are added to congruent segments, then the sums are congruent. (Addition Property)

Theorem 11: If congruent angles are added to congruent angles, then the sums are congruent. (Addition Property)

Subtraction Properties

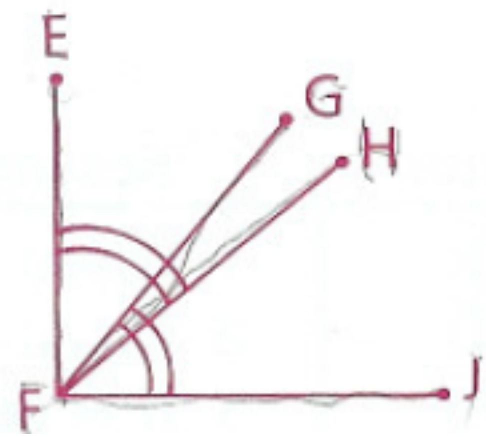
Since subtraction is equivalent to addition of an opposite, we can expect four corresponding subtraction properties.

If AC = BD, is AB = CD?



Let AC = 12 and BC = 3.
How long is $\overline{BD}$?
Is AB = CD?

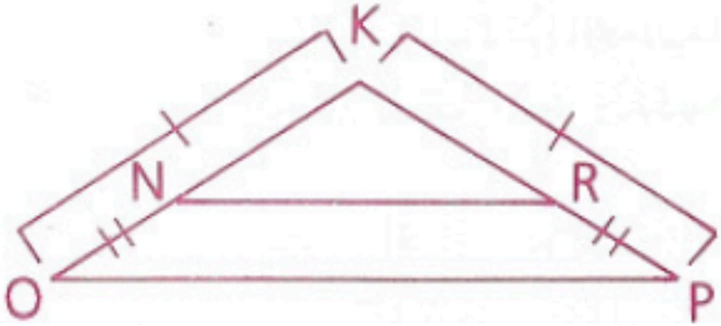
If $\angle EFH \cong \angle GFJ$, is $\angle EFG \cong \angle HFJ$?



Let $m\angle EFH = 50$ and $m\angle GFH = 10$.
How large is $\angle GFJ$?
Is $\angle EFG \cong \angle HFJ$?

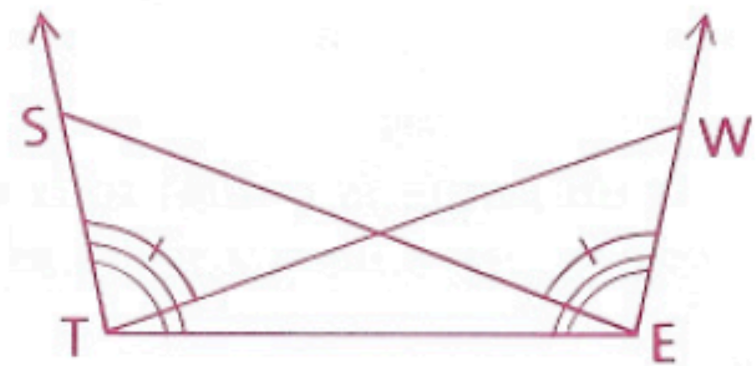
If KO = KP and NO = RP,
is KN = KR?

Try this on your own and see what you think.



If $\angle STE \cong \angle WET$ and $\angle STW \cong \angle WES$,
is $\angle WTE \cong \angle SET$?

Try this on your own.



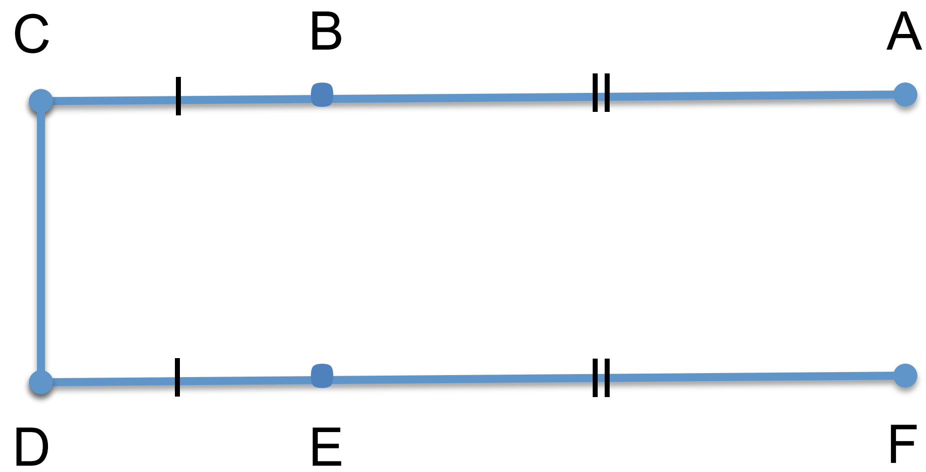
Theorem 12: If a segment (or angle) is subtracted from two congruent segments (or angles), then the differences are congruent. (Subtraction Property)

Theorem 13: If congruent segments (or angles) are subtracted from congruent segments (or angles), then the differences are congruent. (Subtraction Property)

Using the Addition and Subtraction Properties in Proofs

1) An addition property is used when the segments or angles in the conclusion are *greater than* those in the given information.
2) A subtraction property is used when the segments or angles in the conclusion are *smaller than* those in the given information.

Example



Given: $\overline{AB} \cong \overline{FE}$

$\overline{BC} \cong \overline{ED}$

Prove: $\overline{AC} \cong \overline{FD}$

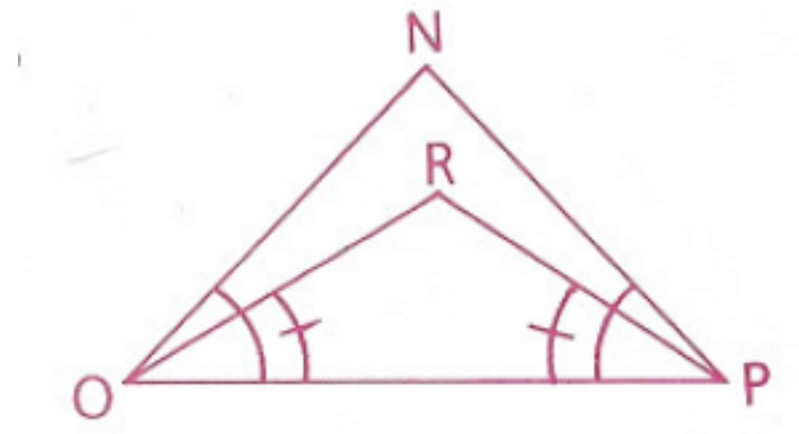
Statements	Reasons
1. $\overline{AB} \cong \overline{FE}$	1. Given
2. $\overline{BC} \cong \overline{ED}$	2. Given
3. $\overline{AC} \cong \overline{FD}$	3. If congruent segments are added to congruent segments, then the sums are congruent (Addition Property)

Example 2

Given: $\angle NOP \cong \angle NPO$

$\angle ROP \cong \angle RPO$

Prove: $\angle NOR \cong \angle NPR$



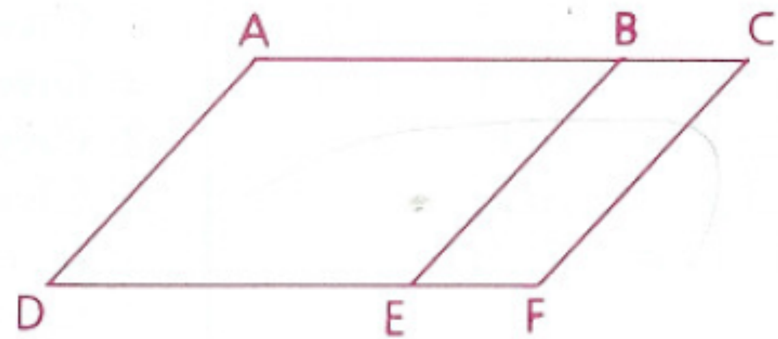
Statements	Reasons
1. $\angle NOP \cong \angle NPO$	1. Given
2. $\angle ROP \cong \angle RPO$	2. Given
3. $\angle NOR \cong \angle NPR$	3. If congruent angles are subtracted from congruent angles, then the differences are congruent (Subtraction Property)

Homework

p. 86-88: 5, 6, 9, 11, 12, 14, 17

5 Given: $\overline{AC} \cong \overline{DF}$,
$\overline{BC} \cong \overline{EF}$

Prove: $\overline{AB} \cong \overline{DE}$

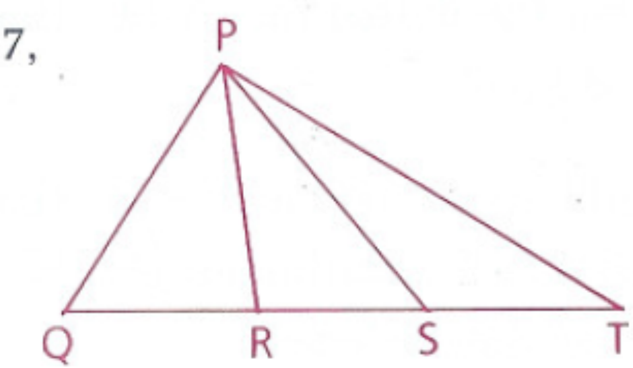


6 Given: $\overline{GH} \cong \overline{JK}$, $GH = x + 10$,
$HJ = 8$, $JK = 2x - 4$

Find: GJ

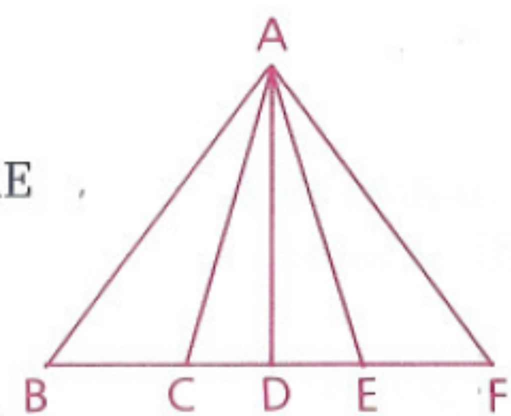
9 Given: $\overline{QR} \cong \overline{ST}$, $QS = 5x + 17$,
$RT = 10 - 2x$, $RS = 3$

Find: QS and QT



11 Given: $\angle BAD \cong \angle FAD$;
$\overrightarrow{AD}$ bisects $\angle CAE$.

Conclusion: $\angle BAC \cong \angle FAE$



12 Given: J and K are trisection points of $\overline{HM}$.
$\overline{GH} \cong \overline{MO}$

Conclusion: $\overline{GJ} \cong \overline{KO}$

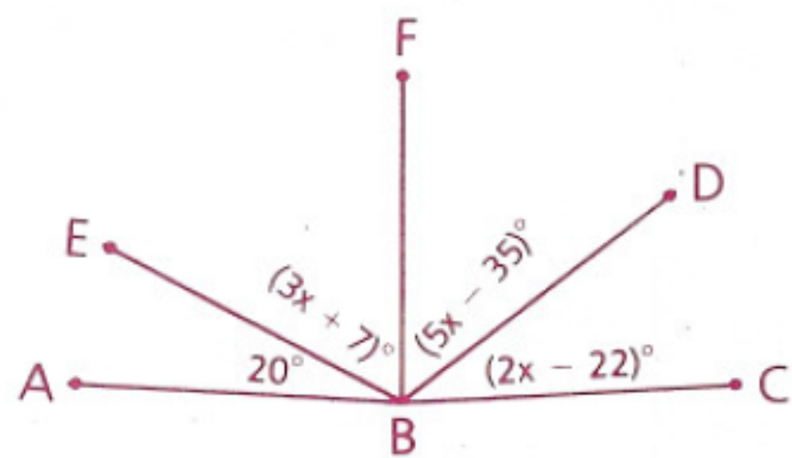
14 Given: $\angle A$ is comp. to $\angle B$.
$\angle C$ is comp. to $\angle B$.
$\angle A = (3x + y)°$,
$\angle B = (x + 4y + 2)°$,
$\angle C = (3y - 3)°$

Find: $m\angle B$

17 $\overrightarrow{BF}$ bisects $\angle DBE$.

a Does $\overrightarrow{BF}$ bisect $\angle CBA$?

b What did you discover about $\angle ABC$ and $\overrightarrow{BF}$?



Objective

Students will be able to understand and apply multiplication and division properties of segments and angles.

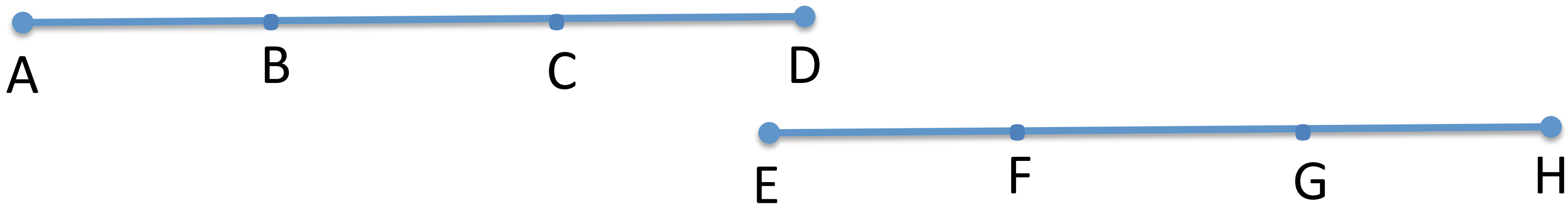
Data for Exercise Lab is due on Monday, 21st November

Science Fair

Remember for your science fair, you need at least 3-4 experimental groups, at least 5 trials for each group, and at least 30 subjects in each group for behavioral science

Create everything in Word and save as PDF

In figure below, B, C, F, and G are trisection points.



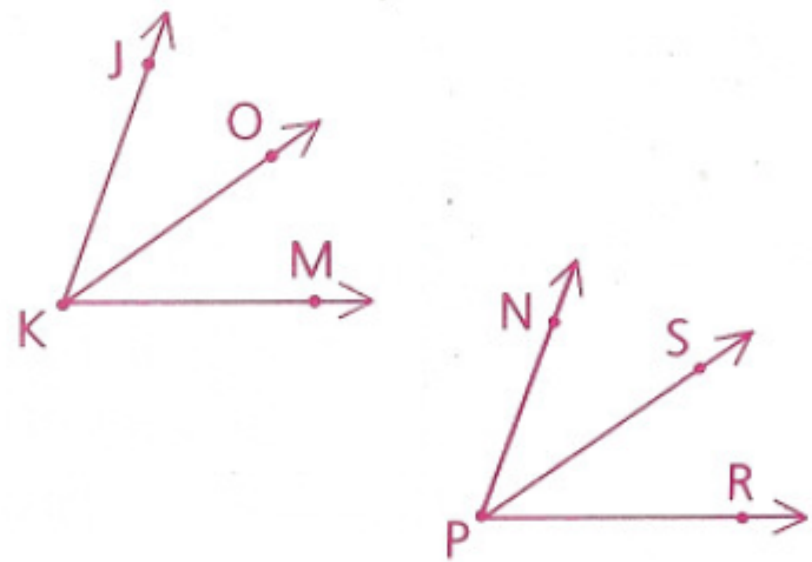
If $AB = EF = 3$, what can we say about $\overline{AD}$ and $\overline{EH}$?

If $\overline{AB} \cong \overline{EF}$, is $\overline{AD}$ congruent to $\overline{EH}$?

In the figure to the right, $\overrightarrow{KO}$ and $\overrightarrow{PS}$ are angle bisectors.

If $m\angle JKO = m\angle NPS = 25$, what can we say about $\angle JKM$ and $\angle NPR$?

If $\angle JKO \cong \angle NPS$, is $\angle JKM$ congruent to $\angle NPR$?



Theorem 14: If segments (or angles) are congruent, then their like multiples are congruent. (Multiplication Property)

Similarly,

Theorem 15: If segments (or angles) are congruent, then their like divisions are congruent. (Division Property)

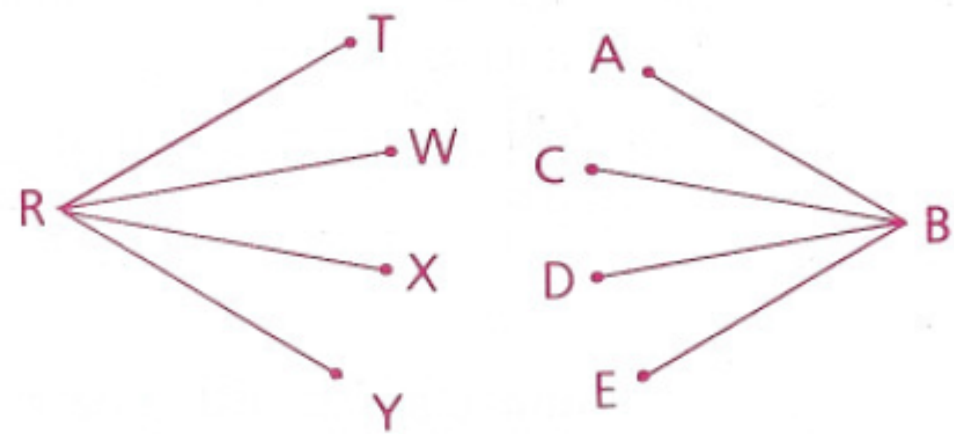
Using the Multiplication and Division Properties in Proofs

1) Look for a double use of the word *midpoint* or *trisect* or *bisects* in the given information.

2) The Multiplication Property is used when the segments or angles in the conclusion are <u>*greater than*</u> those in the given information.

3) The Division Property is used when the segments or angles in the conclusion are <u>*smaller than*</u> those in the given information.

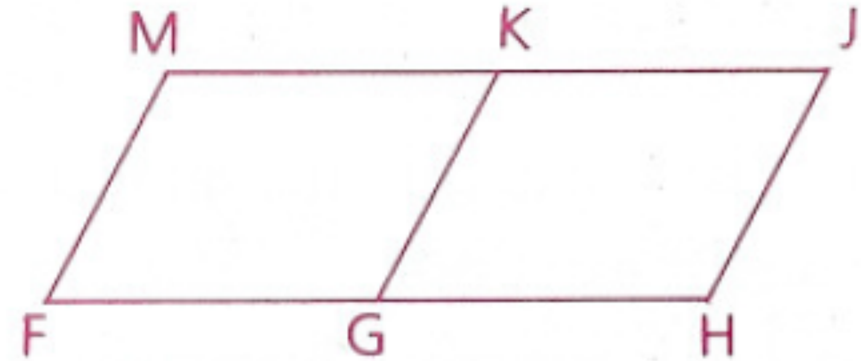
Given: $\angle TRY \cong \angle ABE$;
$\overrightarrow{RW}$ *and* $\overrightarrow{RX}$ *trisect* $\angle TRY$.
$\overrightarrow{BC}$ *and* $\overrightarrow{BD}$ *trisect* $\angle ABE$.
Conclusion: $\angle TRW \cong \angle CBD$



Statements	Reasons
1 $\angle TRY \cong \angle ABE$	1 Given
2 $\overrightarrow{RW}$ and $\overrightarrow{RX}$ trisect $\angle TRY$.	2 Given
3 $\overrightarrow{BC}$ and $\overrightarrow{BD}$ trisect $\angle ABE$.	3 Given
4 $\angle TRW \cong \angle CBD$	4 If angles are $\cong$, their like divisions (thirds) are $\cong$. (Division Property)

Given: $\overline{MK} \cong \overline{FG}$;
$\overline{KG}$ *bisects* $\overline{MJ}$ *and* $\overline{FH}$.

Prove: $\overline{MJ} \cong \overline{FH}$



Statements	Reasons
1 $\overline{MK} \cong \overline{FG}$	1 Given
2 $\overline{KG}$ bisects $\overline{MJ}$ and $\overline{FH}$.	2 Given
3 $\overline{MJ} \cong \overline{FH}$	3 If segments are $\cong$, their like multiples (doubles) are $\cong$. (Multiplication Property)

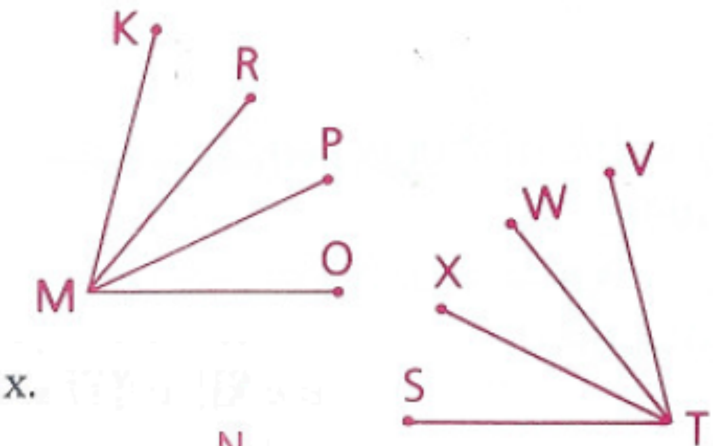
Homework

p. 91-93: 1, 2, 3, 12, 14

Review how to use the multiplication and division properties in proofs as you do the homework

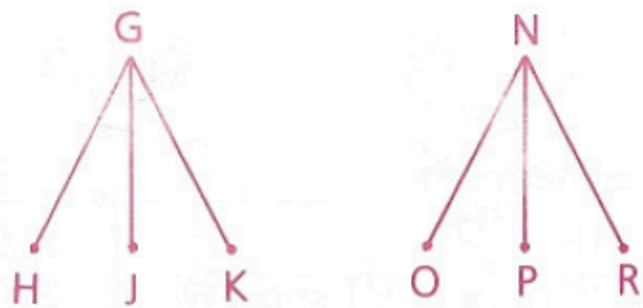
1 Given: $\angle KMR \cong \angle VTW$;
$\overrightarrow{MR}$ and $\overrightarrow{MP}$ trisect $\angle KMO$.
$\overrightarrow{TX}$ and $\overrightarrow{TW}$ trisect $\angle STV$.

Prove: $\angle KMO \cong \angle STV$

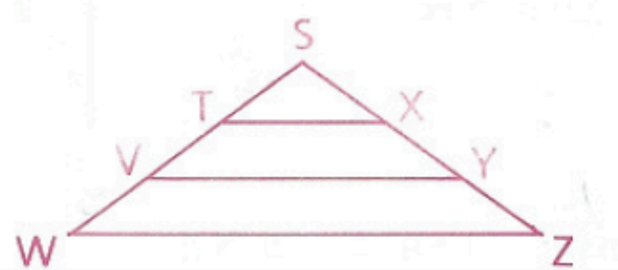


2 Use the given information to find the value of x.

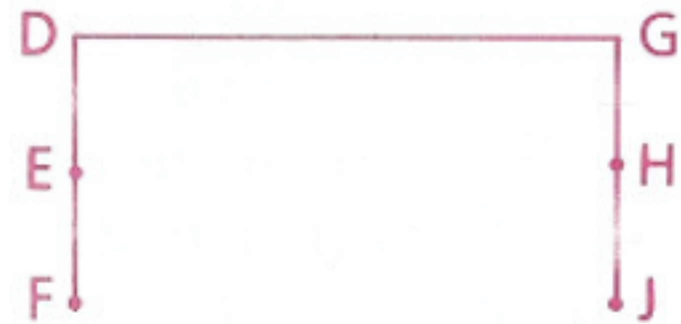
a $\angle HGJ \cong \angle ONP$;
$\overrightarrow{GJ}$ and $\overrightarrow{NP}$ are $\angle$ bisectors.
$\angle HGK = 50°$,
$\angle ONR = (2x + 10)°$



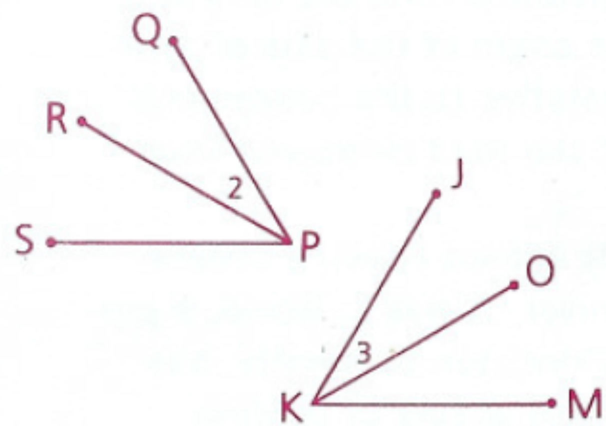
b $\overline{SW} \cong \overline{SZ}$;
$\overleftrightarrow{TX}$ and $\overleftrightarrow{VY}$ trisect $\overline{SW}$ and $\overline{SZ}$.
$ST = 12$,
$YZ = x - 4$



3 Given: $\overline{DF} \cong \overline{GJ}$;
E is the midpoint of $\overline{DF}$.
H is the midpoint of $\overline{GJ}$.
Prove: $\overline{DE} \cong \overline{GH}$



12 Given: $\overrightarrow{PR}$ bisects $\angle QPS$.
$\overrightarrow{KO}$ bisects $\angle JKM$.
$\angle 1$ is supp. to $\angle JKM$.
$\angle 1$ is supp. to $\angle QPS$.
Conclusion: $\angle 2 \cong \angle 3$



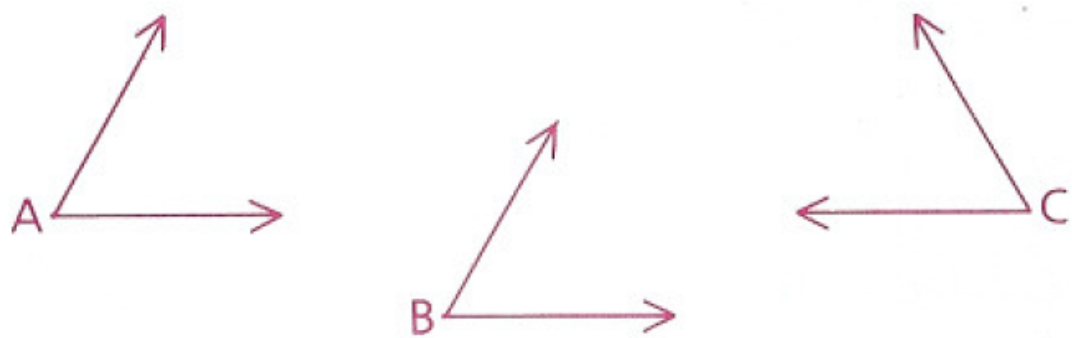
14 If four times the supplement of an angle is added to eight times the angle's complement, the sum is equivalent to three straight angles. Find the measure of the angle that is supplementary to the complement.

Objective

Students will be able to apply the transitive property of angles and segments and apply the substitution property.

Transitive Properties

Suppose that $\angle A \cong \angle B$ and $\angle A \cong \angle C$. Is $\angle B \cong \angle C$?



Theorem 16: If angles (or segments) are congruent to the same angle (or segment), then they are congruent to each other. (Transitive Property)

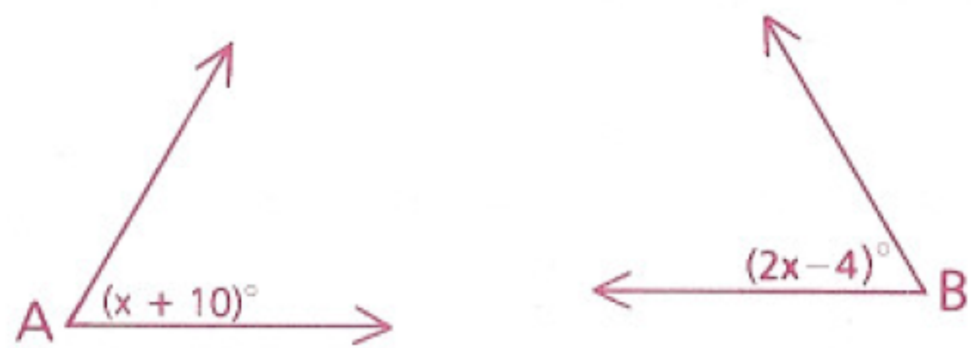
Theorem 16 can be used twice to prove the next theorem.

Theorem 17: If angles (or segments) are congruent to congruent angles (or segments), then they are congruent to each other. (Transitive Property)

Substitution Property

When have you used substitution before in math?

If $\angle A \cong \angle B$, find $m\angle A$.

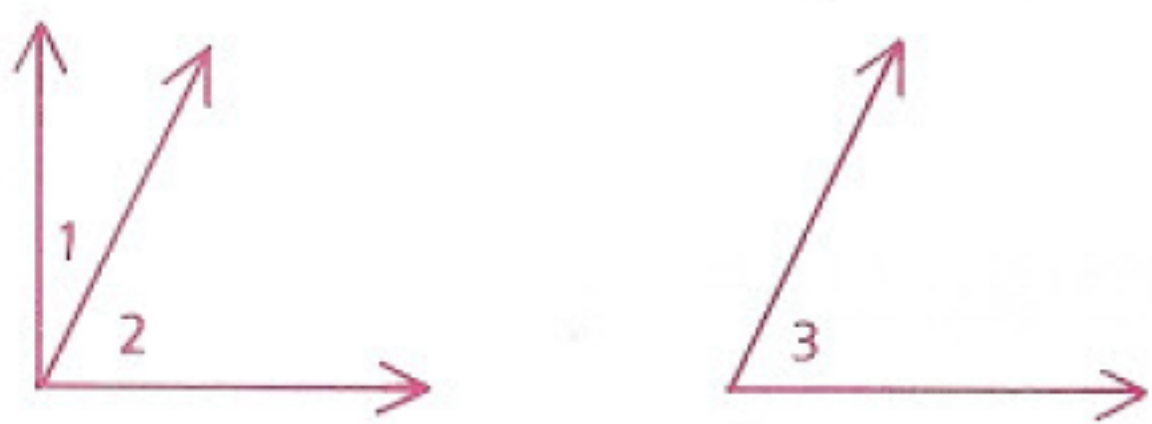


$x + 10 = 2x - 4$

$x = 14$

We substitute 14 for x in $m\angle A = x + 10$ to find that $m\angle A = 14 + 10 = 24$

The Substitution Property can also be applied when no variables are involved.



If $\angle 1$ is comp. to $\angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1$ is comp. to $\angle 3$ by ***substitution.***

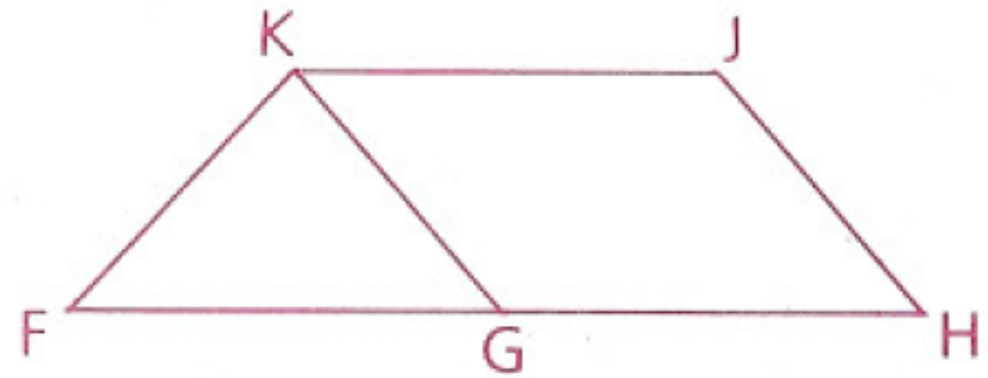
We can use the Transitive and Substitution Property in proofs.

Example

Given: $\overline{FG} \cong \overline{KJ}$

$\overline{GH} \cong \overline{KJ}$

Prove: $\overleftrightarrow{KG}$ bisects $\overline{FH}$



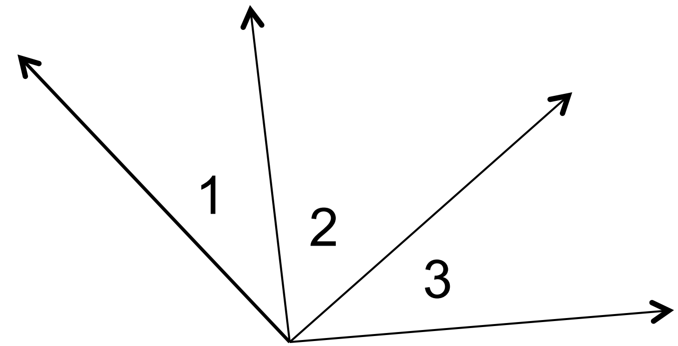
Statements	Reasons
1. $\overline{FG} \cong \overline{KJ}$	1. Given
2. $\overline{GH} \cong \overline{KJ}$	2. Given
3. $\overline{FG} \cong \overline{GH}$	3. If segments are congruent to the same segment, then they are congruent (Transitive Property)
4. $\overleftrightarrow{KG}$ bisects $\overline{FH}$	4. If a line divides a segment into two congruent segments, then it bisects the segment

Example 2

Given: $\angle 1 + \angle 2 = 90°$

$\angle 1 \cong \angle 3$

Prove: $\angle 3 + \angle 2 = 90°$



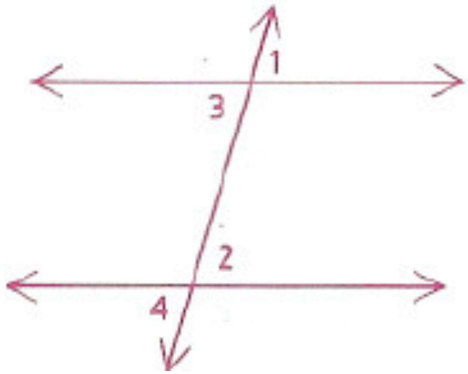
Statements	Reasons
1. $\angle 1 + \angle 2 = 90°$	1. Given
2. $\angle 1 \cong \angle 3$	2. Given
3. $\angle 3 + \angle 2 = 90°$	3. Substitution (step 2 in step 1)

Homework

p. 97-99: 3, 7, 9, 10, 14, 15

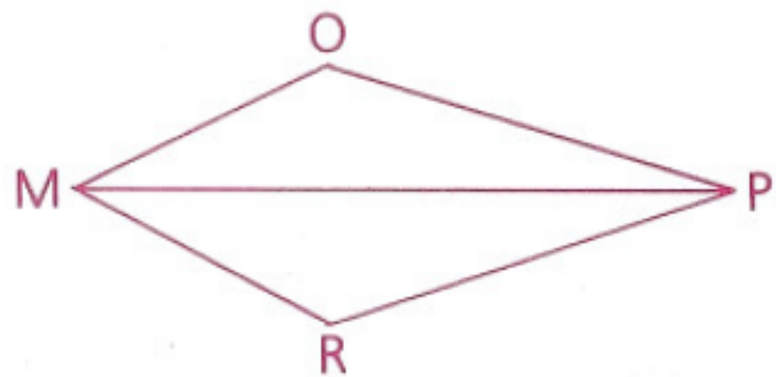
3 Given: $\angle 1 \cong \angle 3$,
$\angle 2 \cong \angle 3$,
$\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 4$



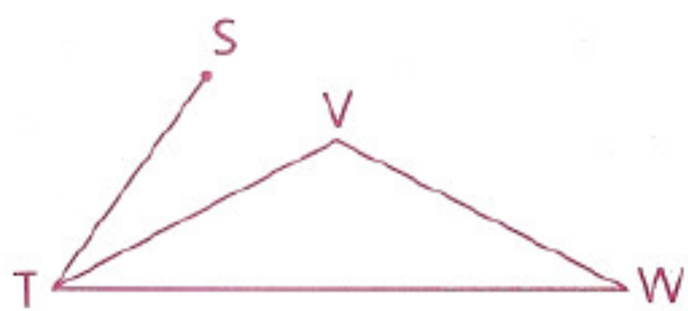
7 Given: $\angle OMP \cong \angle RPM$;
$\overrightarrow{MP}$ bisects $\angle OMR$.
$\overrightarrow{PM}$ bisects $\angle OPR$.

Prove: $\angle OMR \cong \angle OPR$



9 $\angle W \cong \angle STV$;
$\overrightarrow{TV}$ bisects $\angle STW$.
$\angle W = (2x - 5)°$,
$\angle VTW = (x + 15)°$

Find: $m\angle STW$



10 Given: $\overline{VW} \cong \overline{RS}$,
$\overline{XY} \cong \overline{RS}$
Prove: $\overline{VX} \cong \overline{WY}$

14 When one-half the supplement of an angle is added to the complement of the angle, the sum is 120°. Find the measure of the complement.

15 Given: $\angle A$ is a right $\angle$.
$\angle B$ is a right $\angle$.
$\angle B \cong \angle D$
Prove: $\angle A \cong \angle D$