Regular Advanced Algebra w/ Trig Semester 1 Final Review
Chapters 5/6 (Polynomial/Radical Functions), Glawe

Name: Key

Period: ______

1. Simplify the algebraic expressions. (5.1)

a) $\frac{3a^4b^3c^{-2}}{6a^2b^8c^5}$

$c^{-2-5} = c^{-7} = \frac{1}{c^7}$

$\boxed{\frac{1a^2}{2b^5c^7}}$

b) $(x^2y^{-5})^3(xy^3)^4$ add

$(x^6y^{-15})(x^4y^{12}) =$

$x^{10}y^{-3} = \boxed{\frac{x^{10}}{y^3}}$

c) $\frac{(2x)^{-3}}{x^{-5}} =$

$\frac{2^{-3}x^{-3}}{x^{-5}} = \frac{x^2}{2^3} = \boxed{\frac{x^2}{8}}$

$x^{-3-(-5)} = x^{-3+5} = x^2$

d) $\left(\frac{(3y)^{-2}}{xy}\right)^3 = \left(\frac{3^{-2}y^{-2}}{xy}\right)^3 =$

$\left(\frac{1}{3^2xy^3}\right)^3 = \left(\frac{1}{9xy^3}\right)^3 = \frac{1^3}{(9xy^3)^3} = \frac{1}{9^3x^3y^9} = \boxed{\frac{1}{729x^3y^9}}$

$y^{-2-1} = y^{-3} = \frac{1}{y^3}$

2. Divide using synthetic division: (5.5)

a) $(x^3 + 3x^2 + 3x + 2) \div (x - 1)$

$x = 1$

1	1	3	3	2
	↓	1	4	7
	1	4	7	9 ← remainder

$\boxed{x^2 + 4x + 7 + \frac{9}{x-1}}$

b) $(5x^5 - 4x^4 + 2x^2 + 1) \div (x + 2)$

$x = -2$

-2	5	-4	0	2	0	1
	↓	-10	28	-56	108	-216
	5	-14	28	-54	108	-215 ← remainder

$\boxed{5x^4 - 14x^3 + 28x^2 - 54x + 108 + \frac{-215}{x+2}}$

3. Factor the following polynomials completely. (5.4)

a) $n^3 + 5n^2 - 9n - 45$ factor by grouping

$(n^3 + 5n^2) + (-9n - 45) =$

$n^2(n+5) - 9(n+5) =$

difference of two squares $(n^2-9)(n+5) = \boxed{(n+3)(n-3)(n+5)}$

b) $3s^5 - s^3 - 24s$ GCF

$s(3s^4 - s^2 - 24)$ quadratic like

$a \cdot c = 3 \cdot -24 = -72$, $b = -1$: $-9, 8$

$s((3s^4 - 9s^2) + (8s^2 - 24))$

$s(3s^2(s^2-3) + 8(s^2-3)) = \boxed{s(3s^2+8)(s^2-3)}$

4. State the degree of the polynomial, the number of turning points the polynomial has (are they local maximums or local minimums?), and describe the right end and left end behavior of the graph of the following functions: (5.2)

a) $y = -4x^3 + 5x - 2$

degree − 1

Degree: 3 # of turning points: 2

$f(x) \to -\infty$ as $x \to +\infty$

$f(x) \to \infty$ as $x \to -\infty$

# of local mins: 1 # of local maxs: 1

b) $y = 5x^4 - 7x - 7$

Degree: 4 # of turning points: 3

$f(x) \to \infty$ as $x \to +\infty$

$f(x) \to \infty$ as $x \to -\infty$

# of local mins: 2 # of local maxs: 1

5. Add, subtract or multiply the following expressions: (5.3)

a) $(x - 3)(x + 2)(x + 4)$

$(x^2 + 2x - 3x - 6)(x + 4) =$

$(x^2 - x - 6)(x + 4) =$

$x^3 + 4x^2 - x^2 - 4x - 6x - 24 =$

$\boxed{x^3 + 3x^2 - 10x - 24}$

b) $(-2x^4 + 3x^3 - 2x + 1) - (x^4 + 2x^3 - x^2 - 5)$

$-2x^4 + 3x^3 - 2x + 1 - x^4 - 2x^3 + x^2 + 5 =$

$\boxed{-3x^4 + x^3 + x^2 - 2x + 6}$

c) $(x - 4)^2 =$

$(x-4)(x-4) =$

$x^2 - 4x - 4x + 16 =$

$\boxed{x^2 - 8x + 16}$

6. Find all the zeros of the following function: $f(x) = x^3 + x^2 - 4x + 6$. (5.6)

$\pm(1,2,3,6)$ ★graph to see that possible rational zero is -3

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -4 & 6 \\ & \downarrow & -3 & 6 & -6 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

$f(x) = (x+3)(x^2-2x+2)$ prime

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2-4(1)(2)}}{2(1)} = \dfrac{2 \pm \sqrt{4-8}}{2} = \dfrac{2 \pm \sqrt{-4}}{2} = \dfrac{2 \pm 2i}{2} = 1 \pm i$

$\sqrt{-4}$, $\sqrt{-1}\ \sqrt{4}$, $i\ \ 2$

$\boxed{x = -3, 1 \pm i}$

7. Simplify. Write the answer in scientific notation. (5.1)

a) $(7.2 \times 10^9)(9.4 \times 10^8)$

has to be between 1 and 10 → $67.68 \times 10^{17} =$

$\boxed{6.768 \times 10^{18}}$

b) $\dfrac{1.1 \times 10^{-3}}{5.5 \times 10^{-8}}$

$10^{-3-(-8)} = 10^{-3+8} = 10^5$

$.2 \times 10^5$

$\boxed{2 \times 10^4}$

8. Simplify the expression. (6.2)

a) $-6\sqrt[7]{2} + 2\sqrt[7]{256}$

$\sqrt[7]{256}$, $\sqrt[7]{128}\ \sqrt[7]{2}$, $2\sqrt[7]{2}$

$-6\sqrt[7]{2} + 2(2\sqrt[7]{2}) =$

$-6\sqrt[7]{2} + 4\sqrt[7]{2} = \boxed{-2\sqrt[7]{2}}$

b) $5\sqrt[3]{16} - \sqrt[3]{16} = 4\sqrt[3]{16} =$

$4(2\sqrt[3]{2}) = \boxed{8\sqrt[3]{2}}$

$\sqrt[3]{16}$, $\sqrt[3]{8}\ \sqrt[3]{2}$, $2\sqrt[3]{2}$

9. Write the expression in simplest form. Assume that all variables are positive. (6.2)

a) $\sqrt[3]{9x^5y^4} \cdot \sqrt[3]{12x} =$

$\sqrt[3]{108x^6y^4} =$

$\sqrt[3]{3^3 \cdot 4x^3x^3y^3y} =$

$3 \cdot x \cdot x \cdot y\sqrt[3]{4y} = \boxed{3x^2y\sqrt[3]{4y}}$

$\sqrt[3]{108}$, $\sqrt[3]{27}\ \sqrt[3]{4}$, $3\sqrt[3]{4}$

b) $\sqrt[4]{32a^6b^3c^8} =$

$\sqrt[4]{16 \cdot 2 \cdot a^4 \cdot a^2 \cdot b^3c^4c^4} =$

$2 \cdot a \cdot c \cdot c\sqrt[4]{2a^2b^3} =$

$\boxed{2ac^2\sqrt[4]{2a^2b^3}}$

c) $\sqrt[3]{\dfrac{54}{x}}$

10. Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and zeros of -2, 1, 3. Then find the product of the coefficients of the quadratic (x^2) and linear (x) terms. (5.7)

$f(x) = (x+2)(x-1)(x-3)$

$= (x^2+2x-x-2)(x-3)$

$= (x^2+x-2)(x-3)$

$= x^3-3x^2+x^2-3x-2x+6$

$\boxed{f(x) = x^3-2x^2-5x+6}$

$-2(-5) = \boxed{10}$

11. Let $f(x) = 10x^{1/3}$ and $g(x) = 5x^{1/4}$. Find the indicated function. (6.3)

a) $f(x) \cdot g(x)$ add

$(10x^{1/3})(5x^{1/4}) =$

$\boxed{50x^{7/12}}$

$\frac{1(4)}{3(4)} + \frac{1(3)}{4(3)}$

$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

b) $\dfrac{f(x)}{g(x)} = \dfrac{10x^{1/3}}{5x^{1/4}} =$

$\frac{1}{3} - \frac{1}{4} =$

$\frac{4}{12} - \frac{3}{12} = \frac{1}{12}$

$\boxed{2x^{1/12}}$

c) $\dfrac{g(x)}{f(x)} = \dfrac{5x^{1/4}}{10x^{1/3}} = \dfrac{1x^{-1/12}}{2} =$

$\frac{1}{4} - \frac{1}{3} =$

$\frac{3}{12} - \frac{4}{12} = -\frac{1}{12}$

$\dfrac{1}{2x^{1/12}} \cdot \dfrac{x^{11/12}}{x^{11/12}} =$

$\dfrac{x^{11/12}}{2x^{12/12}} = \boxed{\dfrac{x^{11/12}}{2x}}$