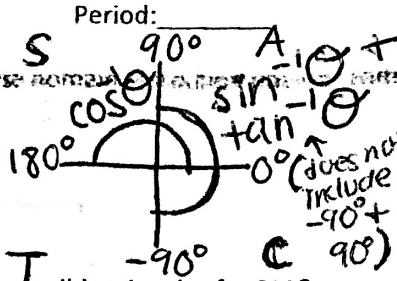
Trigonometric Ratios and Functions (13.4-13.6) Review Name: Key

Advanced Algebra with Trigonometry, Glawe

SOH-CAH-TOA Period:



1) What is the domain restriction (in both radians and degrees) for the follow inverse functions:

a) inverse tangent: $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ $-90° < \theta < 90°$

b) inverse sine: $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ $-90° \le \theta \le 90°$

c) inverse cosine: $0 \le \theta \le \pi$ $0° \le \theta \le 180°$

2) What is the Law of Sines and what types (cases) of triangles do you use it for? What are the possible triangles for SSA?

SSA cases will be given on test

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

AAS, ASA, or SSA

SSA:

If $\angle A$ is obtuse,
- no triangle ($a \le b$)
- one triangle ($a > b$)

If $\angle A$ is acute
- No triangle ($h > a$)
- Two triangles ($h < a < b$)
- One triangle ($h = a$ or $a > b$)

$*h = b \sin A*$

3) What is the Law of Cosines and what types (cases) of triangles do you use it for?

$a^2 = b^2 + c^2 - 2bc \cos A$

SAS or SSS

4) What are the two equations we can use to find the area of a triangle and what is the type (case) of triangle for each?

Area equations will be given on your test

$A = \frac{1}{2} bc \sin A$ SAS

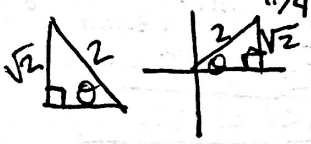
$A = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semi perimeter of the triangle (half perimeter)

$s = \frac{a+b+c}{2}$

Evaluate the following expressions in both radians and degrees (make sure you show all your work):

5) $\sin^{-1}\frac{\sqrt{2}}{2}$ Q1 or Q4

D: $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ or $-90° \le \theta \le 90$

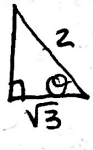


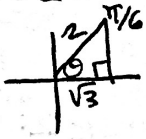
$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\pi}{4}$ or $45°$

6) $\cos^{-1}\frac{\sqrt{3}}{2}$ Q1 or Q2

D: $0 \le \theta \le \pi$ or $0° \le \theta \le 180°$



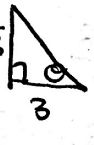


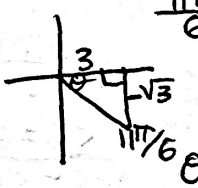
$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$ or $30°$

7) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ Q1 or Q4

D: $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ or $-90° < \theta < 90°$





$\frac{11\pi}{6} - 2\pi =$

$\frac{11\pi}{6} - \frac{12\pi}{6} =$

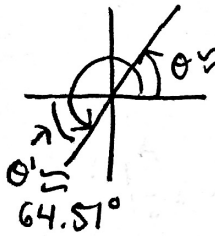
$\frac{-\pi}{6}$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$

$= \frac{-\pi}{6}$ or $-30°$

8) Solve the equation $\tan\theta = 2.1$ where $180° < \theta < 270° \leftarrow$ Q3

$\theta = \tan^{-1}(2.1) \approx 64.54°$

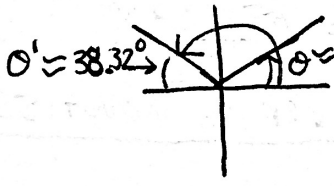


$180° + 64.54° =$

244.54°

9) Solve the equation $\sin\theta = 0.62$ where $90° < \theta < 180° \leftarrow$ Q2

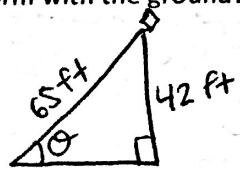
$\theta = \sin^{-1}(0.62) \approx 38.32°$



$180° - 38.32° =$

141.68°

10) A flying kite is attached to the ground by a 65-foot string. The kite is 42 feet above the ground. What angle goes the string form with the ground?



$\sin\theta = \frac{42}{62}$

$\theta = \sin^{-1}\left(\frac{42}{62}\right) \approx$ 40.3°

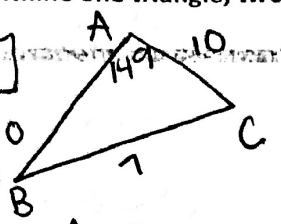
For problems 11-14, state the case (AAS, SSS, SAS, ASA, or SSA) applicable to the given measurements. Then describe whether the measurements determine one triangle, two triangles, or no triangle. *(You do not need to solve ΔABC)*

11) $A = 149°, a = 7, b = 10$

SSA; no triangle

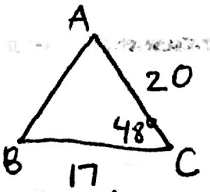
<A → obtuse: NO Δ 7 < 10

+ Δ a < b



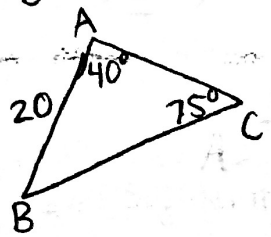
12) $C = 48°, a = 17, b = 20$

SAS; one triangle



13) $A = 40°, C = 75°, c = 20$

AAS; one triangle



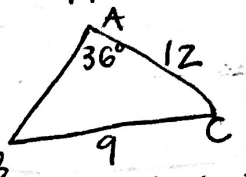
14) $A = 36°, a = 9, b = 12$

SSA; two triangles

<A → acute: ~~NO Δ~~

+ Δ

2 Δ



9 < 12 either NO Δ or 2 Δ

a < b

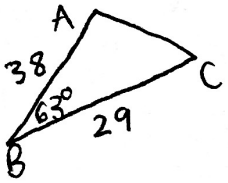
$h = b\sin A = 12\sin 36° \approx 7.05$

$9 > 7.05$

$a > h$

For the following problems, solve ΔABC:

15) $B = 63°, a = 29, c = 38$



$b^2 = a^2 + c^2 - 2ac\cos B$ SAS

$b^2 = 29^2 + 38^2 - 2(29)(38)\cos 63°$

≈ 1284.4 $b \approx 35.8$

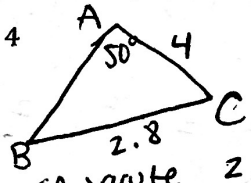
$\frac{\sin A}{29} = \frac{\sin 63°}{35.8}$ $\angle A = \sin^{-1}\left(\frac{29\sin 63}{35.8}\right)$ $\angle A \approx 46.2°$

$180° - 63° - 46.2° = 70.8°$ $\angle C = 70.8°$

16) $A = 50°, a = 2.8, b = 4$

SSA

no triangle



<A → acute

NO Δ

+ Δ

2 Δ

2.8 < 4

a < b

$h = b\sin A = 4\sin 50° \approx 3.06$

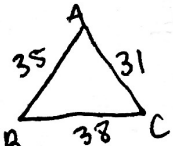
$2.8 < 3.06$

$a < h$

17) $a = 38, b = 31, c = 35$

law of cosines; need to start with <A

SSS



$38^2 = 31^2 + 35^2 - 2(31)(35)\cos A$

$1444 = 961 + 1225 - 2170\cos A$

$1444 = 2186 - 2170\cos A$

$-2186 \quad -2186$

$\frac{-742}{-2170} = \frac{-2170\cos A}{-2170}$

$\cos A = \frac{742}{2170}$

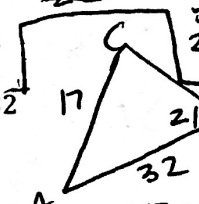
$\angle A = \cos^{-1}\left(\frac{742}{2170}\right)$ $\angle A \approx 70.0°$

$\frac{\sin B}{31} = \frac{\sin 70°}{38}$

$\angle B = \sin^{-1}\left(\frac{31\sin 70}{38}\right)$ $\angle B \approx 50.0°$

$\angle C = 180° - 70° - 50° = 60°$ $\angle C \approx 60°$

18) $B = 21°, b = 17, c = 32$



SSA

<B → acute:

~~NO Δ~~

+ Δ

2 Δ

17 < 32

b < c

$h = c\sin B = 32\sin 21° \approx 11.47$

$17 > 11.47$

$b > h$

Triangle 1: $\frac{\sin C}{32} = \frac{\sin 21°}{17}$

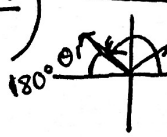
$\sin C = \frac{32\sin 21°}{17}$

$\angle C = \sin^{-1}\left(\frac{32\sin 21°}{17}\right)$ $\angle C \approx 42.4°$

$\angle A = 180° - 21° - 42.4°$ $\angle A \approx 116.6°$

$\frac{a}{\sin 116.6°} = \frac{17}{\sin 21°}$ $a = \frac{17\sin 116.6°}{\sin 21°}$ $a \approx 42.4$

Triangle 2:

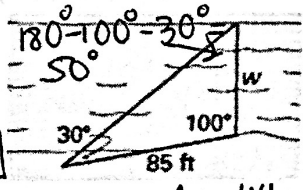


$\angle C = 180° - 42.4°$ $\angle C \approx 137.6°$

$\angle A = 180° - 21° - 131.6°$ $\angle A \approx 21.4°$

$\frac{a}{\sin 21.4°} = \frac{17}{\sin 21°}$ $a = \frac{17\sin 21.4°}{\sin 21°}$ $a \approx 17.3$

19) A surveyor wants to find the width of a river from a particular point on the shoreline for construction of a bridge. The surveyor's measurements are shown in the figure. How wide (w) is the river?



$\frac{w}{\sin 30°} = \frac{85}{\sin 50°}$ $w = \frac{85\sin 30°}{\sin 50°}$ $w \approx 55.48$

The river is about 55.48 feet wide

For the following problems, find the area of ΔABC:

20) $a = 43, b = 59, c = 48$

SSS

$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(43+59+48) = 75$

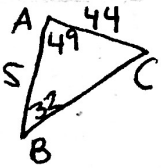
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{75(75-43)(75-59)(75-48)}$

$= \sqrt{75(32)(16)(27)} = \sqrt{1036800}$

$A \approx 1018.23$ units^2

21) $A = 49°, B = 32°, b = 44$

AAS; Need SAS



$\frac{\sin 49°}{a} = \frac{\sin 32°}{44}$

$a = \frac{44\sin 49°}{\sin 32°}$ $a \approx 62.66$

$\angle C = 180° - 32° - 49° = 99°$ $\angle C = 99°$

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(62.66)(44)\sin 99° \approx$ 1361.55 units^2