Rational Exponents and Radical Functions Review (Chapter 6) Name: Key

Advanced Algebra with Trigonometry, Glawe Period:

Perfect Cubes: $2^3 = 8$ $3^3 = 27$ $4^3 = 64$ $5^3 = 125$ $6^3 = 216$	**Perfect Powers of Six:** $2^6 = 64$
Perfect Powers of Four: $2^4 = 16$ $3^4 = 81$ $4^4 = 256$ $5^4 = 625$	**Perfect Powers of Seven:** $2^7 = 128$
Perfect Powers of Five: $2^5 = 32$ $3^5 = 243$	**Perfect Powers of Eight:** $2^8 = 256$

Evaluate the expression.

1) $125^{-2/3}$

$(125^{1/3})^{-2} = ((5^{1/3})^3)^{-2} =$

$5^{-2} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$

2) $100^{5/2} = (100^{1/2})^5 = (\sqrt[2]{100})^5 =$

$10^5 =$

$\boxed{100{,}000}$

3) $32^{3/5} = (32^{1/5})^3 =$

$(\sqrt[5]{32})^3 =$

$2^3 = \boxed{8}$

4) $\sqrt[4]{1134} =$

1134 → 18, 63; 18 → 9, 2; 63 → 7, 9; 9 → 3, 3; 9 → 3, 3

$\sqrt[4]{2 \cdot 3^4 \cdot 7} =$

$3\sqrt[4]{2 \cdot 7} =$

$\boxed{3\sqrt[4]{14}}$

Simplify the expression.

5) $(\sqrt{x} \cdot \sqrt[3]{x})^5 =$

$(x^{1/2} \cdot x^{1/3})^5 =$

$x^{5/2(3/3)} \cdot x^{5/3(2/2)} =$

$x^{15/6} \cdot x^{10/6} = x^{25/6} =$

$\sqrt[6]{x^{25}} = \sqrt[6]{x^6x^6x^6x^6x^1} = \boxed{x^4\sqrt[6]{x}}$

6) $2\sqrt[3]{64y^5} - y\sqrt[3]{8y^2}$

$2\sqrt[3]{4^3y^3y^2} - y\sqrt[3]{2^3y^2} =$

$2 \cdot 4 \cdot y\sqrt[3]{y^2} - y \cdot 2\sqrt[3]{y^2} =$

$8y\sqrt[3]{y^2} - 2y\sqrt[3]{y^2} =$

$\boxed{6y\sqrt[3]{y^2}}$

7) $\sqrt[5]{64x^6y^5} =$

$\sqrt[5]{32 \cdot 2x^5xy^5} =$

$\sqrt[5]{2^5 \cdot 2x^5xy^5} =$

$\boxed{2xy\sqrt[5]{2x}}$

8) $\sqrt[4]{\frac{x^6}{4y^2}} = \frac{\sqrt[4]{x^6}}{\sqrt[4]{4y^2}}$

need to make a perfect fourth

$\frac{\sqrt[4]{x^6}}{\sqrt[4]{4y^2}} \cdot \frac{\sqrt[4]{4y^2}}{\sqrt[4]{4y^2}} =$

$\frac{\sqrt[4]{x^4x^24y^2}}{\sqrt[4]{16y^4}} =$

$\boxed{\frac{x\sqrt[4]{4x^2y^2}}{2y}}$

Solve the following functions. Check for extraneous solutions.

S.M.E.G

9) Solve $(3x + 43)^{2/3} + 22 = 38$

$-22 \quad -22$

$((3x+43)^{2/3})^{3/2} = (16)^{3/2}$

$3x+43 = (\sqrt[2]{16})^3$

$3x+43 = (\pm 4)^3$

$3x+43 = \pm 64$

$3x+43 = 64$, -43 -43, $3x = 21$, $x = 7$ or

$3x+43 = -64$, -43 -43, $\frac{3x}{3} = \frac{-107}{3}$

$\boxed{x = 7 \text{ or } x = -107/3}$

10) Solve $(\sqrt{2x+4})^2 = (x-2)^2$

$2x+4 = (x-2)(x-2)$

$2x+4 = x^2 - 2x - 2x + 4$

$2x+4 = x^2 - 4x + 4$

$-2x - 4 \qquad -2x - 4$

$0 = x^2 - 6x$

$0 = x(x-6)$

$x = 0$ or $\boxed{x = 6}$

extraneous

check:

$x = 0$: $\sqrt{2(0)+4} = 0 - 2$, $\sqrt{4} = -2$, $2 \neq -2$ ✗

$x = 6$: $\sqrt{2(6)+4} = 6 - 2$, $\sqrt{12+4} = 4$, $\sqrt{16} = 4$, $4 = 4$ ✓

Find the inverse of the functions.

11) $f(x) = -\frac{1}{8}x^3$

$y = -\frac{1}{8}x^3$

$(-8)x = -\frac{1}{8}y^3(-8)$

$\sqrt[3]{-8x} = \sqrt[3]{y^3}$

$y = \sqrt[3]{-8x}$ $\quad y = -2\sqrt[3]{x}$

$\boxed{f^{-1}(x) = -2\sqrt[3]{x}}$

12) $h(x) = x^2 - 3$, with the domain $[0, \infty)$

only positive and zero

$y = x^2 - 3$

$x = y^2 - 3$

$+3 \qquad +3$

$\sqrt{x+3} = \sqrt{y^2}$

$y = \pm\sqrt{x+3}$

$y = \sqrt{x+3}$

$\boxed{h^{-1}(x) = \sqrt{x+3}}$

13) The euro is the unit of currency for the European Union. The number of D dollars worth one euro E can be represented by: $E = 1.13621D$ Find the inverse of the function. Then use the inverse to find the number of dollars that could be obtained for 400 euros.

$\frac{E}{1.13621} = \frac{1.13621D}{1.13621}$

$D = 0.88012E$

$D = 0.88012(400)$

try not to round and use long decimal

Inverse equation: $\boxed{D = 0.88012E}$

400 euros is worth \$352.05 dollars.

Let $f(x) = 6x^2$, $g(x) = \frac{2}{x}$, $h(x) = x^{\frac{1}{2}}$, and $j(x) = x^{\frac{5}{2}}$. Perform the indicated operation and state the domain in interval notation.

$h(x) = x^{1/2}$ $\quad j(x) = x^{5/2}$

all nonnegative real numbers

14) $h(x) - j(x)$

$x^{1/2} - x^{5/2} = x^{-4/2} = x^{-2} = \boxed{\frac{1}{x^2}}$

Domain: all real numbers except $x=0$

15) $f(x) \cdot g(x)$

$\frac{6x^2}{1} \cdot \frac{2}{x} = \frac{12x^2}{x} = 12x$

Domain: all real numbers

★need to check domain of denominator too★

16) $\frac{f(x)}{h(x)}$ $= \frac{6x^2}{x^{1/2}} = \boxed{6x^{3/2}}$ ← not zero + nonnegative

$x^{2-1/2} = x^{4/2} - x^{1/2} = x^{3/2}$

Domain: all positive real numbers

★for composition of functions, make sure you check inside function too★

17) $f(g(x))$

$f\left(\frac{2}{x}\right) = 6\left(\frac{2}{x}\right)^2 = 6\left(\frac{2^2}{x^2}\right) = \frac{6}{1}\left(\frac{4}{x^2}\right) = \boxed{\frac{24}{x^2}}$

Domain: all real numbers except $x=0$

18) $h(f(x))$

$h(6x^2) = (6x^2)^{1/2} = 6^{1/2}x^{2/2} = \boxed{6^{1/2}x}$

Domain: all real numbers

19) $g(h(x))$ ↱ nonnegative numbers

$g(x^{1/2}) = \frac{2}{x^{1/2}}$ ← can't leave rational exponent in denominator

$\frac{2}{x^{1/2}} \cdot \frac{x^{1/2}}{x^{1/2}} = \frac{2x^{1/2}}{x^{2/2}} = \boxed{\frac{2x^{1/2}}{x}}$ ← not zero + nonnegative

Domain: all positive real numbers

20) Is the inverse of the function $f(x) = 2x^2 - 3$ also a function? Why or why not?

★you can use a graphing calculator or just think about what a quadratic function looks like★

NO

The inverse is ~~not~~ not a function because it does not pass the horizontal line test.

21) How does the graph of $y = -2\sqrt{x+4} + 5$ compare to the graph of $y = -2\sqrt{x}$?

↳ opposite of h (horizontal shift)
↳ k (vertical shift)

left 4 units, up 5 units

22) Verify that f and f^{-1} inverses using composition of functions: $f(x) = 6x - 2$ and $f^{-1}(x) = \frac{1}{6}x + \frac{1}{3}$

★$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$★

$f(f^{-1}(x)) = f\left(\frac{1}{6}x + \frac{1}{3}\right) = 6\left(\frac{1}{6}x + \frac{1}{3}\right) - 2 = \left(\frac{6}{6}x + \frac{6}{3}\right) - 2 = x + 2 - 2 = x$ ✓

$f^{-1}(f(x)) = f^{-1}(6x-2) = \frac{1}{6}(6x-2) + \frac{1}{3} = \left(\frac{6}{6}x - \frac{2}{6}\right) + \frac{1}{3} = \left(x - \frac{1}{3}\right) + \frac{1}{3} = x$ ✓

Fill out the table of values and graph the following functions. Choose four points that are *integers* (no decimals). Then state the domain and range of the function in **Interval notation**.

23) $y = 4\sqrt[3]{x-1} - 2$

set equal to $\boxed{\text{perfect cubes: } -8, -1, 0, 1, 8}$

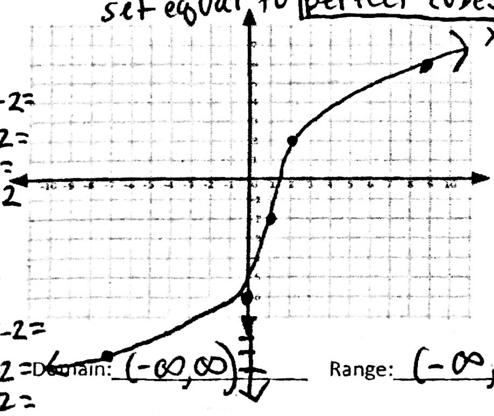
$x-1=-8$, $+1$ $+1$ → $x=-7$: $4\sqrt[3]{-7-1} - 2 = 4\sqrt[3]{-8} - 2 = 4(-2) - 2 = -8-2 = -10$

$x-1=-1$, $+1$ $+1$, $x=0$: $4\sqrt[3]{-1} - 2 = 4(-1) - 2 = -4-2 = -6$

$x-1=0$, $+1$ $+1$, $x=1$: $4\sqrt[3]{1-1} - 2 = 4\sqrt[3]{0} - 2 = 4(0) - 2 = -2$

$x-1=1$, $+1$ $+1$, $x=2$: $4\sqrt[3]{2-1} - 2 = 4\sqrt[3]{1} - 2 = 4(1) - 2 = 4-2 = 2$

$x-1=8$, $+1$ $+1$, $x=9$: $4\sqrt[3]{9-1} - 2 = 4\sqrt[3]{8} - 2 = 4(2) - 2 = 8-2 = 6$



x	y
-7	-10
0	-6
1	-2
2	2
9	6

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

24) $y = -3\sqrt{x+2} + 5$

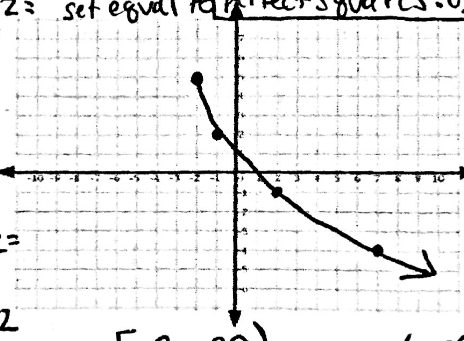
set equal to $\boxed{\text{perfect squares: } 0, 1, 4, 9}$

$x+2=0$, -2 -2, $x=-2$: $-3\sqrt{-2+2} + 5 = -3\sqrt{0} + 5 = -3(0) + 5 = 0 + 5 = 5$

$x+2=1$, -2 -2, $x=-1$: $-3\sqrt{-1+2} + 5 = -3\sqrt{1} + 5 = -3(1) + 5 = -3 + 5 = 2$

$x+2=4$, -2 -2, $x=2$: $-3\sqrt{2+2} + 5 = -3\sqrt{4} + 5 = -3(2) + 5 = -6 + 5 = -1$

$x+2=9$, -2 -2, $x=7$: $-3\sqrt{7+2} + 5 = -3\sqrt{9} + 5 = -3(3) + 5 = -9 + 5 = -4$



x	y
-2	5
-1	2
2	-1
7	-4

Domain: $[-2, \infty)$ Range: $(-\infty, 5]$