Exponential and Logarithmic Functions (Chapter 7) Review
Advanced Algebra with Trig, Glawe

Name: Key
Period: _____

1) What has to be true for function to be an exponential growth? $b>1$

2) What functions are the inverse functions of exponential functions? logarithmic functions

3) What are the three main properties to simplify logarithmic functions? product, quotient, and power

4) $\log y = \log_{10} y$ 5) $\ln y = \log_e y$

Simplify the expression. Give exact answers (no decimals).

6) $8^{\log_8 3x} = \boxed{3x}$

7) $\log_4 64^{2x} = \log_4 (4^3)^{2x} = \log_4 4^{6x} = \boxed{6x}$

8) $3e^2 \cdot 4e^{-6} \quad 12e^{-4} = \boxed{\frac{12}{e^4}}$

$* \; b^{\log_b x} = x$ $* \; \log_b b^x = x$

Expand the following logarithmic expression.

9) $\log_3 \frac{5x^3}{z^2} = \log_3 5x^3 - \log_3 z^2 =$
$\log_3 5 + \log_3 x^3 - \log_3 z^2 =$
$\boxed{\log_3 5 + 3\log_3 x - 2\log_3 z}$

10) $\log_2 \frac{xy}{4z^3} = \log_2 xy - \log_2 4z^3 =$
$\log_2 x + \log_2 y - (\log_2 4 + \log_2 z^3) =$
$\log_2 x + \log_2 y - \log_2 4 - \log_2 z^3 =$
$\boxed{\log_2 x + \log_2 y - \log_2 4 - 3\log_2 z}$

Condense the following logarithmic expression.

11) $2\log_2 4 + \log_2 x - 3\log_2 2$
$\log_2 4^2 + \log_2 x - \log_2 2^3 =$
$\log_2 16 + \log_2 x - \log_2 8 =$
$\log_2 16x - \log_2 8 =$
$\log_2 \frac{16x}{8} = \boxed{\log_2 2x}$

12) $\ln x + 4\ln y - 2\ln 6 - 3\ln z$
$\ln x + \ln y^4 - \ln 6^2 - \ln z^3 =$
$\ln x + \ln y^4 - \ln 36 - \ln z^3 =$
$\ln xy^4 - (\ln 36 + \ln z^3) =$
$\ln xy^4 - \ln 36z^3 = \boxed{\ln \frac{xy^4}{36z^3}}$

Solve the logarithmic or exponential equation. ***Check for extraneous solutions.***

13) $\frac{3e^{2x}}{3} = \frac{15}{3}$
$e^{2x} = 5$
$\log_e 5 = 2x$
$\frac{\ln 5}{2} = \frac{2x}{2}$
$\boxed{x = \frac{\ln 5}{2} \text{ or } 0.805}$

14) $\left(\frac{1}{16}\right)^{x-3} = 64^{2x+1}$
$(4^{-2})^{x-3} = (4^3)^{2x+1}$
$4^{-2x+6} = 4^{6x+3}$
$-2x+6 = 6x+3$
$+2x-3 \quad +2x-3$
$3 = 8x$
$\boxed{x = 3/8}$

15) $2^{3x-3} = 16^{4x-1}$
$2^{3x-3} = (2^4)^{4x-1}$
$2^{3x-3} = 2^{16x-4}$
$3x-3 = 16x-4$
$-3x+4 \quad -3x+4$
$\frac{1}{13} = \frac{13x}{13}$
$\boxed{x = 1/13}$
extraneous
CHECK: $x=-16$: $\log_4(-16+12) + \log_4(-16) = 3$ (negative)

16) $\ln(7x-4) = \ln(2x+11)$
$7x-4 = 2x+11$
$-2x+4 \quad -2x+4$
$\frac{5x}{5} = \frac{15}{5}$
$\boxed{x=3}$

17) $\log 5x + \log(x-1) = 2$
$\log_{10} 5x(x-1) = 2$
$10^{\log_{10} 5x(x-1)} = 10^2$
$5x(x-1) = 100$
$5x^2 - 5x = 100$
$5x^2 - 5x - 100 = 0$
$5(x^2 - x - 20) = 0$
$-20 \mid -1$
$-5, 4$
$5(x-5)(x+4) = 0$
$5 \neq 0 \quad x-5=0 \quad x+4=0$
$\boxed{x=5} \quad x=-4$ extraneous
CHECK: $x=5$: $\log 5(5) + \log(5-1) = 2$
$\log(25)(4) = 2$
$10^2 = 100$ ✓
$x=-4$: $\log 5(-4) + \log(-4-1) = 2$
$\log(-20) + \log(-5) = 2$
negative log

18) $\log_4(x+12) + \log_4 x = 3$
$\log_4(x+12)(x) = 3$
$\log_4(x^2+12x) = 3$
$4^3 = x^2+12$
$64 = x^2+12$
$x^2+12-64=0$
$(x+16)(x-4)=0$
$x=-16 \quad \boxed{x=4}$
$\log_4(4+12) + \log_4(4) = 3$
$\log_4(16) + \log_4(4) = 3$
$\log_4(16)(4) = 3$
$\log_4 64 = 3$ ✓

Fill in a table of values for the translated function *(you can fill it in for the parent function if you would like).* Then identify the domain, range, and asymptote of the function.

19) $y = 3^{x+2} - 1$ ← left 2, down 1

x	y
-1	1/3
0	1
1	3
2	9

×(-2) → | x | y |
|---|---|
| -3 | -2/3 |
| -2 | 0 |
| -1 | 2 |
| 0 | 8 |

Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

Asymptote: $y = -1$, $y = K$

$y = 3^x$ $y = b^x$ $y = b^{x-h} + k$

20) $y = \log_2(x-1) + 2$ ← right 1, up 2

x	y
1/2	-1
1	0
2	1
4	2

×(+1) → | x | y |
|---|---|
| 3/2 | 1 |
| 2 | 2 |
| 3 | 3 |
| 5 | 4 |

Domain: $(1, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 1$, $x = h$

$y = \log_2(x-1) + 2$ (−2, −2)

$y = \log_b x$ $y = \log_b(x-h) + k$ $y - 2 = \log_2(x-1)$

$y = \log_2 x$ $2^{y-2} = x - 1$ $x = 2^{y-2} + 1$

$2^y = x$ (+1, +1)

Find the inverse of the given function.

21) $y = 2e^{x+1}$

$\frac{x}{2} = \frac{2e^{y+1}}{2}$

$e^{y+1} = \frac{x}{2}$

$\log_e(\frac{x}{2}) = y + 1$ (−1, −1)

$\log_e(\frac{x}{2}) - 1 = y$

$y = \ln(\frac{x}{2}) - 1$

22) $y = 4^{x-2}$

$x = 4^{y-2}$

$\log_4 x = y - 2$ (+2, +2)

$y = \log_4 x + 2$

23) $y = \frac{1}{3} \cdot \ln(x+4)$

$3 \cdot x = \frac{1}{3}\ln(y+4) \cdot 3$

$3x = \ln(y+4)$

$3x = \log_e(y+4)$

$e^{3x} = y + 4$ (−4, −4)

$y = e^{3x} - 4$

24) $y = 4\log(x-2)$

$\frac{x}{4} = \frac{4\log(y-2)}{4}$

$\frac{x}{4} = \log_{10}(y-2)$

$10^{x/4} = y - 2$ (+2, +2)

$y = 10^{x/4} + 2$

Exponential Functions: $y = a(1+r)^t$ $A = Pe^{rt}$ $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $y = a(1-r)^t$

25) In 2008, roughly 1.7 million people owned a Smart Phone. In the next 4 years, the number of Smart Phones increased by about 114% each year. About how many people owned a Smart Phone in 2011? 2011: $t = 3$

$y = a(1+r)^t$ $y = 1.7(2.14)^t$

$y = 1.7(1+1.14)^t$ $y = 1.7(2.14)^3 \approx$ **16.7 million people**

26) Your parents bought a car 4 years ago for $31,262. If its value decreases by 16% each year, how much is the car worth now?

$y = a(1-r)^t$ $y = 31262(.84)^t$ $t = 4$

$y = 31262(1-.16)^t$ $y = 31262(.84)^4 \approx$ **$15564.45**

27) You deposit $1500 into a bank account that pays 8% annual interest. Calculate the final amount after 4 years if the interest is...

a) Compounded *Semi-annually*: ↳ two-times a year; $n = 2$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = 1500\left(1 + \frac{.08}{2}\right)^{2(4)} \approx$ **$2052.85**

b) Compounded *Continuously*:

$A = Pe^{rt}$ $A = 1500e^{.08(4)} \approx$ **$2065.69**

28) The *apparent magnitude* of a star is a measure of the brightness of the star as it appears to observers on Earth. The apparent magnitude M of the dimmest star that can be seen with a telescope is given by the function $M = 5\log D + 2$ where D is the diameter (in millimeters) of the telescope's objective lens. If a telescope can reveal stars with a magnitude of 12, what is the diameter of its objective lens?

$M = 5\log D + 2$

$12 = 5\log D + 2$ (−2, −2)

$\frac{10}{5} = \frac{5\log D}{5}$

$2 = \log D$

$\log_{10} D = 2$

$D = 10^2 = 100$ mm