Rating	Frequency
5	Extremely High: 3 or more questions typically appear on every SAT
4	High: at least 2 questions typically appear on every SAT
3	Moderate: at least one question typically appears on every SAT
2	Low: one question typically appears on every two or three SATs
1	Extremely Low: one question appears infrequently and without a pattern

A Frequency Guide is provided for every type of problem so you can prioritize the content you need to study most.

Remember that there are eight solution strategies you can employ on SAT math questions:

1. ANALYZE the Answer Choices
2. BACKPLUG the Answer Choices
3. SUPPLY Numbers
4. TRANSLATE from English to Math
5. RECORD What You Know
6. SPLIT the Question into Parts
7. DIAGRAM the Question
8. SIZE UP the Figures

This chapter will explore the following concepts and explain how they are tested on the SAT:

1. Data Analysis
2. Average, Median, Mode
3. Counting Problems
   A. Combinations
   B. Permutations
4. Probability
5. Sequences
6. Overlapping Groups
7. Logical Reasoning

Average, Median, and Mode

The average, median, and mode are statistical data that occur frequently on the SAT. They are easy to remember if you study the following information.

Required Knowledge and Skill Set

1. The average is always referred to as the "average (arithmetic mean)" on the SAT, so you do not have to memorize that the mean is the average.

2. The formula for averages is needed for every average problem on the SAT:

$$\frac{\text{sum of the numbers}}{\text{number of numbers}} = \text{average}$$

Use a shorthand version of this formula to save time on the test:

$$\frac{\text{sum}}{\text{\# of \#s}} = \text{average}$$

This formula can also be used to find the sum or the number of numbers in a set:

$$\frac{\text{sum}}{\text{\# of \#s}} = \text{average} \quad \rightarrow \quad \text{sum} = (\text{average})(\text{\# of \#s})$$

$$\frac{\text{sum}}{\text{\# of \#s}} = \text{average} \quad \rightarrow \quad \text{\# of \#s} = \frac{\text{sum}}{\text{average}}$$

3. The median is the middle number in a list of numbers placed in ascending or descending order. On the SAT, if the numbers are not listed in ascending or descending order, rearrange them:

2, 12, 18, 4, 11, 7, 5, 9, 14

The Median

4 #s ← 2, 4, 5, 7, 9, 11, 12, 14, 18 → 4 #s

4. Nearly all SAT questions concerning the median will use an odd number of items in the list. If you are presented with an even number of items, the median is the average of the two numbers in the middle:

2, 4, 5, 7, 9, 11, 12, 14

3 #s ← 2, 4, 5, 7, 9, 11, 12, 14 → 3 #s

The Median

$$\frac{7+9}{2} = 8$$

5. In a patterned set of consecutive integers, the average and the median are the same:

   5, 6, 7, 8, 9, 10, 11 — average = 8, median = 8
   Pattern: increase by 1

   36, 38, 40, 42, 44, 46 — average = 41, median = 41
   Pattern: increase by 2

   1055, 1060, 1065 — average = 1060, median = 1060
   Pattern: increase by 5

   –12, –6, 0, 6, 12, 18 — average = 3, median = 3
   Pattern: increase by 6

   The average and the median are not the same if any numbers repeat or if a pattern changes:

   2, 3, 4, 5, 11 — average = 5, median = 4

   36, 38, 38, 40, 42 — average = 38.8, median = 38

6. The mode is the most common number in a series:

   The Mode
   3 occurrences
   
   7, 8, 8, 8, 11, 13, 16, 19, 19, 22

   On the SAT, the mode is tested less often than the average and the median. If the mode is tested, there is usually only one in the series. However, there can be two modes. If another 19 appeared in the list above, the mode would be 8 *and* 19:

   The Mode
   3 occurrences
   
   The Mode
   3 occurrences
   
   7, 8, 8, 8, 11, 13, 16, 19, 19, 19, 22

7. Average, median, and mode questions are often combined with Data Analysis questions, as the series of numbers can be neatly organized in a table.

Counting Problems

Unless you have already taken a statistics course, you probably have not encountered counting problems. These questions are mainly made up of combinations and permutations, which have complex explanations and special formulas. However, on the SAT, they are quite basic, and can be solved without formulas.

Required Knowledge and Skill Set

1. Counting problems require you to do exactly what their name implies—count! The most basic counting problems ask you to count the number of possibilities presented in a word problem. These problems often deal with sums and products, which we will examine more closely in the next section.

2. Permutations and combinations are arrangements of groups of numbers. In a permutation, the order of the items is important; in a combination, the order of the items is not important. There are more possible arrangements in a permutation than a combination.

3. Combinations combine two or more elements. To understand combinations, let's consider an example. At a restaurant, there are three flavors of ice cream and four choices for toppings. If each ice cream sundae consists of one ice cream flavor and one topping, how many different combinations of sundaes are possible?

   Because these are counting problems, you can always just count:

   Three Flavors: 1, 2, and 3
   Four Toppings: A, B, C, and D

1–A	2–A	3–A
1–B	2–B	3–B
1–C	2–C	3–C
1–D	2–D	3–D

   There are 12 combinations. The order of the items is not important; chocolate with sprinkles is the same as sprinkles with chocolate.

   But counting is not the most efficient solution method. To easily find the number of possibilities in a combination, simply multiply the number of elements:

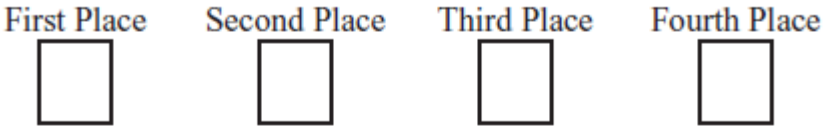
   3 flavors × 4 toppings = 12 combinations

   This works no matter how many elements are present. Say we added 5 syrups to the menu, and each sundae consisted of one flavor of ice cream, one topping, and one syrup. How many combinations are possible now?

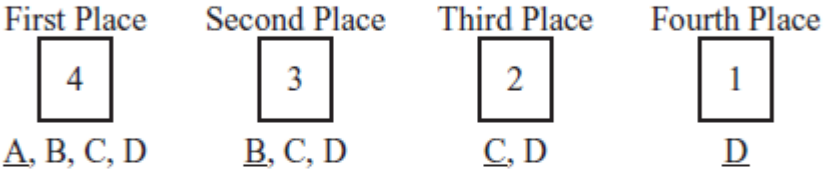
   3 flavors × 4 toppings × 5 syrups = 60 combinations

4. Now look at an example of a permutation. In gym class, four students are running a race. How many different finishing orders are possible at the end of the race?

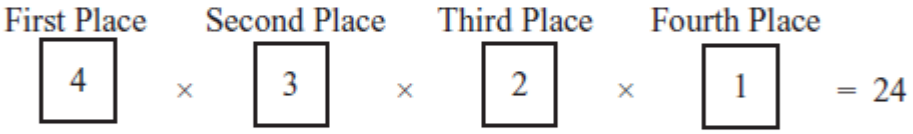
   We often refer to permutations as "card questions," because we use blank "cards" to set up the problems. Draw and label four blank cards, each one representing a specific finishing order:

   Say the four runners are named A, B, C, and D. How many possibilities are there for first place? Four (A, B, C, or D). Assign one of them first place. For the ease of discussion, we will go in alphabetical order. Runner A receives first place. How many runners are now eligible for second place? Three (B, C, or D). If B finishes in second place, how many runners are available for third place? Two (C or D). If C finishes third, only D is left to come in fourth place:

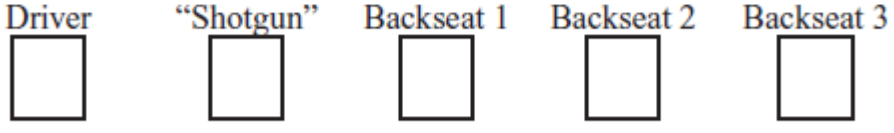
   To find the number of finishing orders, multiply the cards together:

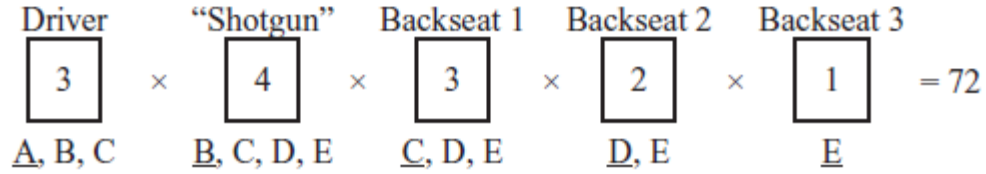
   There are 24 possible finishing orders.

5. Permutations often come with restrictions that dictate rules about the order of the elements. For example, five people are in a car. If only 3 people can drive, how many different seating arrangements are possible?

   Set up five cards, one for each position in the car. Always put the restriction at the front of the list:

   Call the five passengers A, B, C, D, and E. Only A, B, and C can drive:

   There are 72 possible seating arrangements.

Probability

The College Board tests basic probability concepts on the SAT.

Required Knowledge and Skill Set

1. Probability indicates the likelihood that a specific event will occur. The probability of something happening can be expressed using any number from 0 to 1. A probability of 0 means the event will *never* happen. A probability of 1 indicates that an event will *always* happen. A probability of ⅓ signifies that the event has a 1 in 3 chance of occurring.

2. The probability of an occurrence can be expressed by a simple formula:

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

   If there are 6 green socks, 4 blue socks, and 2 red socks in a drawer, what is the probability that you randomly select a blue sock?

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{\text{4 blue socks}}{\text{12 total socks}} \rightarrow \frac{1}{3}$$

   There is a one in three chance that you randomly choose a blue sock.

3. The probability of something *not* occurring is 1 minus the probability that it will occur:

$$\text{Probability of an event } \textit{not} \text{ occurring} = 1 - \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

   If there are 6 green socks, 4 blue socks, and 2 red socks in a drawer, what is the probability that you do *not* select a blue sock? You have two ways to solve this question. You can use the formula for the probability of an event not occurring:

$$\text{Probability of an event } \textit{not} \text{ occurring} = 1 - \frac{\text{favorable}}{\text{possible}}$$

$$1 - \frac{\text{4 blue socks}}{\text{12 total socks}} \rightarrow 1 - \frac{1}{3} \rightarrow \frac{2}{3}$$

   Or you can find the probability of selecting a green or red sock:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{\text{6 green + 2 red}}{\text{12 total socks}} \rightarrow \frac{8}{12} \rightarrow \frac{2}{3}$$

5. Advanced probability questions entail two or more occurrences. These questions are rare, but have on occasion appeared as the most difficult questions in a section.

   The probability of two or more non-related or independent events occurring is the product of the individual probabilities of those events. For example, what is the probability of flipping a penny and getting a "heads" and rolling a standard 6-sided die and getting a 2?

   Find the probability of each independent event:

   $$\text{Probability of flipping heads} = \frac{\text{1 side with heads}}{\text{2 possible sides}} = \frac{1}{2}$$

   $$\text{Probability of rolling a 2} = \frac{\text{1 side with a '2'}}{\text{6 possible sides}} = \frac{1}{6}$$

   And then multiply the individual probabilities to find the probability of both events occurring:

   $$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$ There is a 1 in 12 chance that both events occur.

Sequences

The College Board uses sequence questions to test critical reasoning skills.

Required Knowledge and Skill Set

1. A sequence is a patterned list of numbers. Three types of series are tested on the SAT. In an arithmetic series, each term increases by a constant value. Consider the following sequence:

   3, 7, 11, 15, 19, ...

   In this arithmetic sequence, 4 is added to each term to create the following term.

   Most SAT sequence questions can be solved without using a formula. However, a rare sequence question can benefit from using sequence formulas. For this reason, we recommend that test takers looking to maximize their scores memorize the different formulas for sequences.

   The formula for finding any term of an arithmetic sequence uses several variables:

   $$a_n = a_1 + (n - 1)d$$

   Where:
   a_1 = the first term
   n = the number of terms
   d = constant difference

2. In a geometric sequence, each term increases by a constant ratio:

   4, 8, 16, 32, 64, ...

   In this geometric sequence, 2 is multiplied by each term to create the following term.

   You can find any term in a geometric sequence by using the following formula:

   $$a_n = a_1 \times r^{n-1}$$

   Where:
   a_1 = the first term
   n = the number of terms
   r = constant ratio

3. The majority of sequences on the SAT are not arithmetic or geometric sequences. They are sequences, though, because they have a pattern:

–4, 2, –3, –4, 2, –3, –4, ...	*Repeats the terms –4, 2, –3*
1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, ...	*Adds and then subtracts 1*
1, 1, 2, 3, 5, 8, 13, 21, ...	*Terms are the sum of the two previous terms*

Data Analysis Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 450.

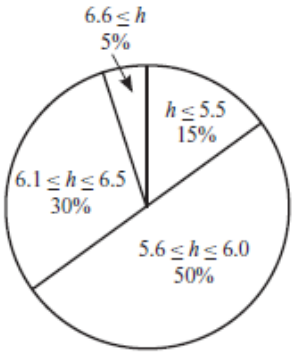
APPROXIMATE CONVERSIONS

Number of Gallons	2	4	x
Number of Liters	7.6	15.2	30.4

1. The table above shows approximate conversions from gallons to liters. What is the value of x ?

   (A) 6
   (B) 8
   (C) 10
   (D) 12
   (E) 16

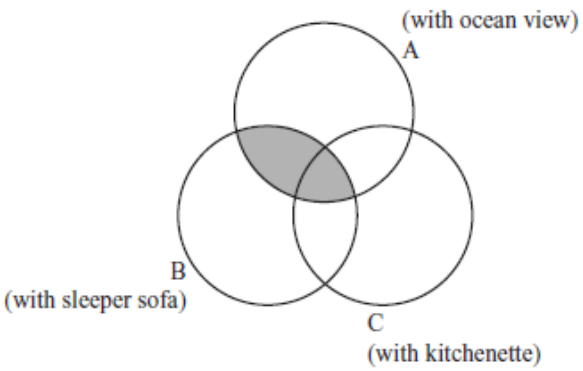
Height of Seniors



2. The seniors at Woodhaven High School are being measured for their caps and gowns for graduation. The figure above shows their height (h), in feet. For example, 30% of the seniors are 6.1 feet to 6.5 feet tall. If there are 760 seniors at Woodhaven High School, how many are 6.0 feet tall or less?

   (A) 114
   (B) 228
   (C) 380
   (D) 494
   (E) 722

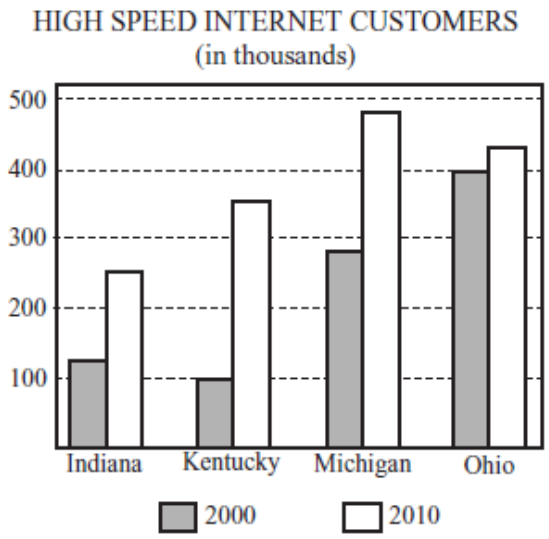
CONDOMINIUMS



3. In the figure above, three circles represent condominiums on the beach. Circle A represents condominiums with an ocean view, Circle B represents condominiums with a sleeper sofa, and Circle C represents condominiums with a kitchenette. What does the shaded region represent?

   (A) Condominiums with an ocean view, sleeper sofa, and kitchenette
   (B) Condominiums with an ocean view and sleeper sofa, but without kitchenettes
   (C) Condominiums with an ocean view and sleeper sofa (some possibly with kitchenettes)
   (D) Condominiums with an ocean view and kitchenette (some possibly with a sleeper sofa)
   (E) Condominiums with a sleeper sofa and kitchenette (some possibly with an ocean view)

Questions 4 and 5 refer to the following figure:



4. The table above shows a cable company's high speed internet customers in four states in 2000 and 2010. The number of customers in Kentucky in 2000 was approximately what percent of the number of customers in Ohio in 2000?

(A) 15%
(B) 20%
(C) 25%
(D) 30%
(E) 35%

---

5. From 2000 to 2010, the total number of high speed internet customers in the four states was increased by approximately what percent?

(A) 25%
(B) 33%
(C) 50%
(D) 58%
(E) 67%

Color	Number of People Choosing Color
Blue	176
Green	x
Red	97
Yellow	y

6. In a survey, 500 people were asked to choose their favorite color among blue, green, red, and yellow. Each person chose exactly one color. The results of the survey are given in the table above. If x and y are positive integers, what is the greatest possible value of x ?

(A) 77
(B) 114
(C) 226
(D) 227
(E) 500

Average, Median, and Mode Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 452.

1. Which answer choice contains a set of numbers in which the median is greater than the average (arithmetic mean)?

   (A) {3, 4, 5, 6, 7}
   (B) {3, 4, 5, 6, 8}
   (C) {3, 5, 5, 5, 7}
   (D) {–2, 4, 5, 6, 7}
   (E) {–2, 4, 5, 6, 12}

2. The sum of five consecutive even integers, a, b, c, d, and e, respectively, is 50. Which of the following is equal to the median of the set?

   (A) $\frac{a+b+c+d+e}{30}$

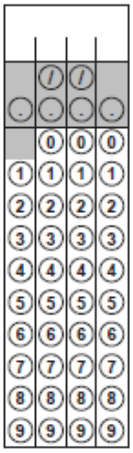
   (B) $\frac{30}{c}$

   (C) $b+2$

   (D) $e-a$

   (E) $\frac{b+d}{5}$

3. Eight consecutive odd integers are arranged in ascending order, from smallest to largest. The sum of the last four integers is 232. What is the sum of the first four integers?



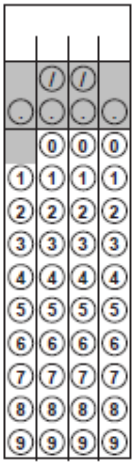
4. Which of the following answer choices is equal to the sum of three consecutive odd integers?

   (A) 153
   (B) 154
   (C) 155
   (D) 156
   (E) 157

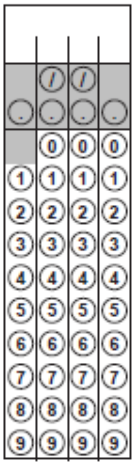
Average, Median, and Mode Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 452.

5. In 7 days, Mario cooked 98 pounds of spaghetti. Each day after the first, he cooked 2 more pounds than he cooked than the day before. What is the difference between the average (arithmetic mean) number of pounds of spaghetti he cooked per day and the median number of pounds he cooked during the 7 days?



6. The average (arithmetic mean) of five different positive integers is 30. What is the greatest possible value of one of these integers?



7. Five numbers, x, $2x$, $2x + 6$, $3x - 1$ and $4x - 8$, are in a set. If the average (arithmetic mean) of the five numbers is 9, what is the value of the mode in this set?

Counting Problems Problem Set

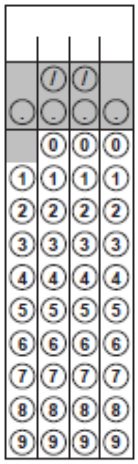
Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 455.

3, 4, 5, 8, 9

1. Two different numbers are selected from the list above and their product is determined. How many different pairs of numbers with a product greater than 30 can be selected?

   (A) 5
   (B) 6
   (C) 7
   (D) 8
   (E) 9

2. A restaurant is offering a new buffet with six types of sandwiches, four sides, and five desserts. If customers are allowed to select one sandwich, one side, and one dessert, how many meal combinations are possible?



3. A hot dog vendor offers three choices of condiments: mustard, ketchup, and horseradish. If a customer can select one, two, or all three condiments, how many different combinations of condiments are possible?

   (A) 5
   (B) 6
   (C) 7
   (D) 8
   (E) 9

4. Five dogs are in a dog show. They are to be lined up in a single row, and the dog with the most ribbons is to be placed in the first position. The two dogs with the fewest ribbons are to be placed in the last two positions. If none of the dogs have the same amount of ribbons, how many different arrangements of dogs are possible?

Counting Problems Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 455.

5. Robbie has to schedule five different meetings during the five day work week. If exactly one meeting is held each day, how many different arrangements of meetings are possible for the five day work week?

6. The sum of the first 50 consecutive positive even integers is x and the sum of the first 50 consecutive positive integers is y. What is x in terms of y?

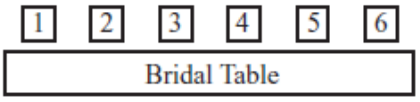
(A) $2y^2$

(B) y^2

(C) $2y$

(D) $\frac{2}{y}$

(E) $\frac{y}{2}$



7. How many different ways can 6 people arrange themselves in the 6 seats at a bridal party table shown above if the bride and groom must be sitting in the two center seats?

Probability Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 458.

1. There are 1096 marbles in a bag. One of the marbles is to be randomly chosen from the bag. If the probability that a red marble will be selected is $\frac{5}{8}$, how many red marbles are in the bag?

NUMBER OF CARS AT WALKER MOTORS

	Coupe	Sedan
Mileage 20,000 or less	10	30
Mileage over 20,000	30	50

2. The table above shows the number of used cars on the lot of Walker Motors. They have been classified by their mileage and style (coupe or sedan). If a sedan is to be randomly selected, what is the probability that the car's mileage is 20,000 miles or less?

(A) $\frac{1}{4}$

(B) $\frac{3}{8}$

(C) $\frac{1}{2}$

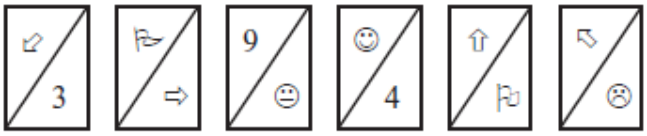
(D) $\frac{5}{8}$

(E) $\frac{3}{4}$

3. A negative even integer x is randomly chosen from the negative integers greater than or equal to –20. What is the probability that $2x + 10 > -10$?

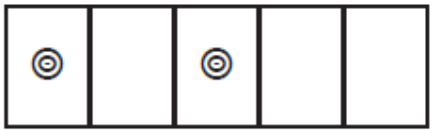
Probability Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 458.



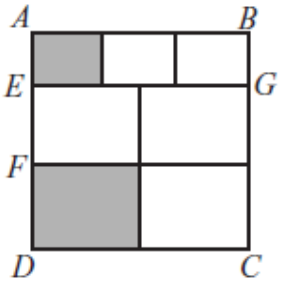
4. The six cards above are laid face down on a table. If one is to be picked at random, which of the following types of cards has the greatest probability of being chosen?

(A) A card with an arrow
(B) A card with a flag
(C) A card with a number
(D) A card with a face
(E) A card with both an arrow and a flag



5. The figure above shows five lockers. Five students, including Jud and Remy, will be randomly assigned one of the lockers, one student per locker. What is the probability that Jud and Remy will each be given a locker marked by a bullseye?

(A) $\frac{1}{25}$
(B) $\frac{1}{20}$
(C) $\frac{2}{25}$
(D) $\frac{1}{10}$
(E) $\frac{1}{5}$



6. In the figure above, $ABCD$ is a square with an area of 576. Line segment AE is one-fourth of AD, and $ABGE$ is divided into three equal rectangles. Line segment FD is one-half of ED, and $EGCD$ is divided into 4 equal rectangles. If a point is randomly chosen from $ABCD$, what is the probability that the point will be from a shaded region?

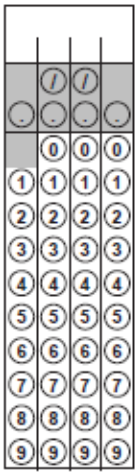
(A) $\frac{1}{12}$
(B) $\frac{3}{16}$
(C) $\frac{11}{24}$
(D) $\frac{7}{36}$
(E) $\frac{13}{48}$

Sequences Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 460.

$t, 5t, \ldots$

1. In the sequence above, the first term is t. Each term after the first is 5 times the preceding term and the sum of the first four terms is 936. What is the value of t ?

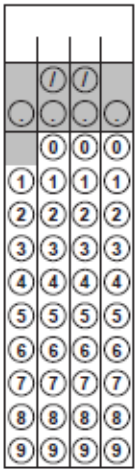


2. In a sequence of positive integers, the ratio of each term to the term immediately following it is 1 to 4. What is the ratio of the 2nd term to the 5th term?

(A) 1 to 16
(B) 1 to 32
(C) 1 to 64
(D) 1 to 128
(E) 1 to 256

$-3, -1, 0, 1, 5$

3. In the sequence above, the first 5 numbers repeat continuously. What is the sum of the first 30 numbers of this sequence?



$-4, 4, 0, \ldots$

4. In the sequence above, the first term is -4. Each even-numbered term is -1 times the previous and each odd-numbered term, after the first, is 4 less than the previous term. For example, the second term is -4×-1 and the third term is $4 - 4$. What is the 45th term of the sequence?

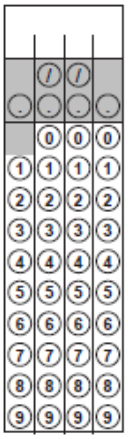
(A) -8
(B) -4
(C) 0
(D) 4
(E) 8

Overlapping Groups Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers are on page 462.

1. On a game show, there are 100 sealed boxes. Each box contains dollar bills only, coins only, or both dollar bills and coins. If 76 of the boxes contain dollar bills and 52 of the boxes contain coins, how many contain both dollar bills and coins?

2. At a clothing store, 35 shirts have stripes, 12 shirts have polka dots, and 5 shirts have both stripes and polka dots. If 63 shirts have neither stripes nor polka dots, how many total shirts are in the clothing store?

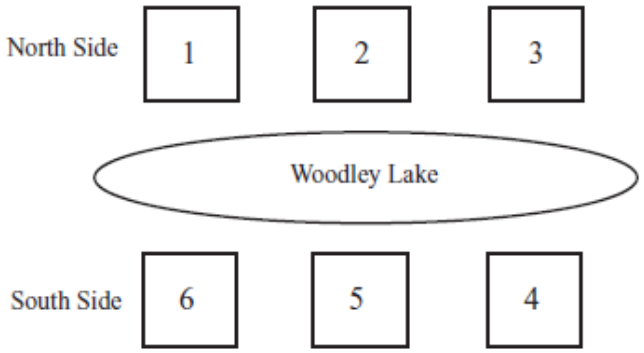


Logical Reasoning Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 462.

1. Raj just bought a pet from a pet store that only sells birds and snakes. Of the following, which must be true?

   (A) The pet is a bird.
   (B) The pet is a snake.
   (C) The pet is not a yellow bird.
   (D) The pet is not a brown dog.
   (E) The pet is not a snake with fangs.



2. In the diagram above, six cabins are shown on Woodley Lake. Three of the cabins are on the north side of the lake, and three cabins are on the south side, each one directly across from another cabin, as shown above. Five people—Ron, Sue, Tom, Val, and Will—are each assigned to one of the cabins given the following conditions:

   - One cabin will remain unoccupied.
   - Tom and Val will be assigned to cabins on the north side of the lake. Val's cabin is next to Tom's cabin but no other cabin.
   - Sue will be in cabin 5
   - Ron and Will will be in cabins on opposite sides of the lake, directly across from each other.

   If Val is assigned to cabin 1, who among the following could be assigned to cabin 3?

   I. Ron
   II. Tom
   III. Will

   (A) I only
   (B) III only
   (C) I and III only
   (D) II and III only
   (E) I, II, and III