Objectives

Students will be able to recognize and work with ratios, proportions, and geometric means.

Students will be able to understand and derive the means-extremes products theorem and the means-extremes ratio theorem.

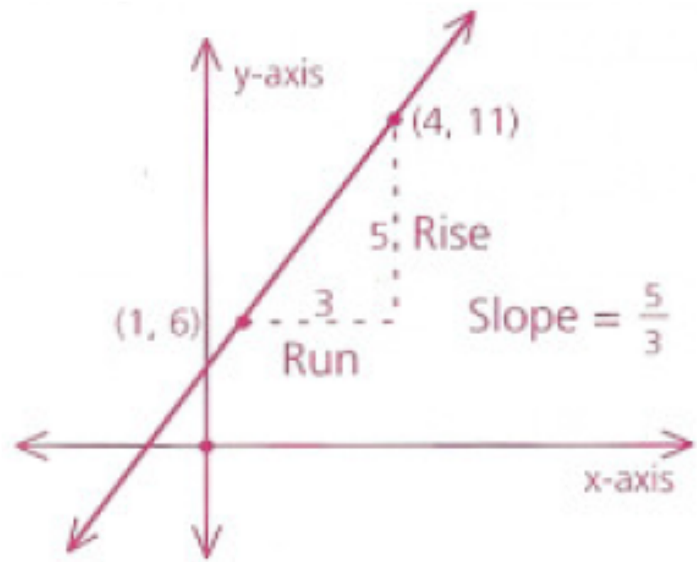
Ratio

A <u>ratio</u> is a quotient of two numbers.

The ratio of 5 meters to 3 meters can be written in any of the following ways:

5/3 5:3 5 to 3 5 ÷ 3

The slope of a line is the ratio of the rise between any two points on the line to the run between the two points.



always reduce ratios!!!

Proportion

A <u>proportion</u> is an equation stating that two or more ratios are equal.

Examples: $\frac{1}{2}=\frac{5}{10}$ $\quad 5:15=15:45 \quad$ $\frac{4}{6}=\frac{10}{15}=\frac{x}{y}=\frac{2}{3}$

Most are written as

$$\text{first term} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{third term}$$

$$\text{second term} \rightarrow b \qquad d \leftarrow \text{fourth term}$$

or $a:b=c:d$

a is the first term
b is the second term
c is the third term
d is the fourth term

Find the ratio of x to y.

Goal: Isolate x/y

1) $3x = 8y$

$$\frac{x}{y} = \frac{8}{3}$$

2) $2(x + 6) = 3(5y + 4)$

$$\frac{x}{y} = \frac{15}{2}$$

The Product and Ratio Theorems

$$\text{first term} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{third term}$$

$$\text{second term} \rightarrow \quad \leftarrow \text{fourth term}$$

the first and fourth terms are the extremes
the second and third terms are the means

Means-Extremes Products Theorem:

In a proportion, the product of the means is equal to the product of the extremes

*cross multiplication

If $\frac{a}{b} = \frac{c}{d}$ then, $ad = bc$

product of extremes = product of means

Examples

1) $\frac{7}{3} = \frac{x}{10}$

x = 70/3

2) $\frac{y}{20} = \frac{5}{y}$

y = 10, -10

3) $\frac{8}{m-3} = \frac{10}{7}$

m = 43/5

4) Find the 4th term (or 4th proportional) of a proportion if the first 3 terms are 2, 5, and 6.

x = 15

Means-Extremes Ratio Theorem

If the product of a pair of nonzero numbers is equal to the product of another pair of nonzero numbers, then either pair of numbers may be made the extremes, and the other pair the means, of a proportion.

Example: Given that $pq = rs$ we can create

proportions such as $\frac{p}{r} = \frac{s}{q}$, $\frac{p}{s} = \frac{r}{q}$, and $\frac{r}{p} = \frac{q}{s}$.

All these proportions are equivalent forms, since multiplying them out yields equivalent equations.

Geometric Mean

If the means in a proportion are equal, either mean is called a <u>geometric mean</u>, or <u>mean proportional</u>, between the extremes.

The means are the same.

Example: $\frac{1}{4} = \frac{4}{16}$ $\frac{a}{x} = \frac{x}{r}$

4 is a geometric mean between 1 and 16. What is the mean proportional in the second example?

<u>Arithmetic mean</u>- the average of numbers (another kind of mean between the numbers)

Find the geometric and arithmetic means between 3 and 27.

Arithmetic Mean:

Average = $\frac{3+27}{2}$

$= 15$

Geometric Mean:

Write a proportion, using 3 and 27 as the extremes and x as both mean.

$$\frac{3}{x} = \frac{x}{27}$$

$$x^2 = 81$$

$$x = \pm 9$$

Find the mean proportional(s) between 4 and 16.

$$\frac{4}{x} = \frac{x}{16} \qquad x^2 = 64 \qquad x = \pm 8$$

Find the mean proportional(s) between 6 and 8.

$$\frac{6}{x} = \frac{x}{8} \qquad x^2 = 48 \qquad x = \pm 4\sqrt{3}$$

Homework p. 329: 3, 6, 11, 15, 17, 19, 20

3 Solve each proportion for x.

a $\frac{3}{x} = \frac{12}{16}$ **b** $\frac{x}{18} = \frac{3}{7}$ **c** $\frac{7}{x - 4} = \frac{3}{5}$

6 Find the ratio of x to y if

a $2x = 3y$ **b** $6(y + 3) = 2(x + 9)$ **c** $\frac{3}{x + 5} = \frac{9}{y + 15}$

11 Find the geometric mean(s) between each pair of extremes.

a 4 and 25 **b** 3 and 5 **c** a and b

15 If 4 is a mean proportional between 6 and a number, what is the number?

17 The ratio of the measure of the supplement of an angle to the measure of the complement of the angle is 5:2. Find the measure of the supplement.

19 If $x(a + b) = y(c + d)$, find the ratio of x to y.

20 If $ex - fy = gx + hy$, find the ratio of x to y.

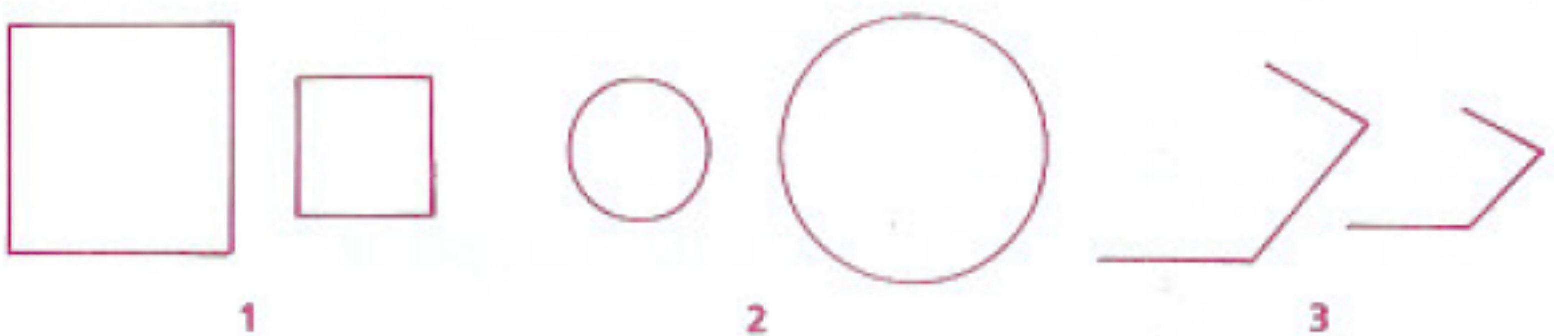
Objective

Students will be able to identify the characteristics of similar figures.

Similar Figures

Similar figures have the same shape but not necessarily the same size.

- Dilation (enlargement) or Reduction
- Symbol: ~

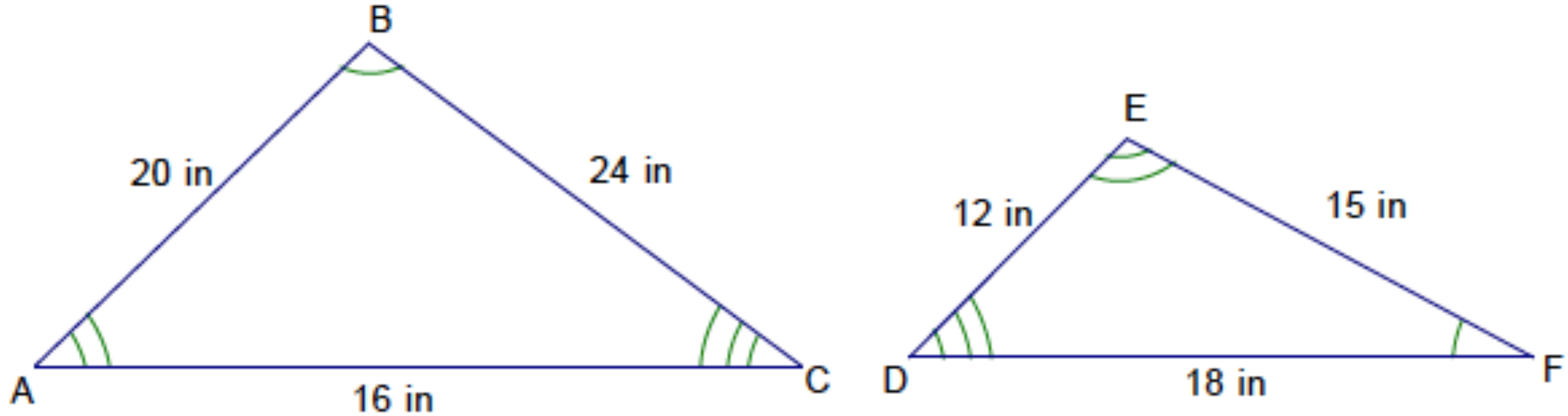


Similar Polygons

In order for two polygons to be similar, they must meet the following criteria:

1) The ratio of the measures of ALL corresponding sides are equal (proportional corresponding sides)
2) ALL corresponding angles are congruent

The ratio of the corresponding sides is called the <u>similarity ratio</u>



In order to say that $\triangle ABC \sim \triangle EFD$, we need to have:

1) Similarity Ratio

$$\frac{20}{15}=\frac{24}{18}=\frac{16}{12} \qquad \frac{4}{3}=\frac{4}{3}=\frac{4}{3} \quad \checkmark$$

2) Congruent Angles

$$\angle A \cong \angle E;\ \angle B \cong \angle F;\ \angle C \cong \angle D \quad \checkmark$$

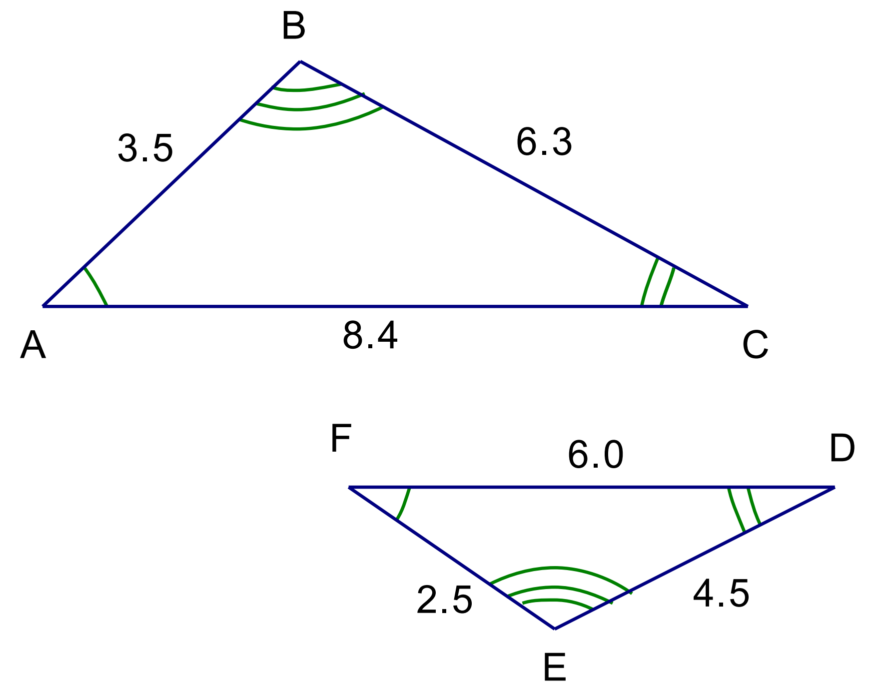
$\triangle ABC \sim \triangle EFD$

Are the polygons similar?

If they are, write a similarity statement and give the similarity ratio. If they are not, explain.

$$\frac{FE}{AB} = \frac{DE}{CB} = \frac{FD}{AC}$$

$$\frac{2.5}{3.5} = \frac{4.5}{6.3} = \frac{6.0}{8.4}$$



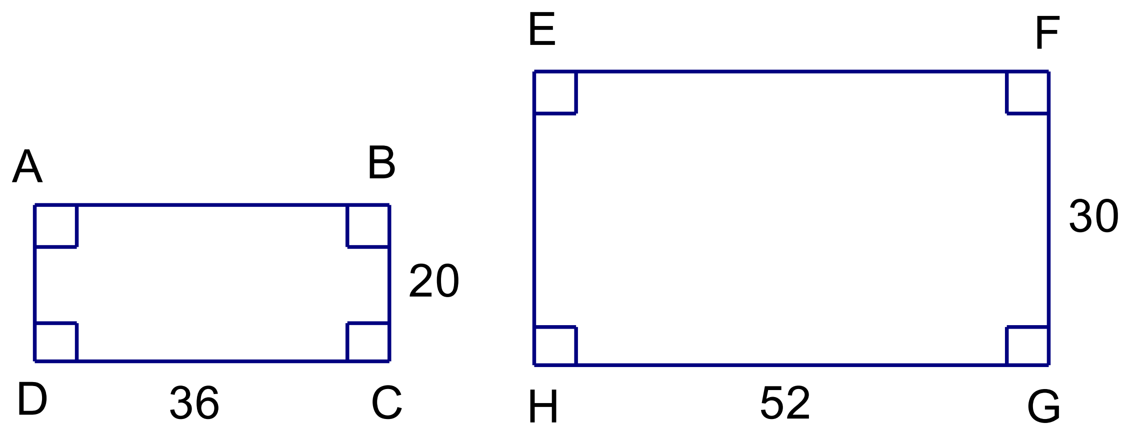
Ratios are equal

Angles are congruent

$\triangle ABC \sim \triangle FED$

Are the polygons similar?

If they are, write a similarity statement and give the similarity ratio. If they are not, explain.



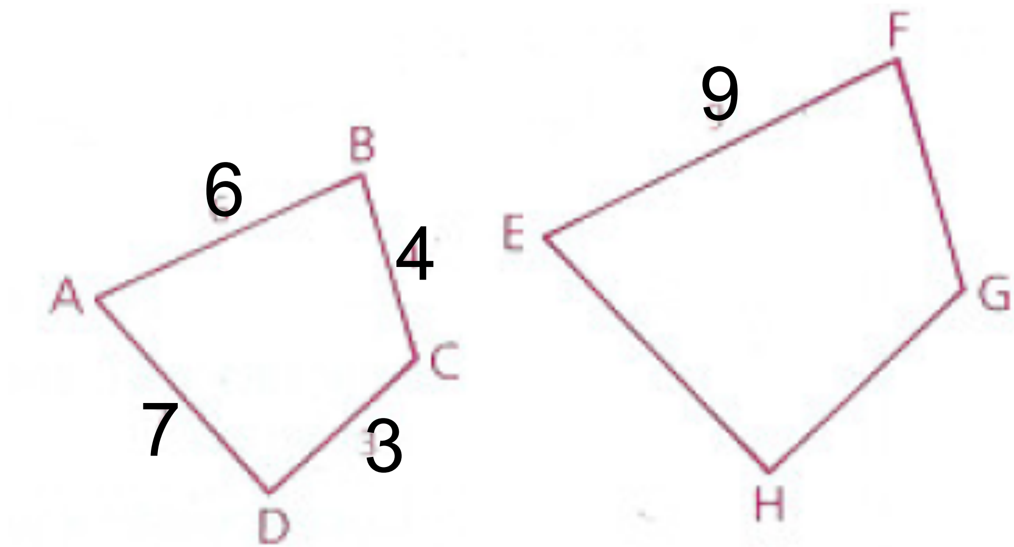
$$\frac{AD}{EH} = \frac{AB}{EF} = \frac{BC}{FG} = \frac{DC}{HG} \quad \frac{20}{30} = \frac{36}{52} = \frac{20}{30} = \frac{36}{52} \quad \frac{2}{3} = \frac{9}{13} = \frac{2}{3} = \frac{9}{13}$$

ratio of sides are not equal

Therefore, they are not similar

Given ABCD ~ EFGH, with measures as shown.

$$\frac{AB}{EF} = \frac{6}{9} = \frac{2}{3}$$



1) Find FG, GH, and EH

To find FG: $\frac{AB}{EF} = \frac{BC}{FG}$ FG = 6 GH = 4.5 EH = 10.5

2) Find the ratio of the perimeter of ABCD to the perimeter of EFGH

ratio – 2/3

perimeter of ABCD = 20

perimeter of EFGH = 30

Theorem 61: The ratio of the perimeters of two similar polygons equals the ratio of any pair of corresponding sides.

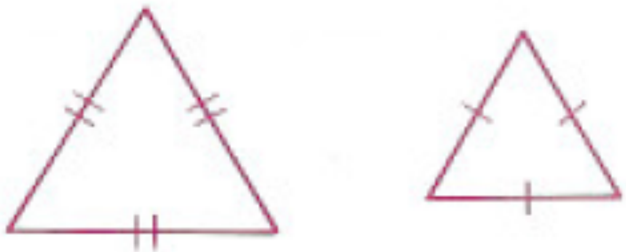
Homework

p.336: 2, 3, 5, 10, 11, 14

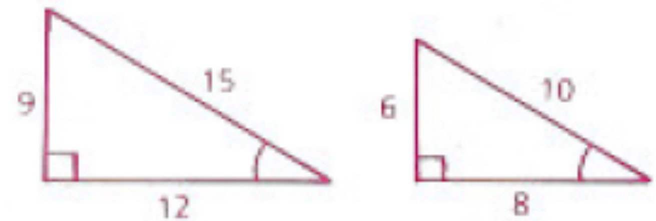
2 Which pairs of polygons *can be proved* to be similar?

a

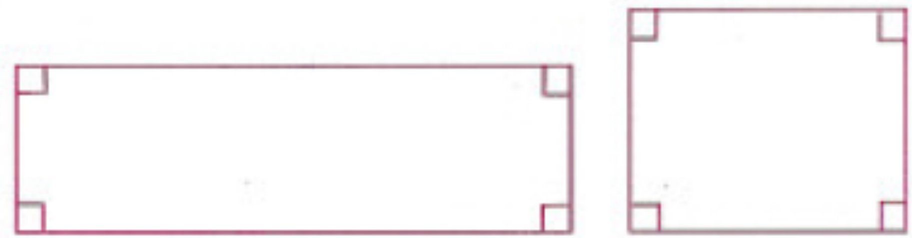
b



c

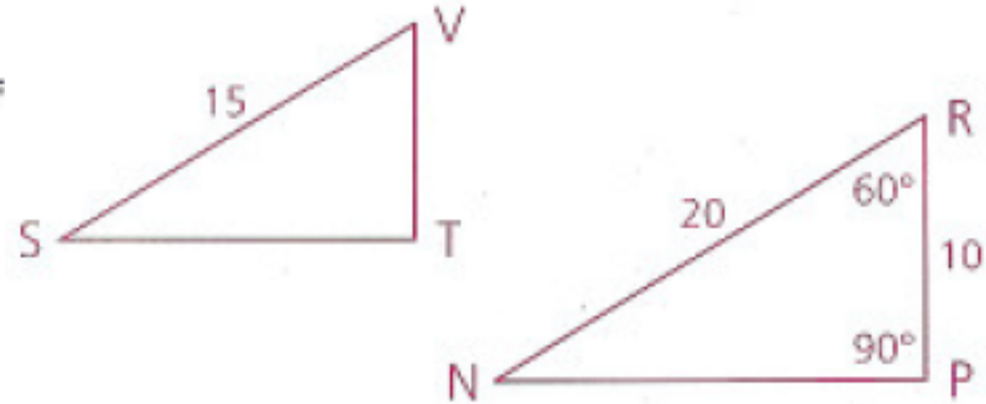


d



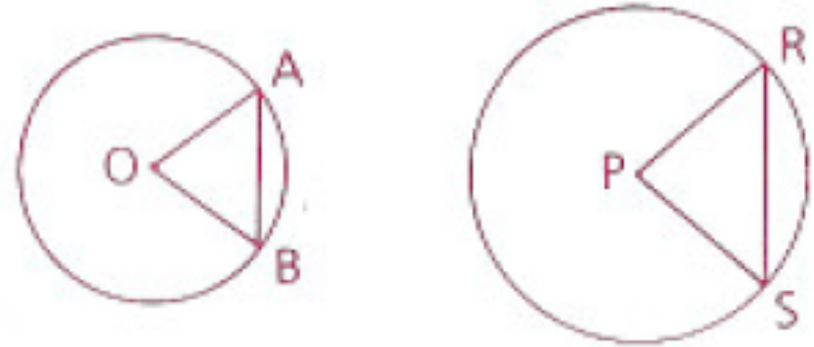
3 Given: $\triangle NPR \sim \triangle STV$,
$m\angle P = 90$, $m\angle R = 60$,
$SV = 15$, $NR = 20$, $RP =$

Find: $m\angle T$, $m\angle S$, and VT



5 Given: $\odot O$, $\odot P$, $\triangle AOB \sim \triangle RPS$,
$OA = 2$, $AB = 3$, $PR = 6$

Find: PS and RS



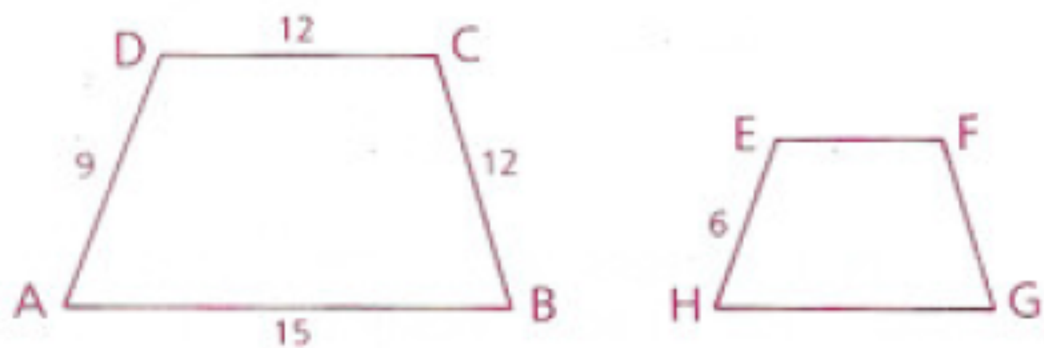
10 Given: Quad ABCD ~ quad HGFE, with measures as shown

Find: **a** The ratio of lengths of corresponding sides

b EF

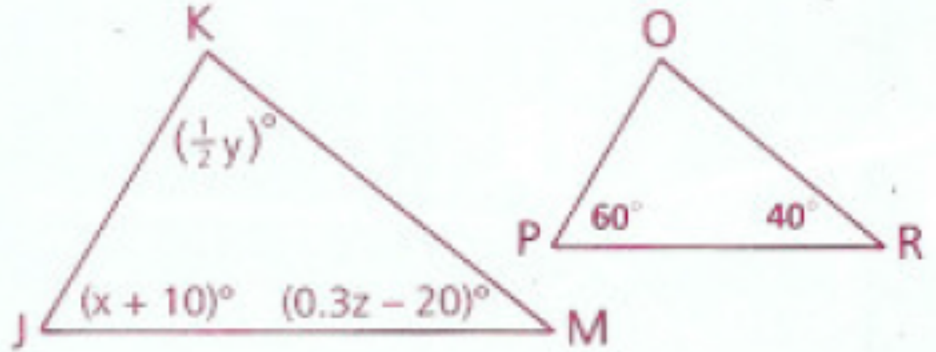
c The perimeter of EFGH

d The ratio of the perimeters

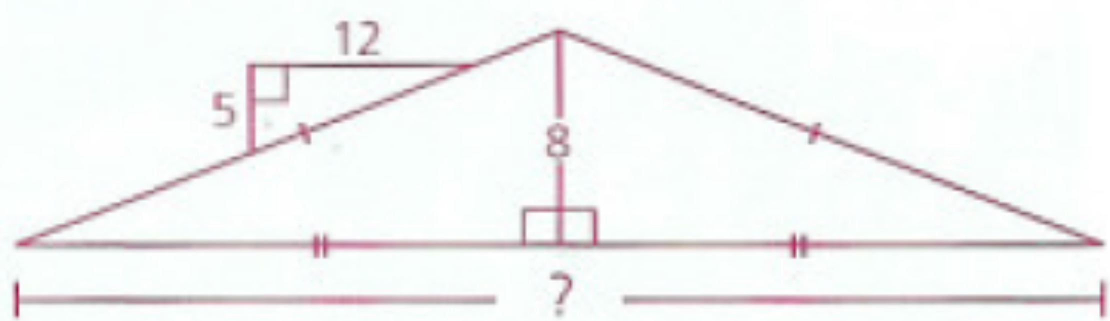


11 Given: $\triangle KJM \sim \triangle OPR$,
with angles as shown

Find: $\dfrac{x + y + z}{2}$



14 The roof of a house has a slope of $\frac{5}{12}$.
What is the width of the house if the height of the roof is 8 ft?



Objective

Students will be able to use several methods to prove that triangles are similar.

Theorem: If two pairs of corresponding angles are congruent, then the triangles are similar (AA~)

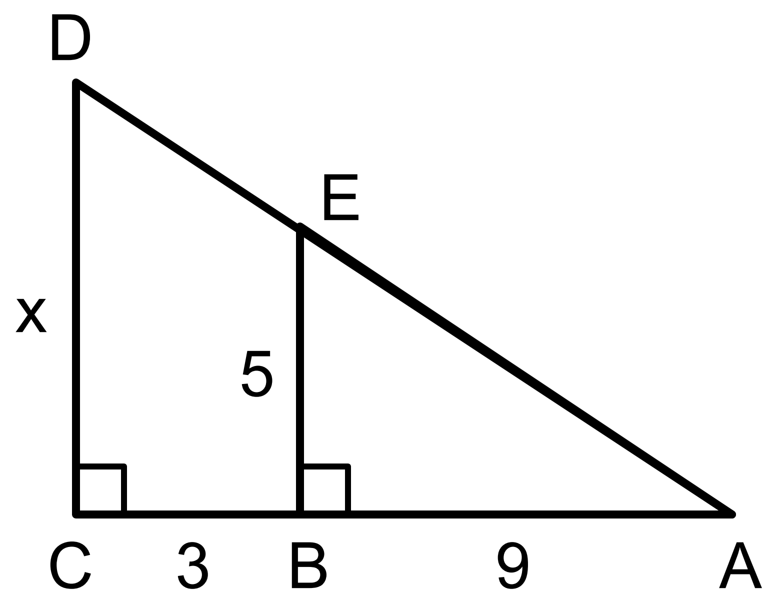
Explain why $\triangle ABE \sim \triangle ACD$. Then find CD.

$\angle A \cong \angle A$ by reflexive property

$\angle B \cong \angle C$ right angles are congruent

$\triangle ABE \sim \triangle ACD$ by AA~

$\frac{AB}{AC} = \frac{BE}{CD} \quad \frac{9}{12} = \frac{5}{CD} \quad CD = \frac{20}{3}$



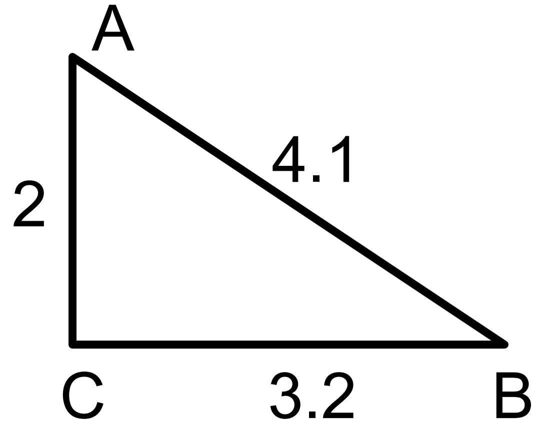
Theorem: If all three pairs of corresponding sides of two triangles have the same ratio, then the triangles are similar (SSS~)

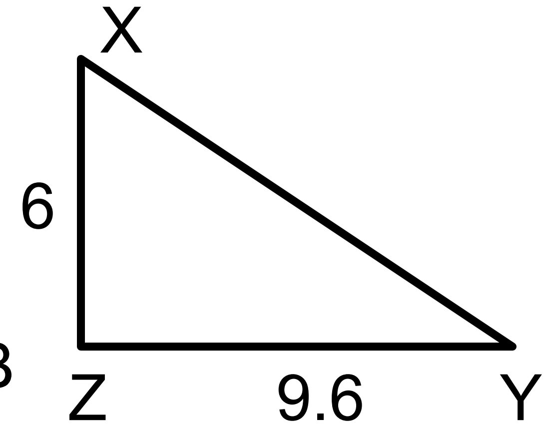
What does XY have to equal in order to prove $\triangle ABC \sim \triangle XYZ$?

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

$$\frac{4.1}{XY} = \frac{3.2}{9.6} = \frac{2}{6}$$

$$\frac{4.1}{XY} = \frac{1}{3} = \frac{1}{3}$$

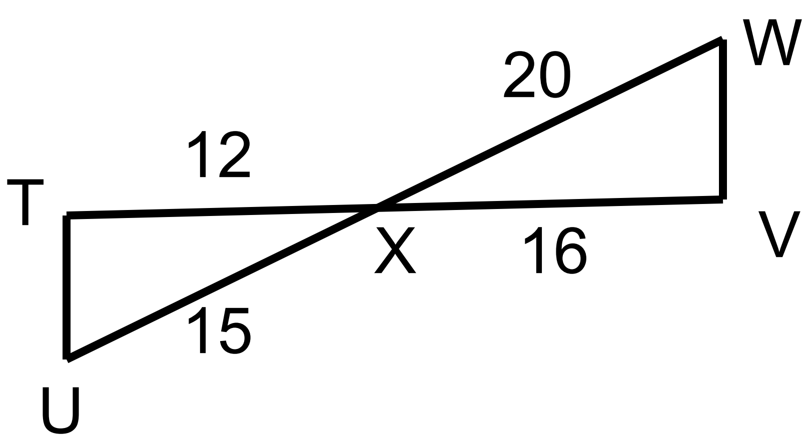




$$XY = 12.3$$

Theorem: If two pairs of corresponding sides of two triangles have the same ratio AND the included angles are congruent, then the triangles are similar (SAS~)

Explain why $\triangle TXU \sim \triangle VXW$.



$\angle TXU \cong \angle VXW$ b/c vertical angles are congruent

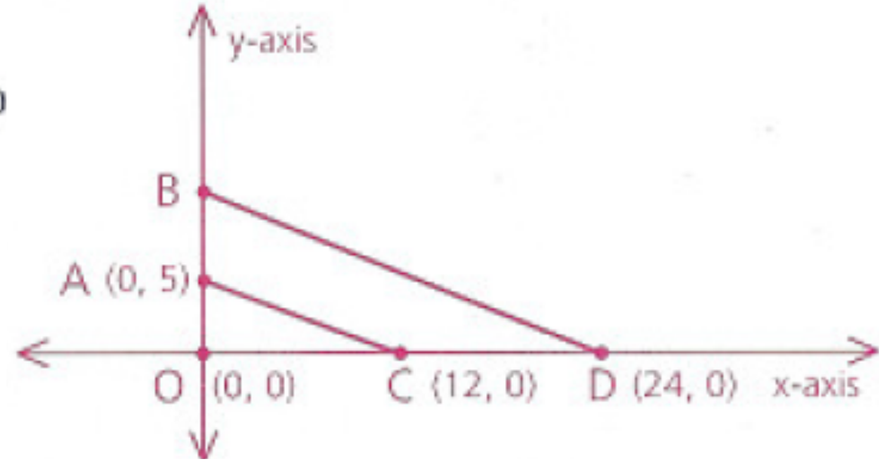
$$\frac{TX}{XV} = \frac{XW}{XU} \qquad \frac{12}{16} = \frac{15}{20}$$

$$\frac{3}{4} = \frac{3}{4}$$

Same similarity ratio and angle inside those sides is congruent so $\triangle TXU \sim \triangle VXW$ by SAS~

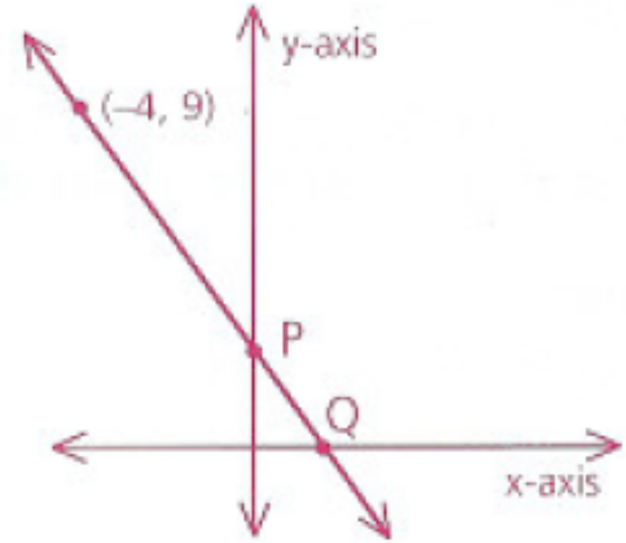
Homework p.341: 6, 8, 10, 12, 19

6 Find the coordinates of B if $\triangle OAC \sim \triangle OBD$. Then write a paragraph proof to show that $\triangle OAC \sim \triangle OBD$. Challenge: Can you find the length of $\overline{BD}$?



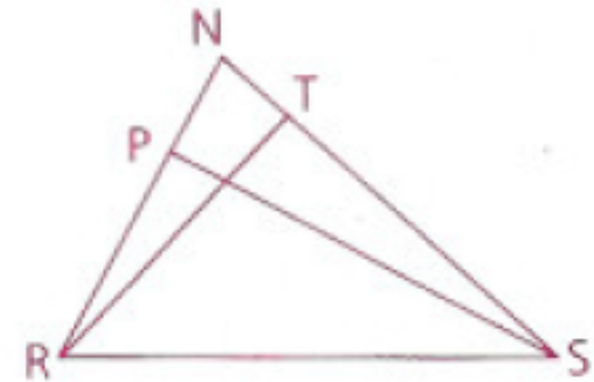
8 In $\triangle FGH$, $FG = 6$, $GH = 8$, and $FH = 12$. $\triangle FGH$ is projected onto a wall, and the image, $\triangle F'G'H'$, has sides $F'G' = 15$, $G'H' = 20$, and $F'H' = 30$. Is $\triangle FGH$ similar to $\triangle F'G'H'$? Explain.

10 The slope of line PQ is $-\frac{3}{2}$. Find the coordinates of P and Q.



12 Given: $\overline{SP}$ is the altitude from S to $\overline{NR}$.
$\overline{RT}$ is the altitude from R to $\overline{NS}$.

Conclusion: $\triangle NRT \sim \triangle NSP$



19 Given: Figure as shown

a Is $\triangle PQT \sim \triangle PRS$? Justify your reasoning.

b Is $\overline{QT}$ parallel to $\overline{RS}$? Justify your reasoning.