

# Objectives

Students will be able to recognize and work with ratios, proportions, and geometric means.

Students will be able to understand and derive the means-extremes products theorem and the means-extremes ratio theorem.

# Ratio

A ratio is a quotient of two numbers.

The ratio of 5 meters to 3 meters can be written in any of the following ways:

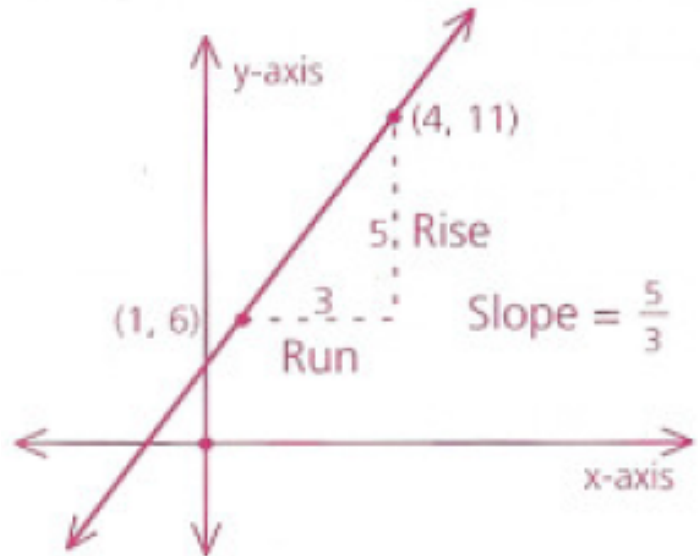
$$5/3$$

$$5:3$$

$$5 \text{ to } 3$$

$$5 \div 3$$

The slope of a line is the ratio of the rise between any two points on the line to the run between the two points.



**always reduce ratios!!!**

# Proportion

A proportion is an equation stating that two or more ratios are equal.

Examples:  $\frac{1}{2} = \frac{5}{10}$        $5:15 = 15:45$        $\frac{4}{6} = \frac{10}{15} = \frac{x}{y} = \frac{2}{3}$

Most are written as

$$\begin{array}{ccccc} \text{first term} & \rightarrow & a & = & c \leftarrow \text{third term} \\ & & \frac{\quad}{b} & = & \frac{\quad}{d} \leftarrow \text{fourth term} \end{array} \quad \text{or} \quad a:b = c:d$$

$a$  is the first term

$c$  is the third term

$b$  is the second term

$d$  is the fourth term

Find the ratio of  $x$  to  $y$ .

Goal: Isolate  $x/y$

$$1) 3x = 8y$$

$$\frac{x}{y} = \frac{8}{3}$$

$$2) 2(x + 6) = 3(5y + 4)$$

$$\frac{x}{y} = \frac{15}{2}$$

# The Product and Ratio Theorems

$$\begin{array}{ccccc} \text{first term} & \rightarrow & a & = & c \leftarrow \text{third term} \\ & & \underline{\quad} & & \underline{\quad} \\ \text{second term} & \rightarrow & b & = & d \leftarrow \text{fourth term} \end{array}$$

the first and fourth terms are the extremes  
the second and third terms are the means

## Means-Extremes Products Theorem:

In a proportion, the product of the means is equal to the product of the extremes

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then, } ad = bc$$

\*cross multiplication

product of extremes = product of means

# Examples

$$1) \quad \frac{7}{3} = \frac{x}{10}$$

$$x = 70/3$$

$$2) \quad \frac{y}{20} = \frac{5}{y}$$

$$y = 10, -10$$

$$3) \quad \frac{8}{m-3} = \frac{10}{7}$$

$$m = 43/5$$

4) Find the 4<sup>th</sup> term (or 4<sup>th</sup> proportional) of a proportion if the first 3 terms are 2, 5, and 6.

$$x = 15$$

# Means-Extremes Ratio Theorem

If the product of a pair of nonzero numbers is equal to the product of another pair of nonzero numbers, then either pair of numbers may be made the extremes, and the other pair the means, of a proportion.

Example: Given that  $pq = rs$  we can create

proportions such as  $\frac{p}{r} = \frac{s}{q}$ ,  $\frac{p}{s} = \frac{r}{q}$ , and  $\frac{r}{p} = \frac{q}{s}$ .

All these proportions are equivalent forms, since multiplying them out yields equivalent equations.

# Geometric Mean

If the means in a proportion are equal, either mean is called a geometric mean, or mean proportional, between the extremes.

The means are the same.

Example:  $\frac{1}{4} = \frac{4}{16}$        $\frac{a}{x} = \frac{x}{r}$

4 is a geometric mean between 1 and 16.

What is the mean proportional in the second example?

Arithmetic mean- the average of numbers (another kind of mean between the numbers)



Find the geometric and arithmetic means between 3 and 27.

Arithmetic Mean:

$$\begin{aligned}\text{Average} &= \frac{3 + 27}{2} \\ &= 15\end{aligned}$$

Geometric Mean:

Write a proportion, using 3 and 27 as the extremes and  $x$  as both mean.

$$\frac{3}{x} = \frac{x}{27}$$

$$x^2 = 81$$

$$x = \pm 9$$

Find the mean proportional(s)  
between 4 and 16.

$$\frac{4}{x} = \frac{x}{16} \quad x^2 = 64 \quad x = \pm 8$$

Find the mean proportional(s)  
between 6 and 8.

$$\frac{6}{x} = \frac{x}{8} \quad x^2 = 48 \quad x = \pm 4\sqrt{3}$$

# Homework

p. 329: 3, 6, 11, 15, 17, 19, 20

**3** Solve each proportion for  $x$ .

**a**  $\frac{3}{x} = \frac{12}{16}$

**b**  $\frac{x}{18} = \frac{3}{7}$

**c**  $\frac{7}{x-4} = \frac{3}{5}$

**6** Find the ratio of  $x$  to  $y$  if

**a**  $2x = 3y$

**b**  $6(y + 3) = 2(x + 9)$

**c**  $\frac{3}{x+5} = \frac{9}{y+15}$

**11** Find the geometric mean(s) between each pair of extremes.

**a** 4 and 25

**b** 3 and 5

**c**  $a$  and  $b$

**15** If 4 is a mean proportional between 6 and a number, what is the number?

**17** The ratio of the measure of the supplement of an angle to the measure of the complement of the angle is 5:2. Find the measure of the supplement.

**19** If  $x(a + b) = y(c + d)$ , find the ratio of  $x$  to  $y$ .

**20** If  $ex - fy = gx + hy$ , find the ratio of  $x$  to  $y$ .

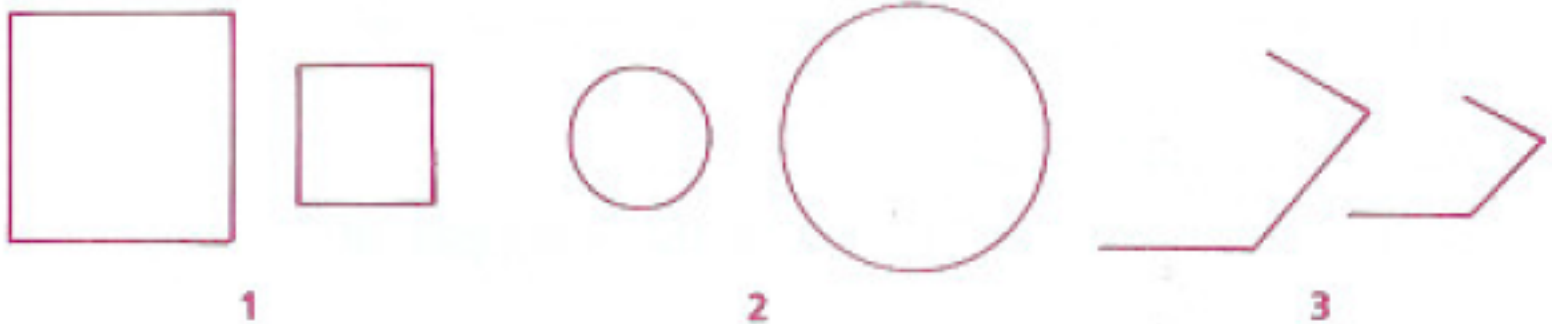
# Objective

Students will be able to identify the characteristics of similar figures.

# Similar Figures

Similar figures have the same shape but not necessarily the same size.

- Dilation (enlargement) or Reduction
- Symbol:  $\sim$

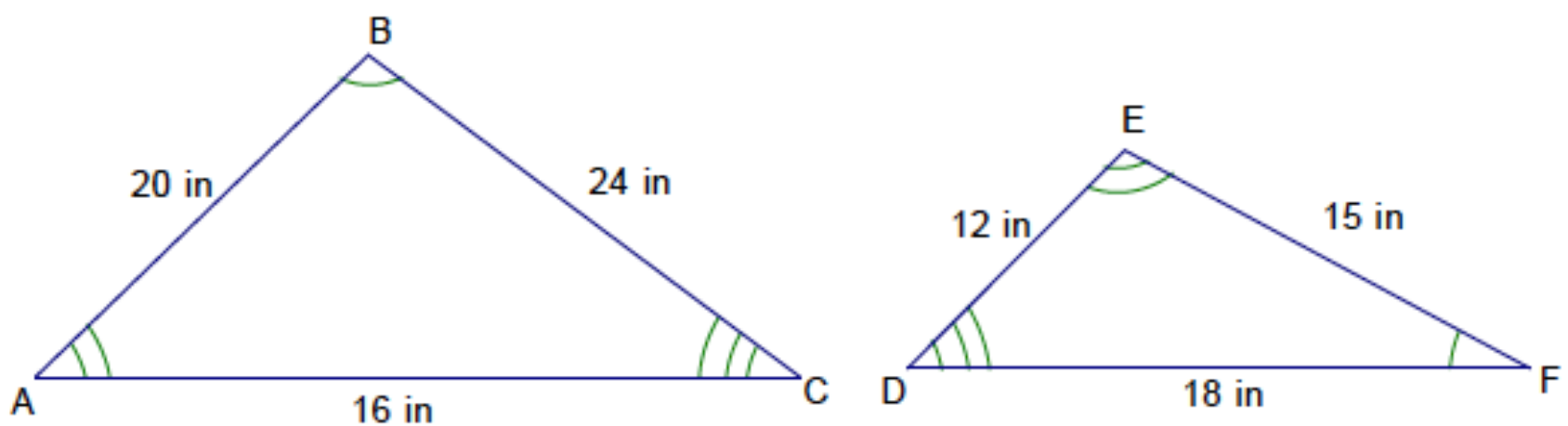


# Similar Polygons

In order for two polygons to be similar, they must meet the following criteria:

- 1) The ratio of the measures of ALL corresponding sides are equal (proportional corresponding sides)
- 2) ALL corresponding angles are congruent

The ratio of the corresponding sides is called the similarity ratio



In order to say that  $\triangle ABC \sim \triangle EFD$ , we need to have:

1) Similarity Ratio

$$\frac{20}{15} = \frac{24}{18} = \frac{16}{12}$$

$$\frac{4}{3} = \frac{4}{3} = \frac{4}{3}$$

✓

2) Congruent Angles

$$\angle A \cong \angle E; \angle B \cong \angle F; \angle C \cong \angle D$$

✓

$$\triangle ABC \sim \triangle EFD$$

# Are the polygons similar?

If they are, write a similarity statement and give the similarity ratio. If they are not, explain.

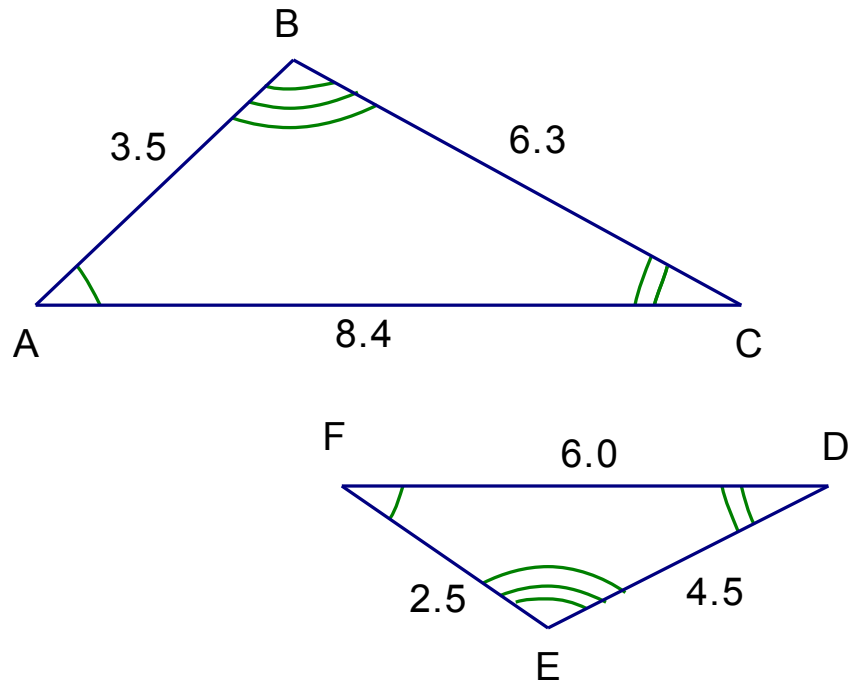
$$\frac{FE}{AB} = \frac{DE}{CB} = \frac{FD}{AC}$$

$$\frac{2.5}{3.5} = \frac{4.5}{6.3} = \frac{6.0}{8.4}$$

Ratios are equal

Angles are congruent

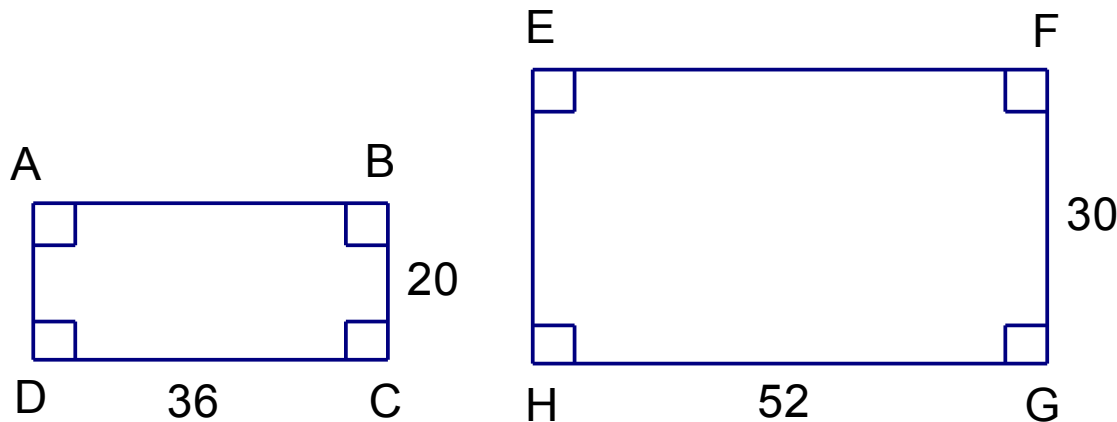
$$\triangle ABC \sim \triangle FED$$





# Are the polygons similar?

If they are, write a similarity statement and give the similarity ratio. If they are not, explain.



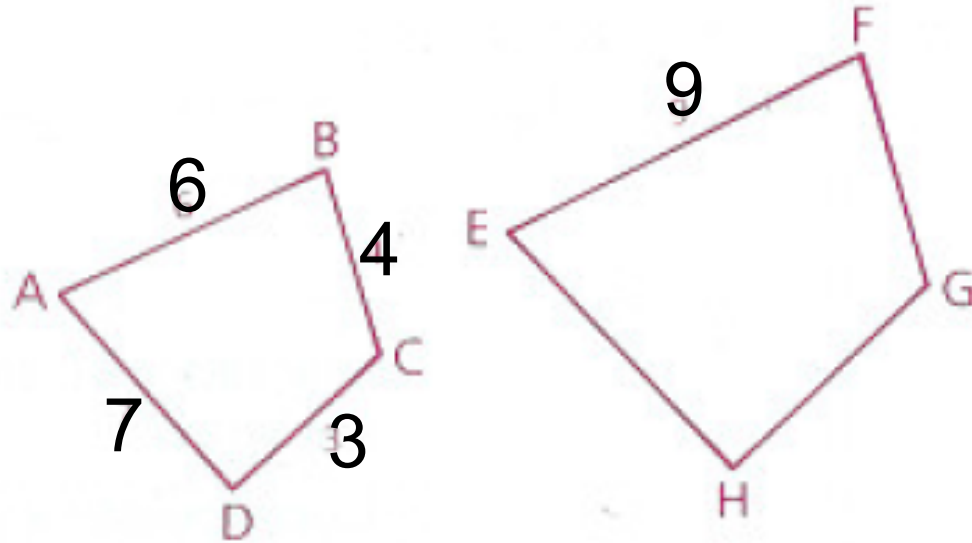
$$\frac{AD}{EH} = \frac{AB}{EF} = \frac{BC}{FG} = \frac{DC}{HG} \quad \frac{20}{30} = \frac{36}{52} = \frac{20}{30} = \frac{36}{52} \quad \frac{2}{3} = \frac{9}{13} = \frac{2}{3} = \frac{9}{13}$$

ratio of sides are  
not equal

Therefore, they are not similar

Given  $ABCD \sim EFGH$ , with measures as shown.

$$\frac{AB}{EF} = \frac{6}{9} = \frac{2}{3}$$



1) Find FG, GH, and EH

To find FG:  $\frac{AB}{EF} = \frac{BC}{FG}$        $FG = 6$        $GH = 4.5$        $EH = 10.5$

2) Find the ratio of the perimeter of ABCD to the perimeter of EFGH

perimeter of ABCD = 20

perimeter of EFGH = 30

ratio –  $\frac{2}{3}$

Theorem 61: The ratio of the perimeters of two similar polygons equals the ratio of any pair of corresponding sides.

# Homework

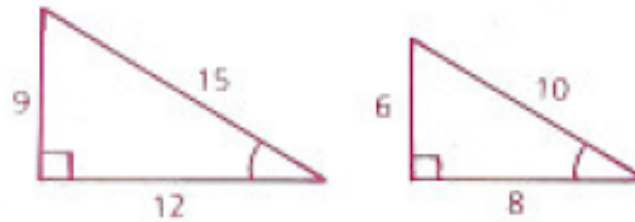
p.336: 2, 3, 5, 10, 11, 14

2 Which pairs of polygons can be proved to be similar?

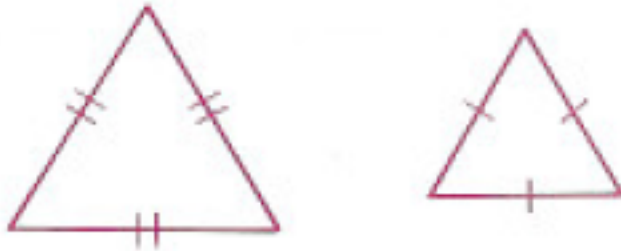
a



c



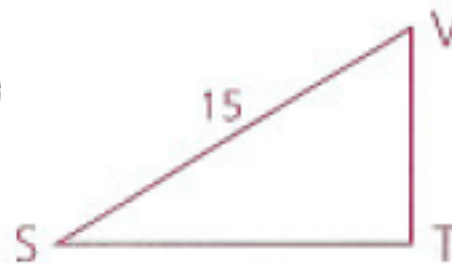
b



d

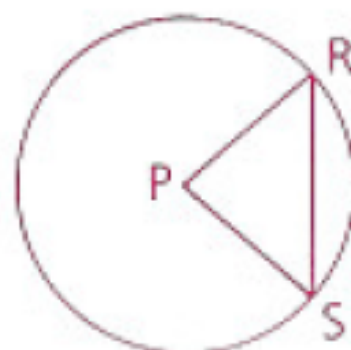


3 Given:  $\triangle NPR \sim \triangle STV$ ,  
 $m\angle P = 90$ ,  $m\angle R = 60$ ,  
 $SV = 15$ ,  $NR = 20$ ,  $RP =$   
Find:  $m\angle T$ ,  $m\angle S$ , and  $VT$



- 5 Given:  $\odot O$ ,  $\odot P$ ,  $\triangle AOB \sim \triangle RPS$ ,  
 $OA = 2$ ,  $AB = 3$ ,  $PR = 6$

Find:  $PS$  and  $RS$



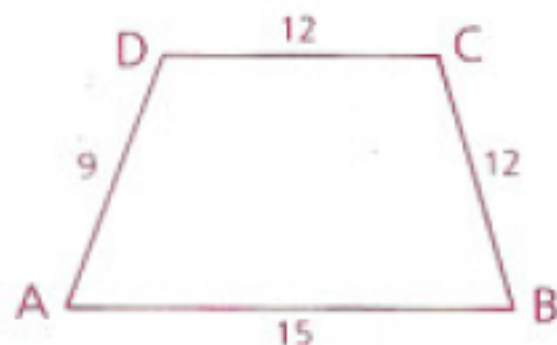
- 10 Given: Quad  $ABCD \sim$  quad  $HGFE$ ,  
with measures as shown

Find: **a** The ratio of lengths of corresponding sides

**b**  $EF$

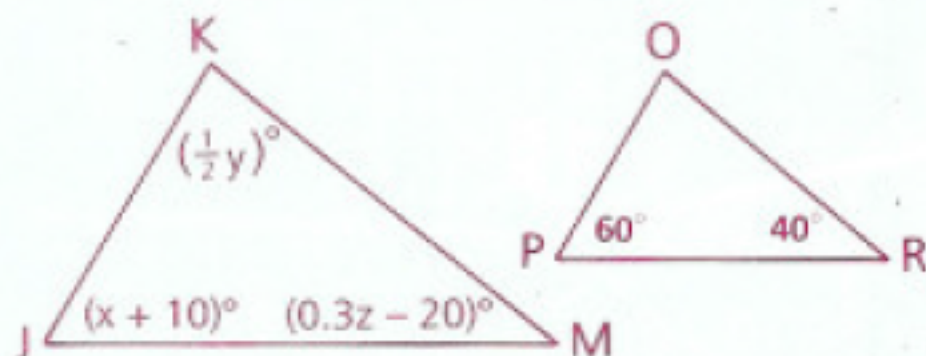
**c** The perimeter of  $EFGH$

**d** The ratio of the perimeters



- 11 Given:  $\triangle KJM \sim \triangle OPR$ ,  
with angles as shown

Find:  $\frac{x + y + z}{2}$



- 14 The roof of a house has a slope of  $\frac{5}{12}$ .  
What is the width of the house if the  
height of the roof is 8 ft?

