

Find the discriminant of the function, the number and types of solutions, and solve using the quadratic formula.

1) $y = 4x^2 + 4x + 1$

$b^2 - 4ac$

$4^2 - 4(4)(1) =$

$16 - 16 = 0$

one real solution

$$X = \frac{-4 \pm \sqrt{0}}{2(4)} = \frac{-4}{8} = -\frac{1}{2}$$

$$X = -\frac{1}{2}$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2) $y = 2x^2 + 6x + 3$

$6^2 - 4(2)(3)$

$36 - 24 = 12$

two real solutions

$$\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

$$X = \frac{-6 \pm \sqrt{12}}{2(2)} = \frac{-6 \pm 2\sqrt{3}}{4} =$$

$$X = \frac{-3 \pm \sqrt{3}}{2}$$

Convert each equation from standard form of the quadratic function into vertex form by completing the square.

3) $y = x^2 + 36x - 17$

$\left(\frac{36}{2}\right)^2 = 18^2 = 324$

$y + 17 = x^2 + 36x$

$y + 17 + 324 = x^2 + 36x + 324$

$y + 341 = (x + 18)^2$

$y = (x + 18)^2 - 341$

4) $y = 5x^2 - 70x + 163$

$\left(\frac{14}{2}\right)^2 = (7)^2 = 49$

$y - 163 = 5x^2 - 70x$

$y - 163 = 5(x^2 - 14x)$

$y - 163 + 5(49) = 5(x^2 - 14x + 49)$

$y + 82 = 5(x - 7)^2$

$y = 5(x - 7)^2 - 82$

Solve the following equations by completing the square.

5) $x^2 + 4x = 10$

$\left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$x^2 + 4x + 4 = 10 + 4$

$\sqrt{(x+2)^2} = \sqrt{14}$

$x + 2 = \pm \sqrt{14}$

$x = -2 \pm \sqrt{14}$

6) $4x^2 - 40x + 12 = 0$

$4x^2 - 40x = 12$

$4(x^2 - 10x) = 12$

$4(x^2 - 10x + 25) = 12 + 4(25)$

$4(x - 5)^2 = 112$

$\sqrt{(x-5)^2} = \sqrt{28}$

$x - 5 = \pm \sqrt{28}$

$x = 5 \pm 2\sqrt{7}$

Solve the following equations by using square roots.

7) $\frac{144}{25} = \frac{25x^2}{25}$

$\sqrt{x^2} = \sqrt{\frac{144}{25}}$

$x = \pm \sqrt{\frac{144}{25}}$

$x = \pm \frac{12}{5}$

8) $25 = x^2 - 8x + 16$

$\sqrt{25} = \sqrt{(x-4)^2}$

$\pm 5 = x - 4$

$+4 \quad +4$

$x = 4 \pm 5$

$x = 4 + 5 = 9$

$4 - 5 = -1$

$x = 9, -1$

9) $7(x - 4)^2 - 18 = 10$

$7(x - 4)^2 = 28$

$\sqrt{(x-4)^2} = \sqrt{4}$

$x - 4 = \pm 2$

$+4 \quad +4$

$x = 4 \pm 2$

$x = 4 + 2 = 6$

$x = 4 - 2 = 2$

$x = 6, 2$

Solve the following equations by factoring.

10) $144 = 25x^2$
 $-144 \quad -144$

$$25x^2 - 144 = 0$$

$$(5x-12)(5x+12) = 0$$

$$5x-12=0 \text{ or } 5x+12=0$$

$$5x=12 \text{ or } 5x=-12$$

$$x=12/5 \text{ or } x=-12/5$$

11) $25 = x^2 - 8x + 16$

$$x^2 - 8x - 9 = 0$$

$$(x-9)(x+1) = 0$$

$$x-9=0 \text{ or } x+1=0$$

$$x=9 \text{ or } x=-1$$

12) $7x^2 - 56x + 84 = 10$

$$7x^2 - 56x + 84 = 0$$

$$7(x^2 - 8x + 12) = 0$$

$$7(x-6)(x-2) = 0$$

$$x-6=0 \text{ or } x-2=0$$

$$x=6 \text{ or } x=2$$

13) $12x^2 + 5x + 5 = 7$

$$12x^2 + 5x - 2 = 0$$

$$12x^2 + 8x - 3x - 2 = 0$$

$$(12x^2 + 8x) + (-3x - 2) = 0$$

$$4x(3x+2) - 1(3x+2) = 0$$

$$(4x-1)(3x+2) = 0$$

$$4x-1=0 \text{ or } 3x+2=0$$

$$4x=1 \text{ or } 3x=-2$$

$$x=1/4 \text{ or } x=-2/3$$

Solve the inequality algebraically.

14) $x^2 + 2x - 3 > 0$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x+3=0 \text{ or } x-1=0$$

$$x=-3 \text{ or } x=1$$

Test $x=0$:
 $0^2 + 2(0) - 3 > 0$

$$x < -3 \text{ or } x > 1$$

15) $x^2 - 3x \leq 10$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x-5=0 \text{ or } x+2=0$$

$$x=5 \text{ or } x=-2$$

$$-10 \quad -3$$

Test

$$x=0:$$

$$0^2 - 3(0) \leq 10$$

$$0 \leq 10 \checkmark$$

$$-2 \leq x \leq 5$$

$$[-2, 5]$$

16) A monthly magazine has 28,000 subscribers when it charges \$10 per annual subscription. For each \$1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

Annual Revenue = (number of subscribers)(subscription price)

$$R(x) = (28,000 - 2000x)(10 + x)$$

$$R(x) = (-2000x + 28,000)(x + 10)$$

$$R(x) = -2000(x-14)(x+10)$$

$$0 = -2000(x-14)(x+10)$$

$$0 \neq -2000 \quad x-14=0 \text{ or } x+10=0$$

$$x=14 \text{ or } x=-10$$

Find the average:

$$\frac{14 + (-10)}{2} = \frac{4}{2} = 2$$

Each subscription should be \$2 + \$10 = \$12

Max annual revenue:

$$R(x) = (28,000 - 2000(2))(10 + 2)$$

$$R(x) = (24,000)(12) = \$288,000$$

17) The height y (in feet) of a baseball t seconds after it is hit is given by the function $y = -16t^2 + 80t + 2$.

a) Find the maximum height of the baseball. TWO WAYS TO SOLVE

completing the square: $(-5)^2 = \frac{25}{4}$

$$y-2 = -16t^2 + 80t$$

$$y-2 = -16(t^2 - 5t)$$

$$y-2 - 16(\frac{25}{4}) = -16(t^2 - 5t + \frac{25}{4})$$

$$y-2 - \frac{400}{4} = -16(t-5)^2$$

$$y-102 = -16(t-5)^2$$

$$y = -16(t-5)^2 + 102$$

max height:
 (102 feet)

vertex:
 (5, 102)

finding vertex in standard form:

$$x = \frac{-b}{2a} = \frac{-80}{2(-16)} = \frac{-80}{-32} = \frac{80}{32} = \frac{5}{2}$$

$$y = -16(\frac{5}{2})^2 + 80(\frac{5}{2}) + 2 = -16(\frac{25}{4}) + \frac{400}{2} + 2 =$$

$$-\frac{400}{4} + 200 + 2 = -100 + 200 + 2 = 102$$

102 feet