Objective

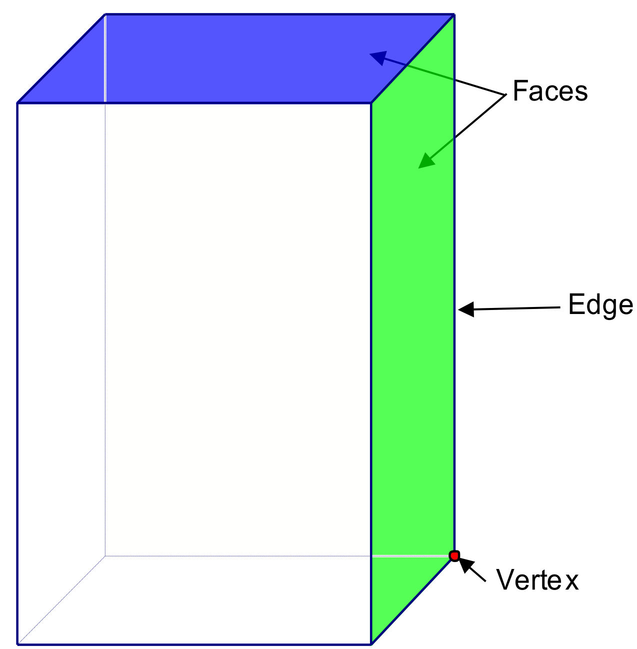
Students will be able to find the surface areas of prisms.

Polyhedra

A polyhedra is a three-dimensional figure whose surfaces are polygons.

What are some types of polyhedra you can name?

Every polyhedron has the following characteristics:



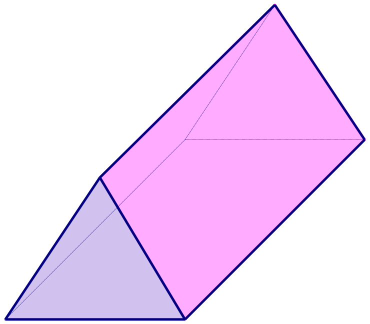
1) Faces – the polygons

2) Edge – a segment that is the intersection of two faces

3) Vertex – a point where edges intersect

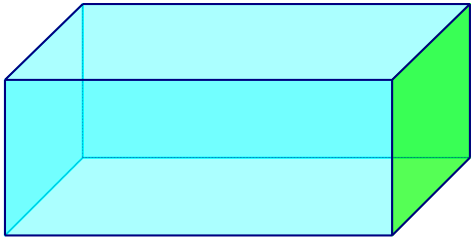
Prism

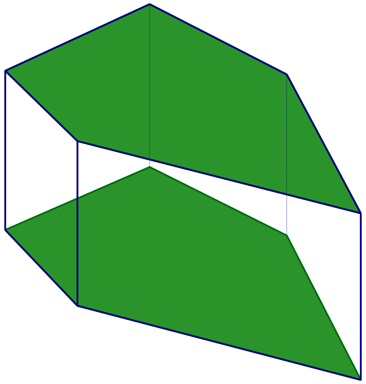
A <u>prism</u> is a polyhedron with two congruent, parallel <u>bases</u>. The remaining faces are called <u>lateral faces</u>. Each prism is specifically named for the shape of its bases.



Triangular Prism

Rectangular Prism

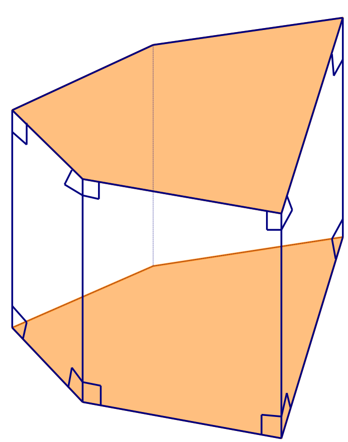


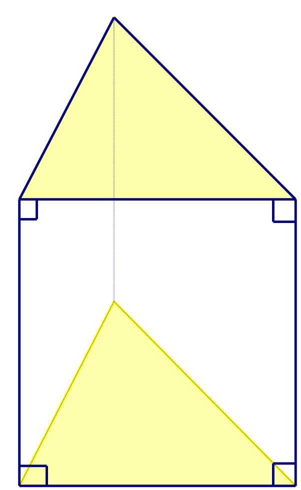


Pentagonal Prism

Right Prisms

If the lateral edges are perpendicular to the bases, then the lateral faces will be rectangles. These are called right prisms.



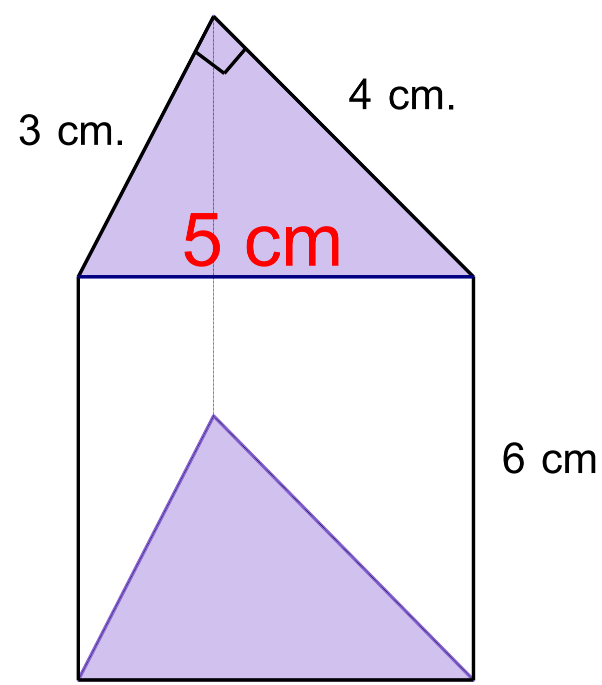


Surface Area

If we were to add up the areas of all the lateral faces of a prism, we could find the lateral surface area (L.A.). If we add the area of the two bases with lateral surface area, then we could find the total surface area (S.A.). This allows us to find how much space is on the outside of a prism, much like the perimeter of a two-dimensional figure.

Find the lateral surface area and total surface area of the prism.

To find the lateral surface area, we need to find the area of the faces.

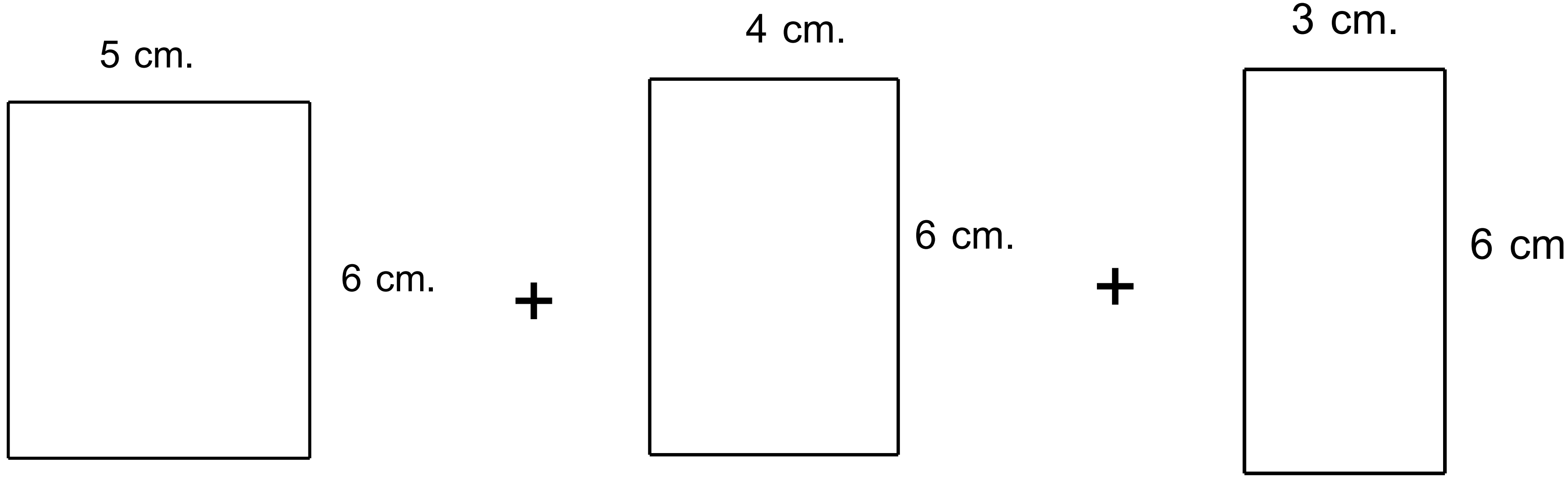


These are congruent rectangles, but we do not know the length of the base.

We can find the missing side of the triangle

Now we can find the area of the rectangles:

Lateral surface area =



$$A = bh$$
$$= 5(6)$$
$$= 30cm^2$$

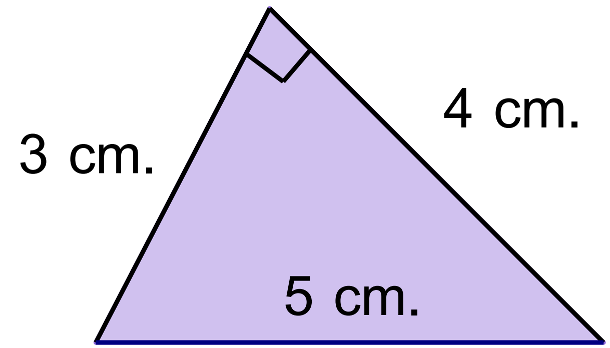
$$A = bh$$
$$= 4(6)$$
$$= 24cm^2$$

$$A = bh$$
$$= 3(6)$$
$$= 18cm^2$$

L.A = 30 + 24 + 18 = 72 cm^2

Now, we know that the bases are congruent in all prisms, so we just need the area of one triangle.

$$A = \frac{1}{2}bh \ = \frac{1}{2}(4)(3) \ = 6cm^2$$



To find the total surface area, we add two bases to the lateral area:

$$\begin{aligned} S.A. &= L.A. + 2(6) \\ &= 72 + 2(6) \\ &= 84cm^2 \end{aligned}$$

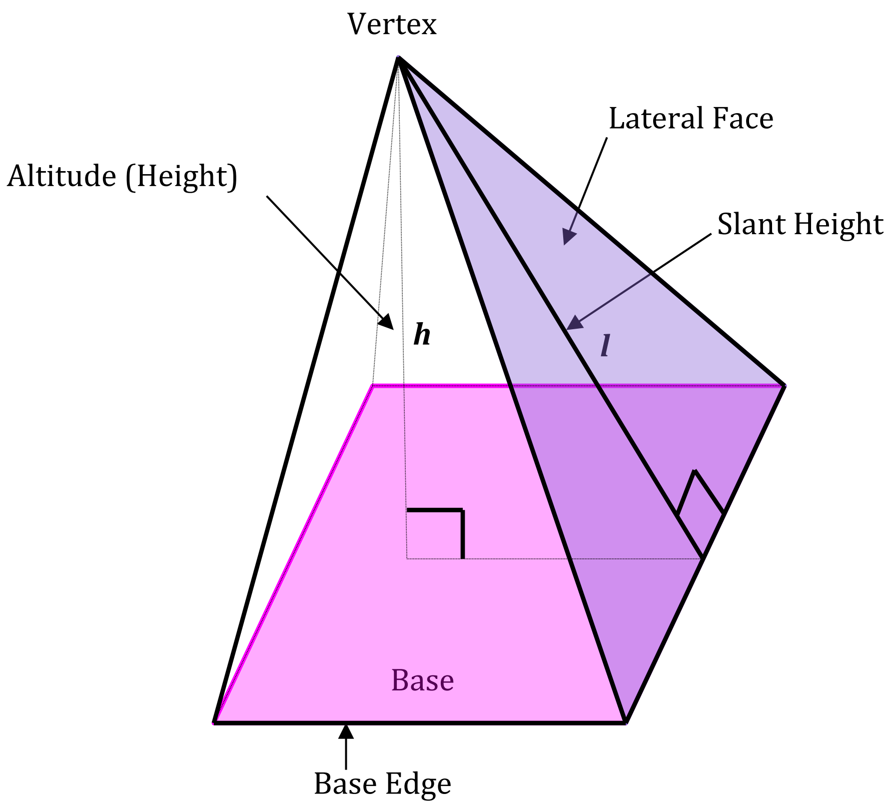
Objective

Students will be able to find the surface areas of pyramids.

Surface Area (12.1-12.3) Quiz on Friday!

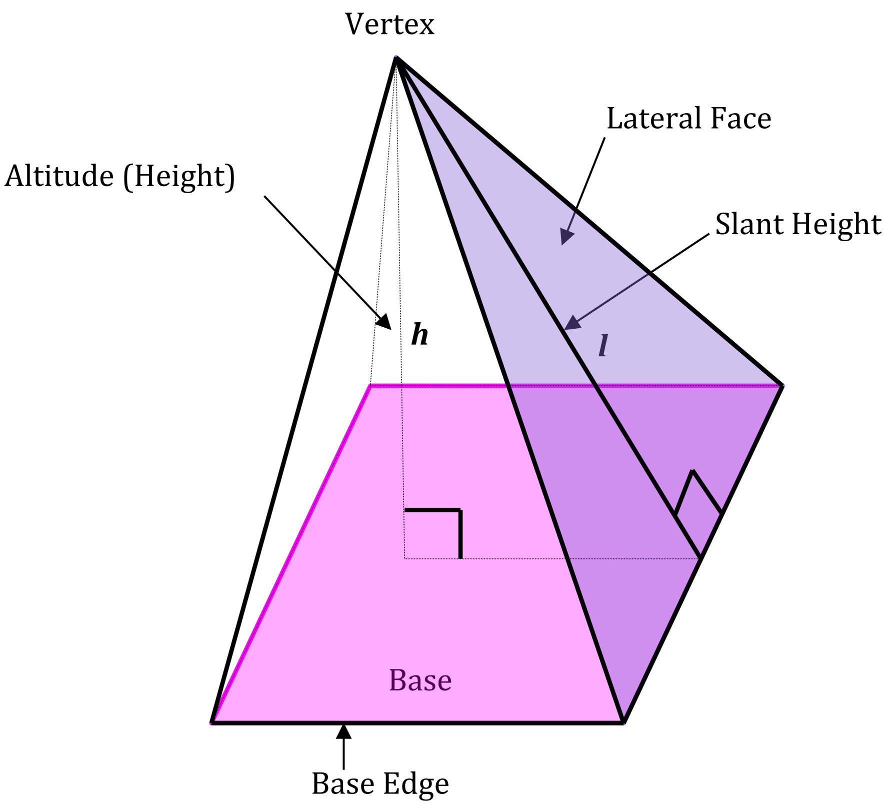
Bring an orange to class on Thursday!

Pyramids



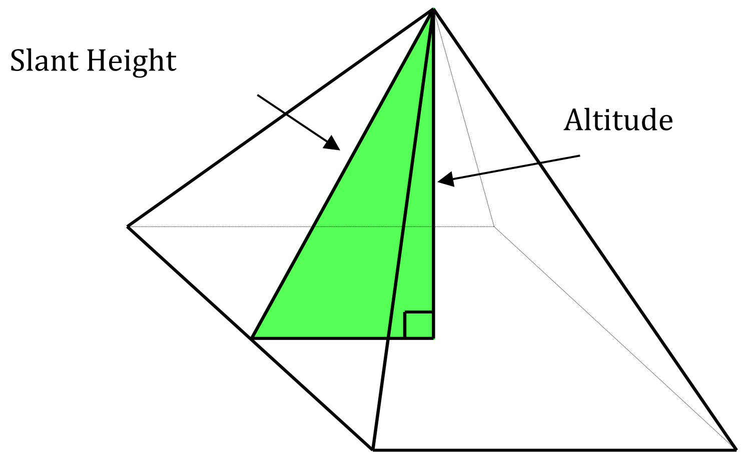
A pyramid is a polyhedron in which one face (the base) can be any polygon and the other faces (the lateral faces) are triangles that meet at a common vertex. You can name a pyramid by the shape of its base. The altitude of the pyramid is the perpendicular segment from the vertex to the base. The length of the altitude is the height, *h*, of the pyramid.

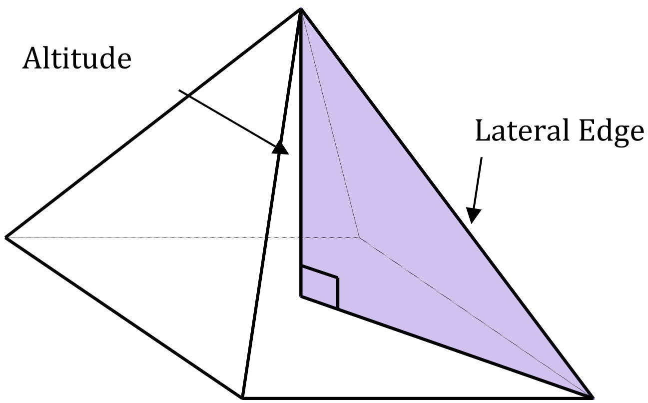
Regular Pyramids



A <u>regular pyramid</u> is a pyramid whose base is a regular polygon. The lateral faces are congruent isosceles triangles. The <u>slant height</u>, *l*, is the length of the altitude of a lateral face of the pyramid. You can make the assumption that all pyramids we work with are regular.

Surface Area



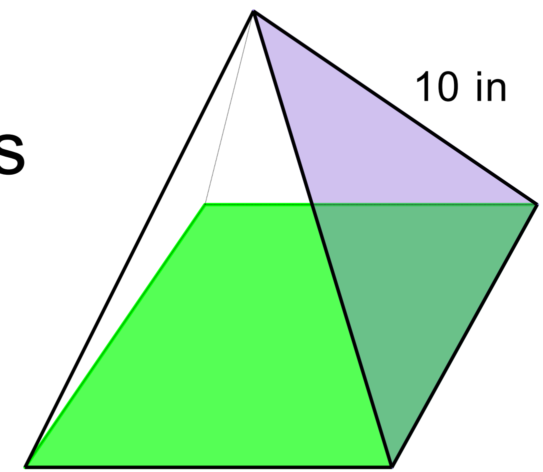


The altitude and a slant height determine a right triangle. Also, the altitude and a lateral edge determine a right triangle.

To find the <u>lateral surface area</u>, we add up the area of the triangles. To find the <u>total surface area</u>, we add the area of the base to the lateral surface area.

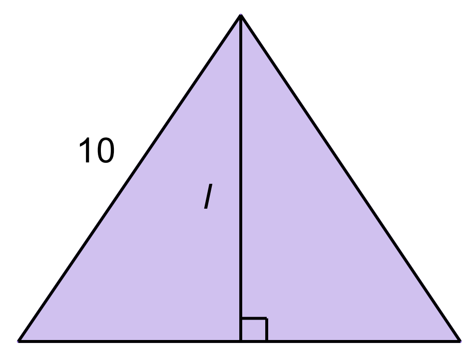
Find the lateral area and surface area of the pyramid.

The lateral area is the sum of the areas of the four congruent isosceles triangles.



To find the area, we have to draw in the slant height.

$$6^2 + l^2 = 10^2 \qquad l = 8$$



The area of one triangle is:

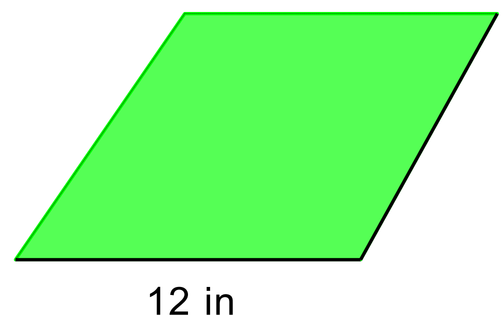
$$A = \frac{1}{2}bh \quad = \frac{1}{2}(12)(8) \quad = 48in^2$$

So, to find the lateral area, add the area of all four congruent triangles.

$$L.A. = 4(48) \quad = 192in^2$$

Now, to find the total surface area, we need to add the area of the base. Since the base is a square, the area is:

$$A = bh \quad = 12(12) \quad = 144in^2$$



Therefore, the surface area is:

$$S.A. = L.A. + A_{base}$$

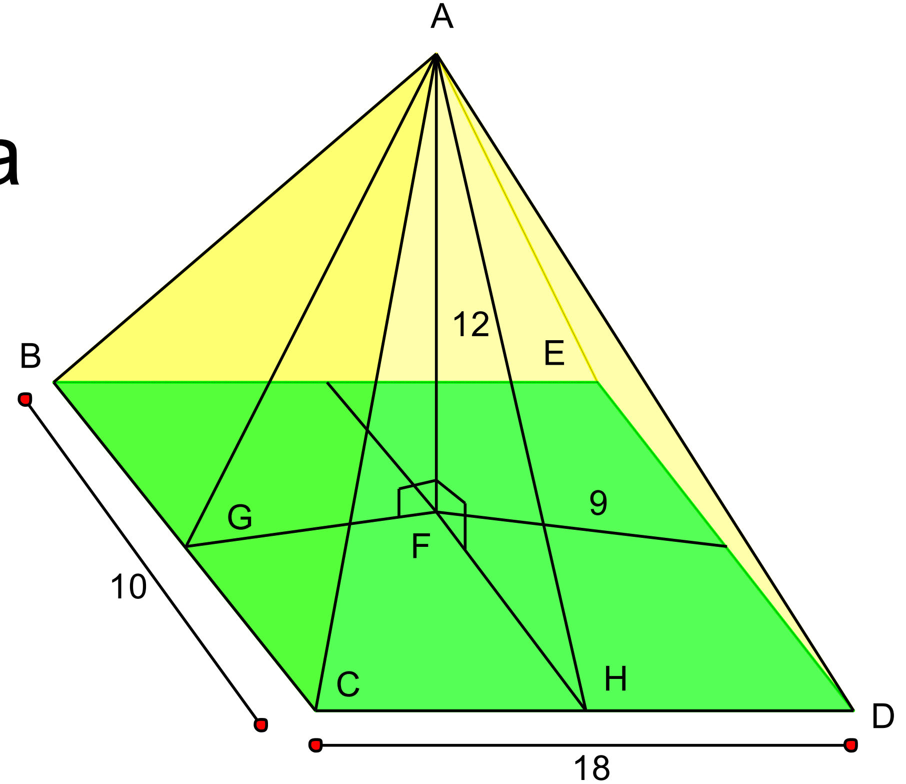
$$= 192 + 144 \quad = 336in^2$$

The base of pyramid ABCDE is 10 by 18. The altitude is 12. The lateral edges are congruent.

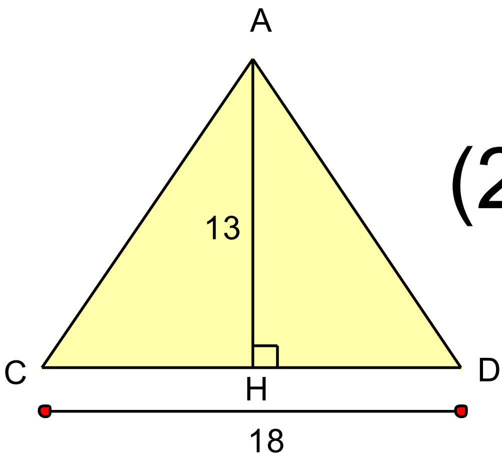
a) Why is ABCDE not a regular pyramid?

Since the base is not regular, the pyramid is not regular

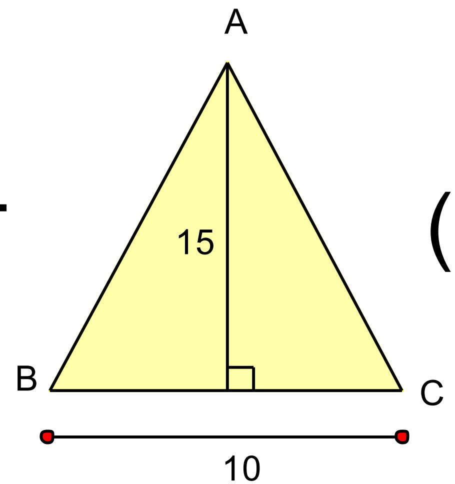
b) Find the pyramid's total surface area.



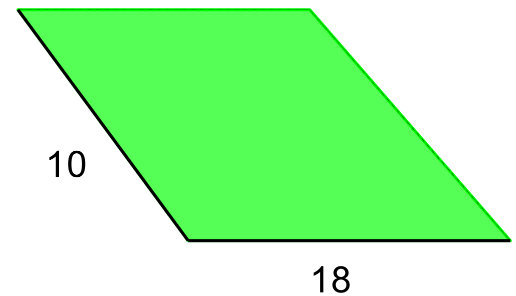
Surface Area =



(2) +



(2) +



$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(13)(18)(2)$$
$$= 234 units^2$$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(10)(15)(2)$$
$$= 150 units^2$$

$$A = bh$$
$$= 18(10)$$
$$= 180 units^2$$

$$= 234 + 150 + 180 = 564 units^2$$

Homework

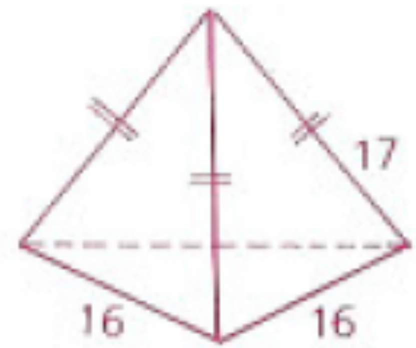
p. 567: 2, 6-8

2 The pyramid shown is regular and has a triangular base. What is

a The area of each lateral face?

b The area of the base?

c The total area?



6 A regular pyramid has a slant height of 8. The area of its square base is 25. Find its total area.

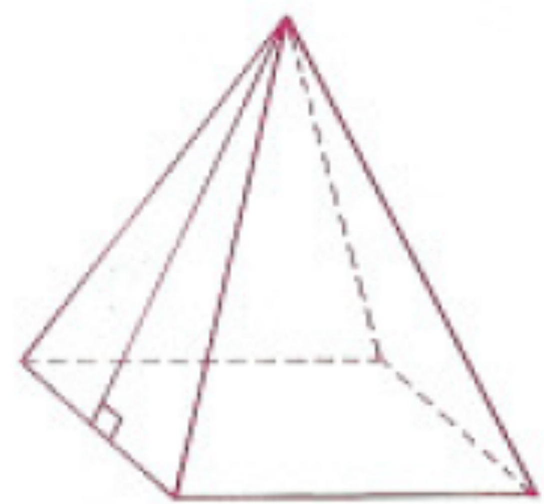
7 A regular pyramid has a slant height of 12 and a lateral edge of 15. What is

a The perimeter of the base?

b The pyramid's lateral area?

c The area of the base?

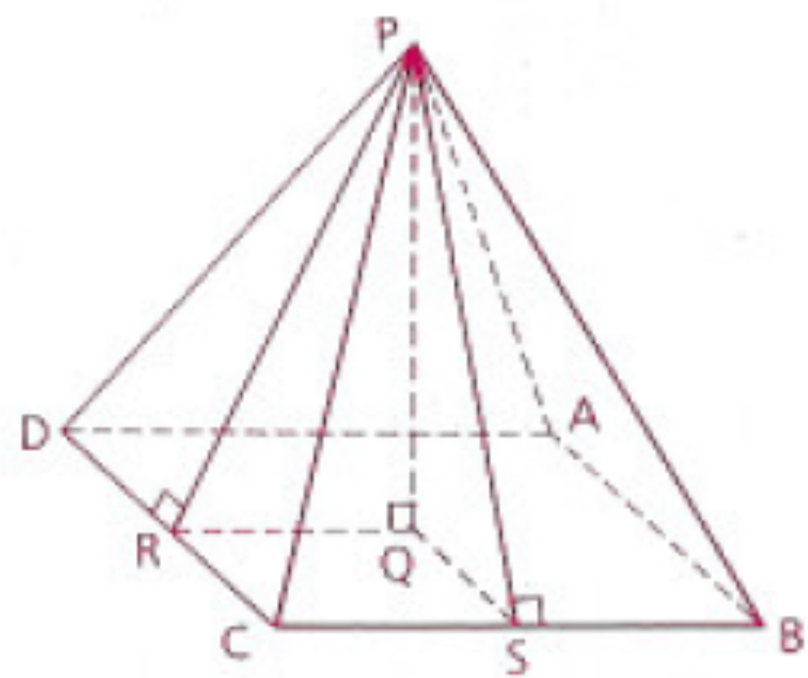
d The pyramid's total area?



8 PABCD is a regular square pyramid.

a If each side of the base has a length of 14 and the altitude (PQ) is 24, find the pyramid's lateral area and total area.

b If each slant height is 17 and the altitude is 15, find the pyramid's lateral area and total area.



Objective

Students will be able to find the area of circular solids.

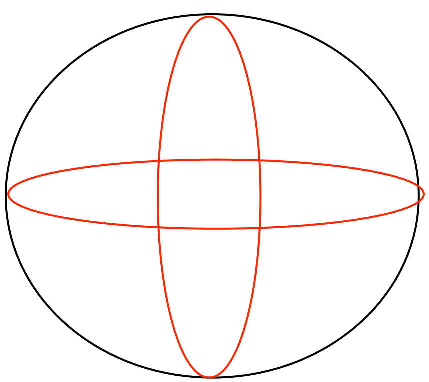
Surface Area (12.1-12.3) Quiz on Friday!

Bring an orange to class <u>tomorrow</u>!

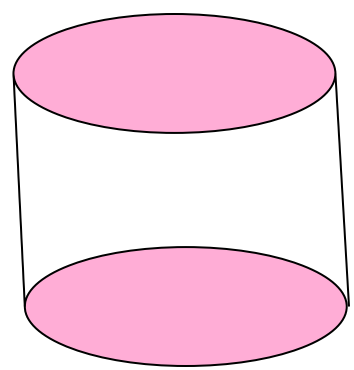
Circular Solids

There are three circular solids that are not polyhedrons, meaning they don’t have polygons on all sides.

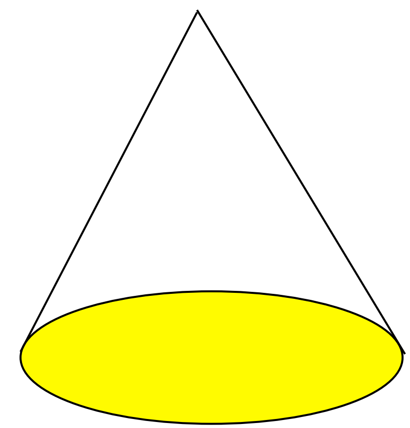
Can you name them?



Sphere

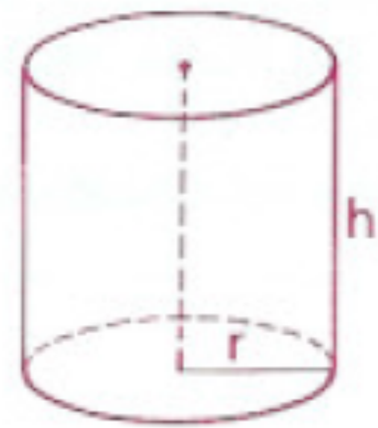


Cylinder



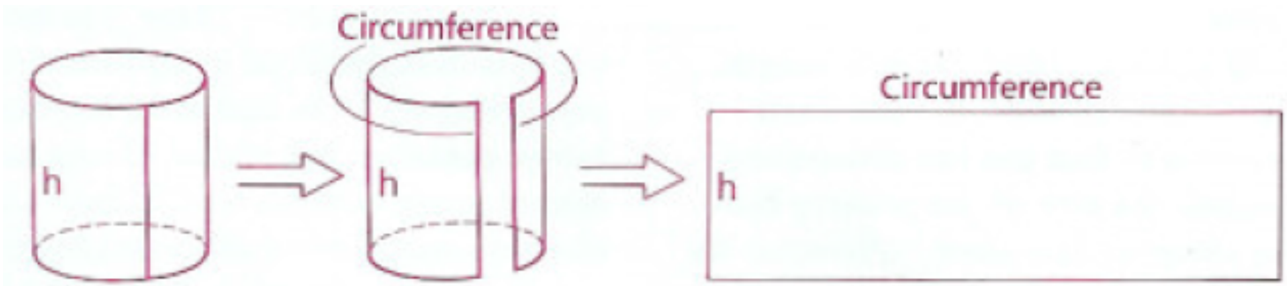
Cone

Cylinder

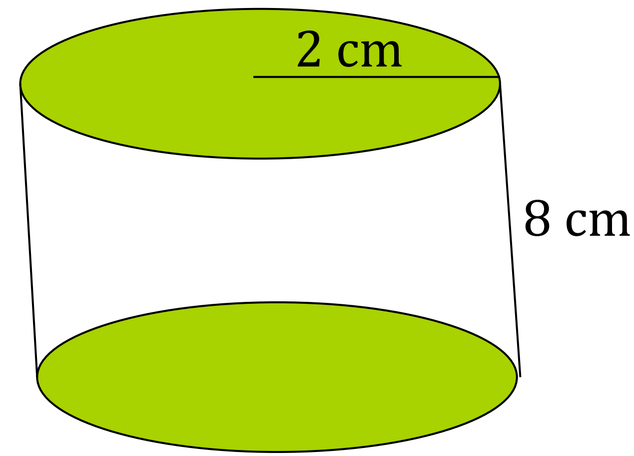


A <u>cylinder</u> is a lot like a prism, with two bases that are circles. The cylinder has an <u>altitude</u>, *h*, the perpendicular segment that joins the planes of the bases.

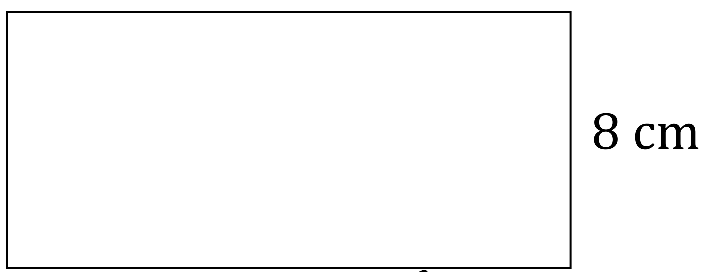
If you were to open up the "lateral face" of a cylinder it would be in the shape of a rectangle, whose bases is the circumference of the circle and the height is the altitude of the cylinder.



Find the lateral area and surface area of a cylinder with height 8 cm and radius 2 cm.



To find lateral area, we need to find the area of the rectangle:



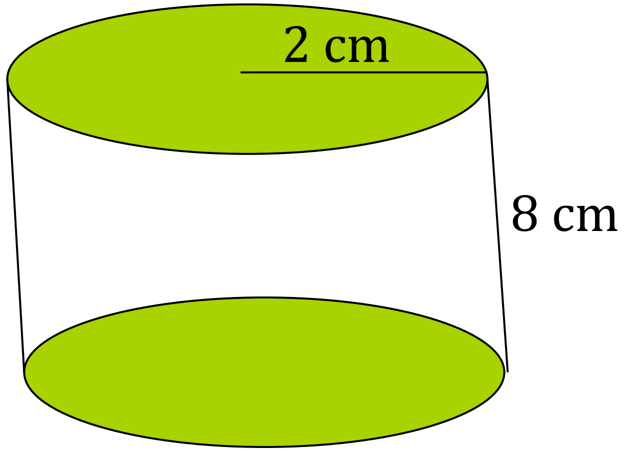
$$C = 2\pi r = 2\pi(2) = 4\pi$$

$$L.A. = bh = (4\pi)8 = 32\pi cm^2$$

To find surface area, we need to add in the area of the bases (circles):

$$A_{circle} = \pi r^2 = \pi(2)^2$$

$$= 4\pi$$



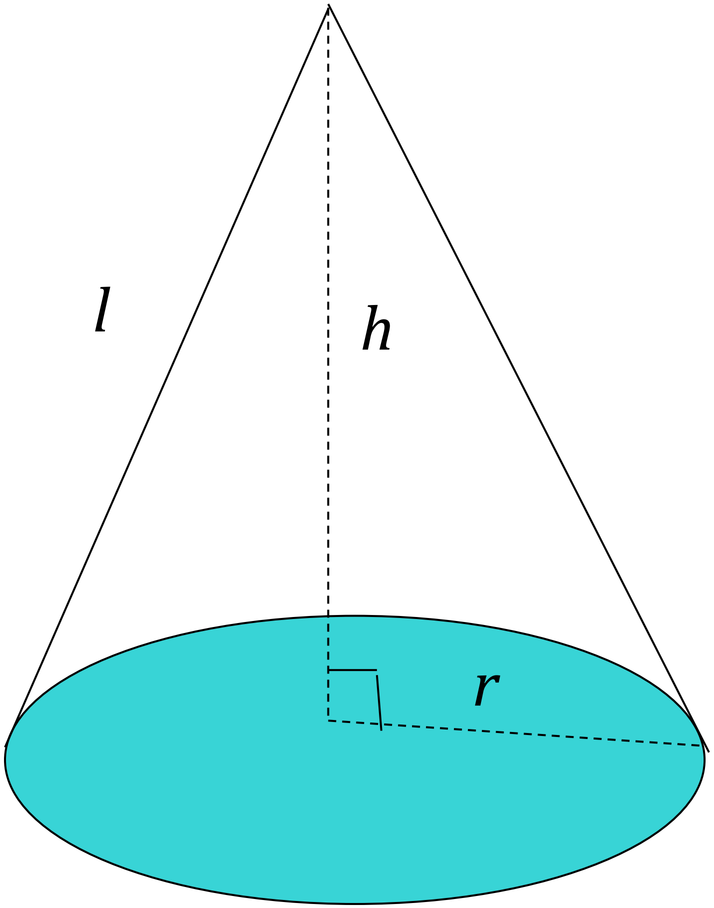
$$S.A = L.A + 2(A_{circle})$$

$$= 32\pi + 2(4\pi)$$

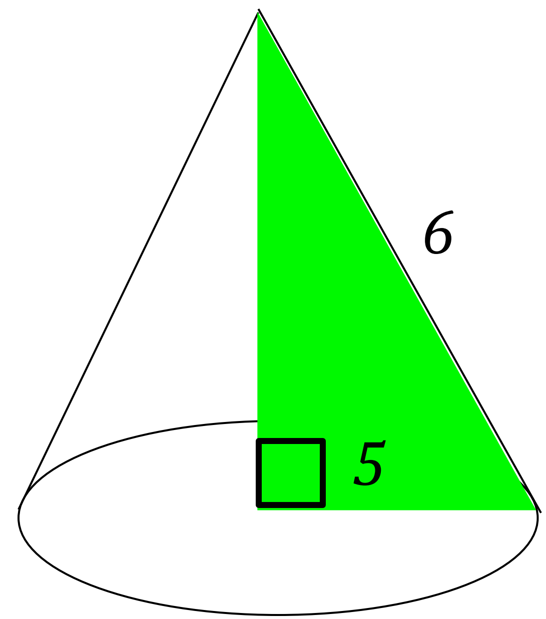
$$= 40\pi cm^2$$

Cone

A cone is very similar to a pyramid, except the base is a circle. In a right cone, the altitude is a perpendicular segment from the vertex to the center of the base. The height, *h*, is the length of the altitude. The slant height, *l*, is the distance from the vertex to a point on the edge of the base.

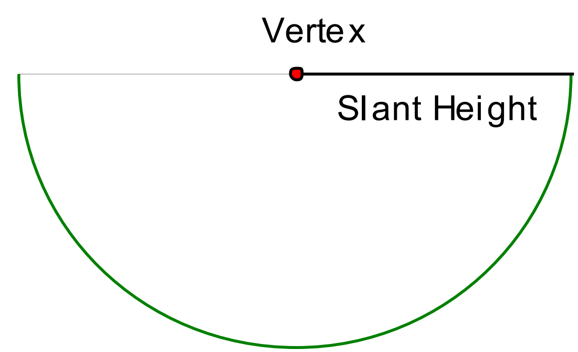


Find the lateral area and surface area of a cone with slant height 6 cm and radius 5 cm.



Just like in a pyramid, the slant height, radius, and altitude create a right triangle inside the cone.

If we open up the lateral face of this cone, you will find that it equals half a circle whose radius is equal to the slant height.



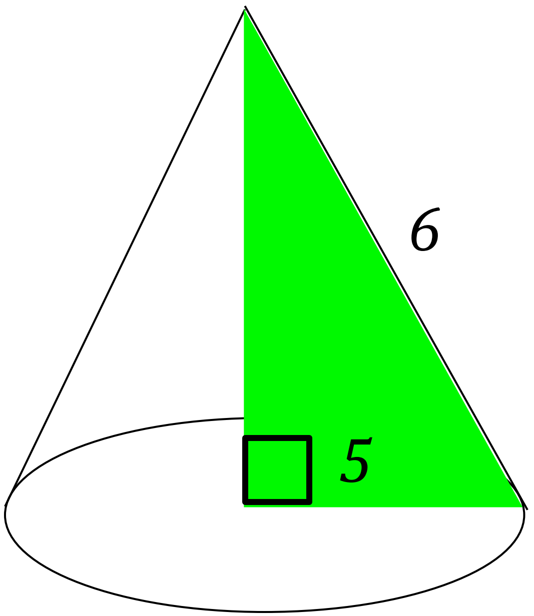
So the lateral area is…

$$L.A. = \frac{1}{2}Cl$$

$$= \frac{1}{2}(2\pi r)l$$

$$= \frac{1}{2}(10\pi)6$$

$$= 30\pi cm^2$$



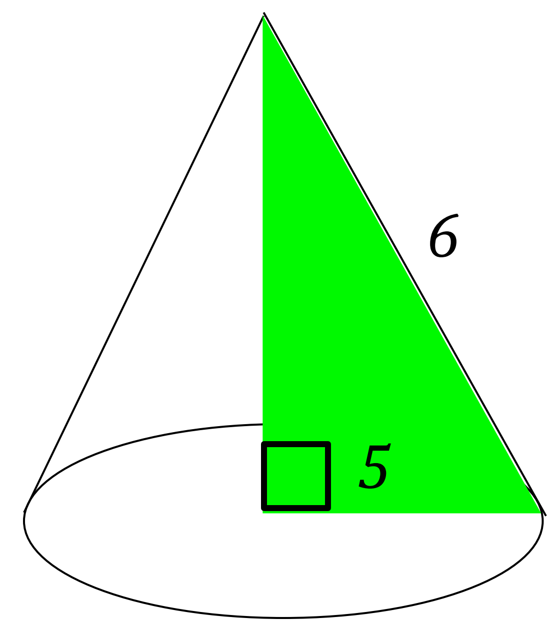
Then, to find the surface area, we just need to add the area of the base.

$$S.A. = L.A. + \pi r^2$$

$$= 30\pi + \pi(5)^2$$

$$= 30\pi + 25\pi$$

$$= 55\pi cm^2$$

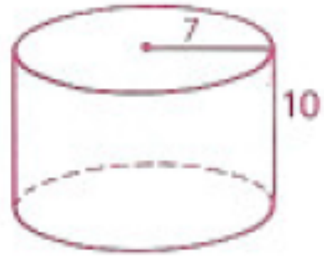


Homework p. 572: 2, 6, 7, 8, 9

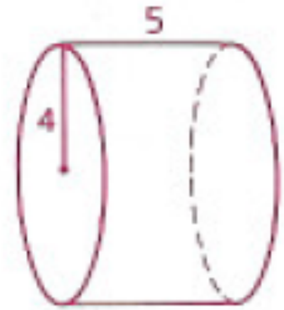
2 Find the lateral area and the total area of each solid.

a

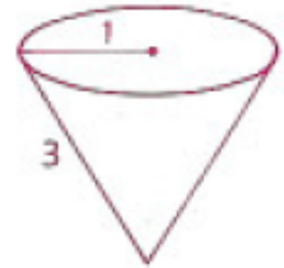
b



c



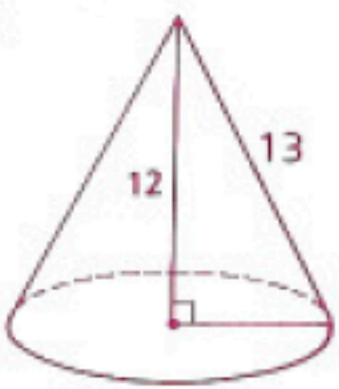
d



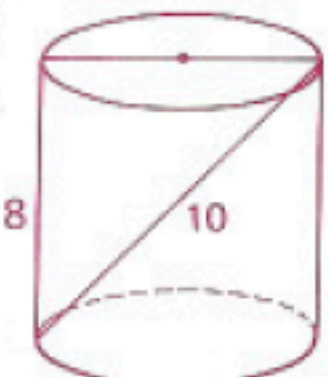
6 Find the total (including the rectangular face) surface area of a half cylinder with a radius of 5 and a height of 2.

7 Find the total area of each solid.

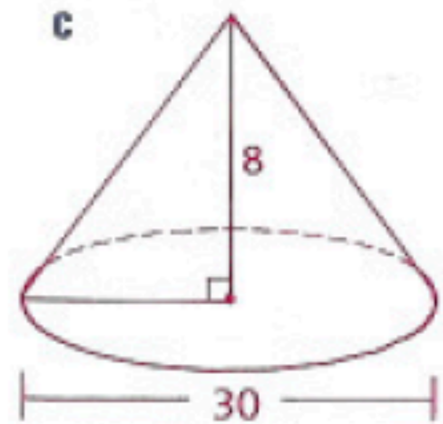
a



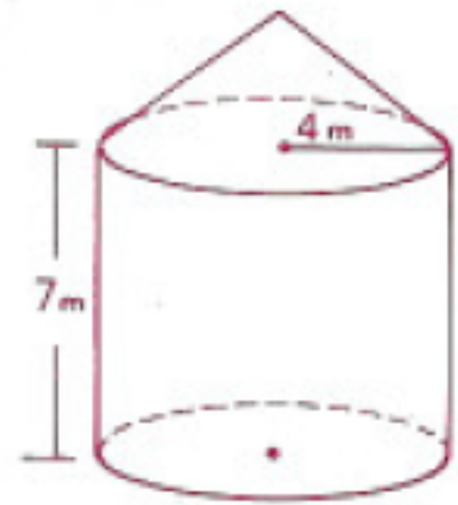
b



c



8 The total height of the tower shown is 10 m. If one liter of paint will cover an area of 10 sq m, how many 1-L cans of paint are needed to paint the entire tower? (Hint: First find the total area to be painted, using 3.14 for π.)



9 What size label (length and width) will just fit on a can 8 cm in diameter and 14 cm high?

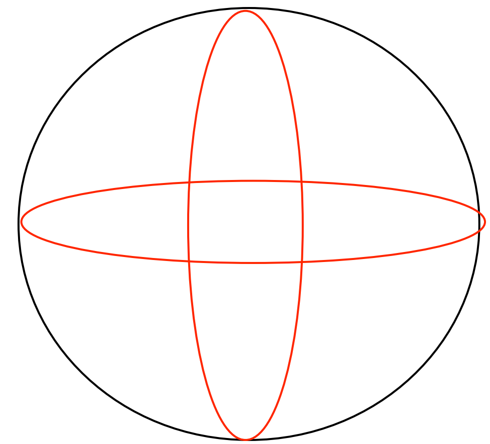
Objective

Students will be able to find the area of circular solids.

Surface Area (12.1-12.3) Quiz <u>tomorrow</u>!

Sphere

A <u>sphere</u> is a set of all points in space equidistant from a given point, its <u>center</u>. The <u>radius</u> of a sphere is a segment that as one endpoint at the center and the other endpoint on the sphere.



A sphere has no bases or faces, so there is no lateral area for a sphere.

Orange Activity

1) Trace around your sphere (orange) to create 5 circles on a separate sheet a paper. The circles should not overlap and should have the same radius as the sphere.

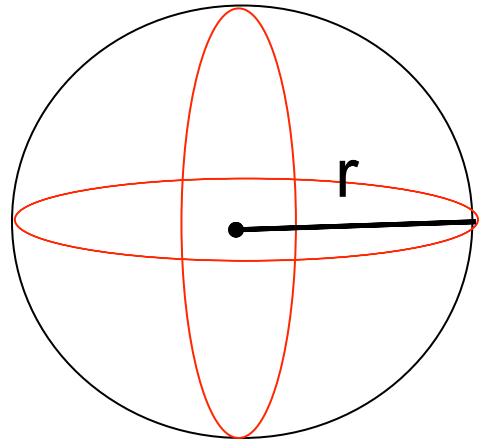
2) Peel your orange (sphere).

3) Fill in as many circles as you can, completely using the peels. Be sure that the peels are flat!

4) Based on the results of step 3, write a sentence that describes the relationship between the surface area of a sphere and the area of the circles of the sphere (orange).

5) Try to come up with an equation for the S.A. of a sphere

Surface Area of a Sphere



$$S.A. = 4\pi r^2$$

where r is the sphere's radius

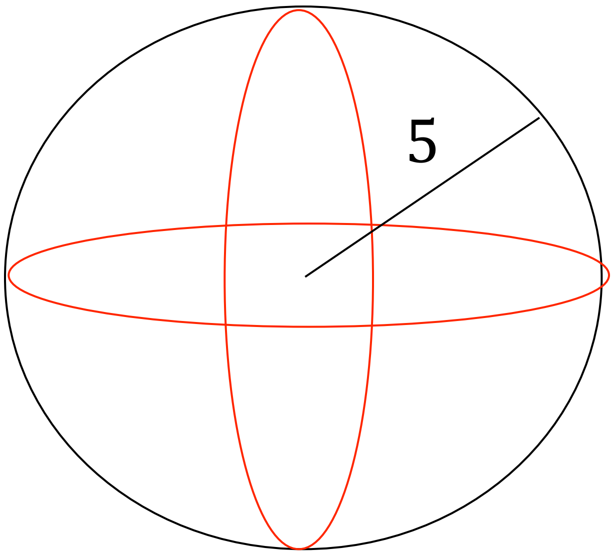
Find the surface area of a sphere with radius 5 cm.

$$S.A. = 4\pi r^2$$

$$= 4\pi(5)^2$$

$$= 4\pi 25$$

$$= 100\pi cm^2$$



Homework

p. 572: 1, 3, 4, 9, 10

1 What is the total area of a sphere having

a A radius of 7?

b A radius of 3?

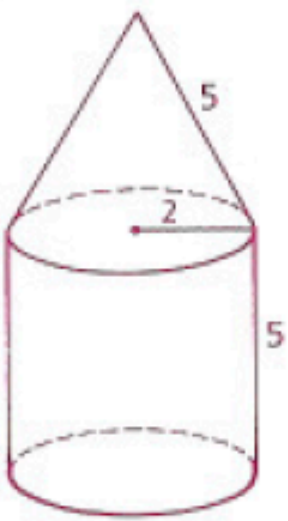
c A diameter of 6?

d A diameter of 5?

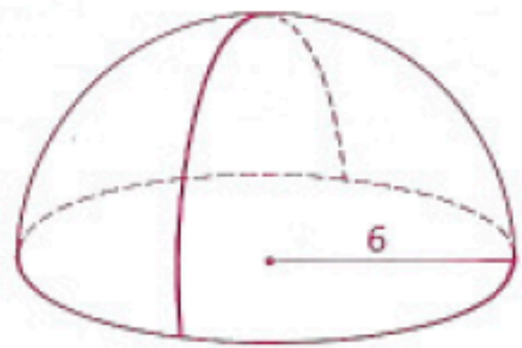
3 Find the radius of a sphere whose surface area is 144π.

4 Find the total area of each solid. (Hint: Be sure that you include only outside surfaces and that you do not miss any.)

a



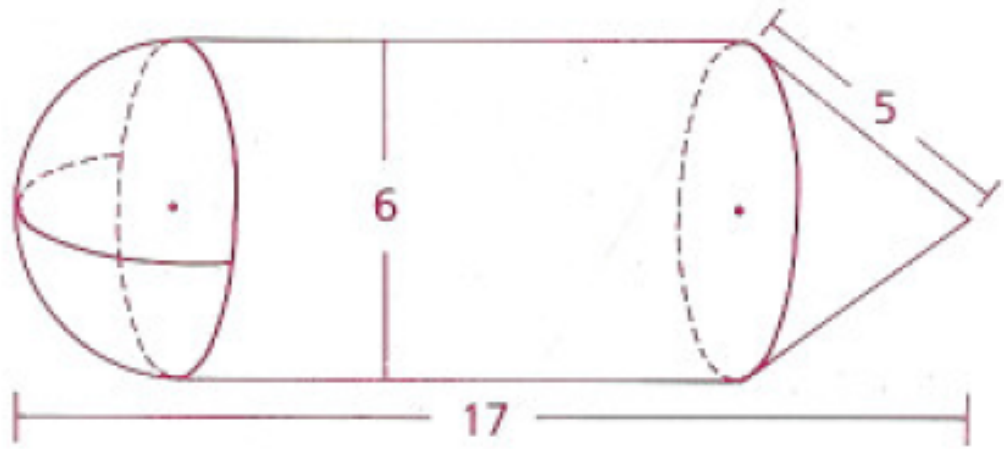
b



This is a *hemisphere* ("half sphere"). The T.A. includes the area of the circular base

9 What size label (length and width) will just fit on a can 8 cm in diameter and 14 cm high?

10 Find the total area of the solid.



Objective

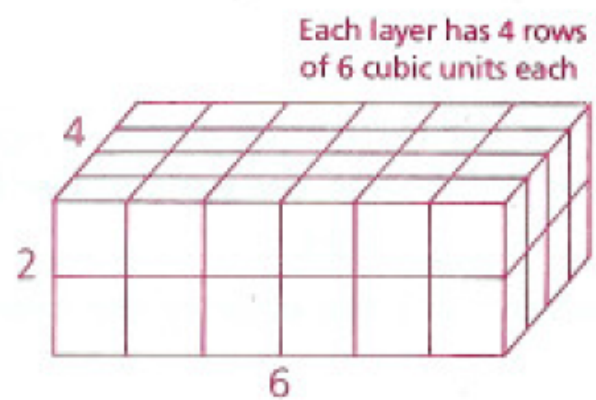
Students will be able to find the volumes of prism and cylinders.

Volume

The measure of the space enclosed by a solid is called the solid's volume.
The <u>volume</u> of a solid is the number of cubic units of space contained by the solid.

A cubic unit is the volume of a cube with edges one unit long.

One Cubic Unit



Rectangular Box

contains 48 cubic units

Volume of a Prism

$$V_{prism} = Bh$$

where *B* is the area of the base and *h* is the height (length of the altitude)

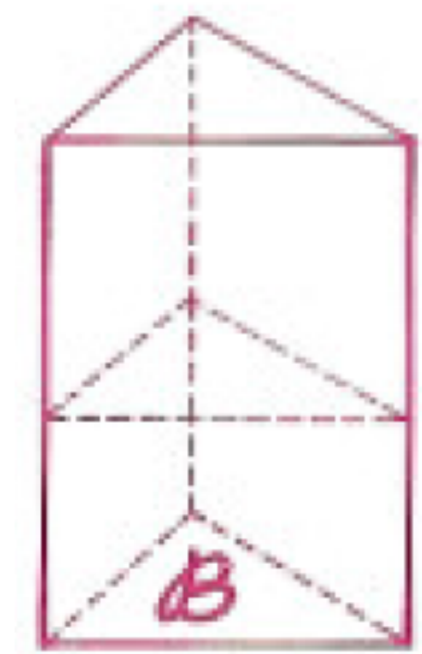
Volume of a Cylinder

$$V_{cylinder} = Bh$$

where *B* is the area of the base and *h* is the height (length of the altitude)

Cross Section of a Prism or a Cylinder

When we visualize a prism or a cylinder as a stack of sheets, all the sheets are congruent, so the area of any one of them can be substituted for *B*. Each of the sheets between the bases is an example of a <u>cross section</u> (the intersection of a solid with a plane).



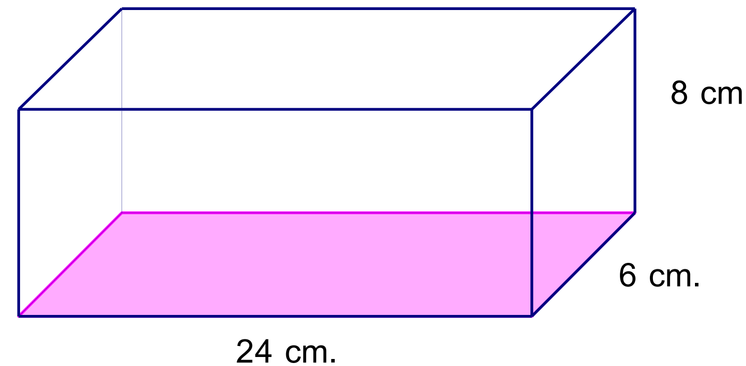
Volume of a Prism or a Cylinder

$$V = Ch$$

where *C* is the area of a cross section and *h* is the height (length of the altitude).

Find the volume of the following:

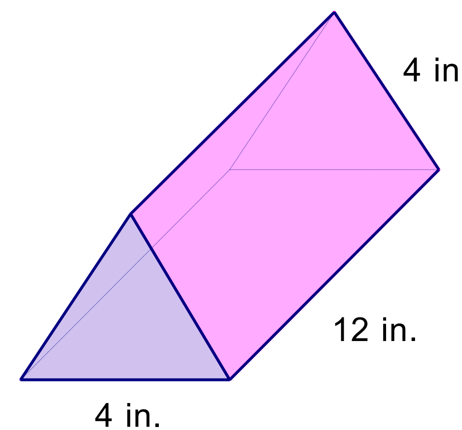
1) Rectangular Prism



$$V = Bh = (24 \cdot 6)8$$

$$= 1152cm^3$$

2) Regular Triangular Prism

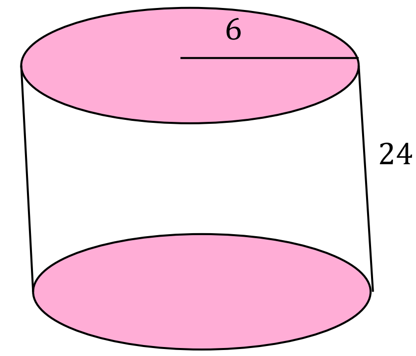


Since this is a regular prism, the bases are equilateral triangles and we can use the equilateral triangle formula.

$$V = Bh = \left(\frac{4^2\sqrt{3}}{4}\right)12 = \left(4\sqrt{3}\right)12 = 48\sqrt{3}in^3$$

Find the volume of the following:

3) Cylinder with radius of 6 and height of 24

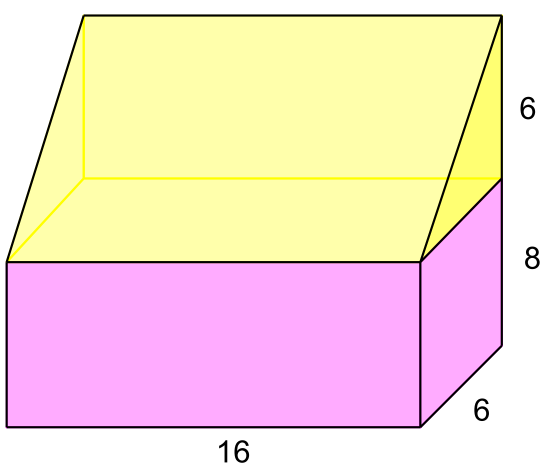


$$V = Bh = (\pi r^2)h = (\pi 6^2)24$$

$$= 864\pi units^3$$

4) Right Prism

Volume of Triangular Prism + Volume of Rectangular Prism



$$V = Bh = \left(\frac{1}{2}bh\right)h$$

$$= \left(\frac{1}{2}6(6)\right)16$$

$$= 288 units^3$$

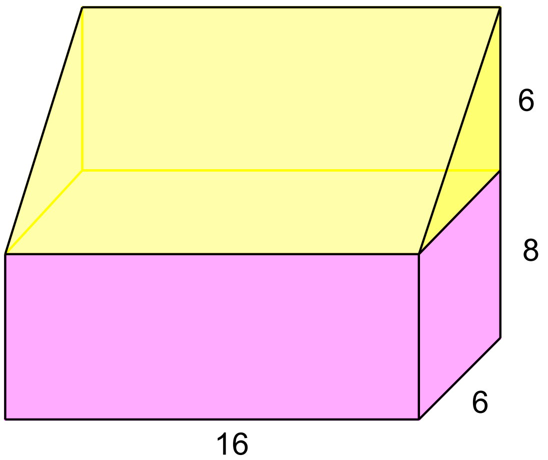
$$V = Bh = (bh)h$$

$$= (6 \cdot 8)16$$

$$= 768 units^3$$

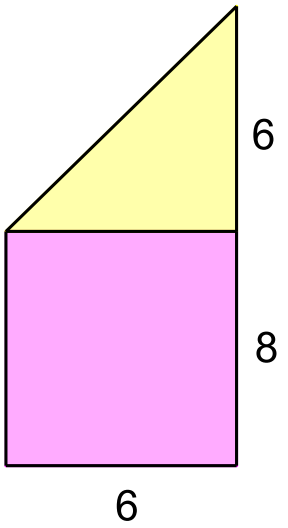
$$288 + 768 = 1056 units^3$$

4) Right Prism



Method 2: Cross Section Area

With this method, you can find the area of a cross section and multiply it by height.



$$A_{\Delta} = \frac{1}{2}bh \quad + \quad A_{sq} = bh$$

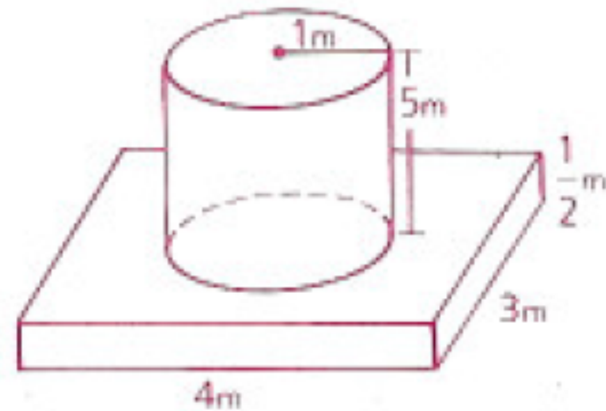
$$A_{\Delta} = \frac{1}{2}6 \cdot 6 \quad + \quad A_{sq} = 6 \cdot 8 \quad = 18 + 48 = 66$$

$$V = Ch = 66(16) = 1056 units^3$$

Homework

p. 579: 2, 7, 12, 15, 16, 18

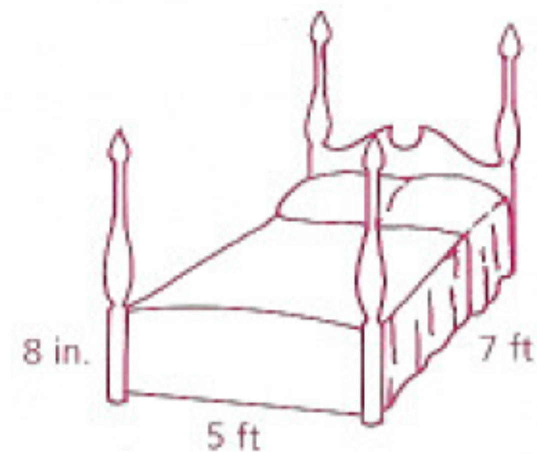
2 Find the volume of cement needed to form the concrete pedestal shown. (Leave your answer in π form.)



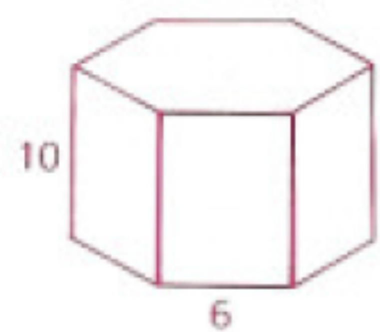
7 Traci's queen-size waterbed is 7 ft long, 5 ft wide, and 8 in. thick.

a Find the bed's volume to the nearest cubic foot.

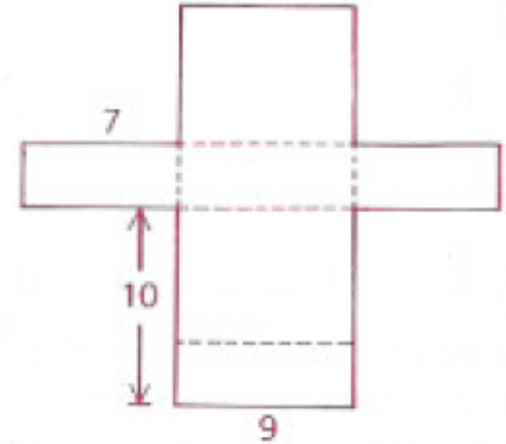
b If 1 cu ft of water weighs 62.4 lb, what is the weight of the water in Traci's bed to the nearest pound?



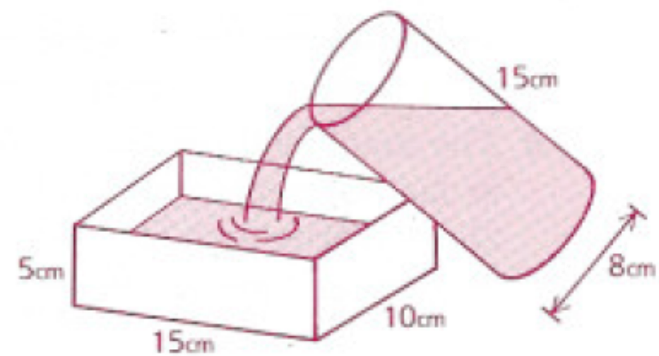
12 Find the volume and the surface area of the regular hexagonal right prism.



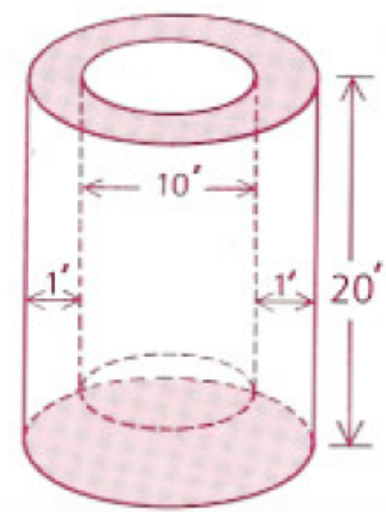
15 A rectangular container is to be formed by folding the cardboard along the dotted lines. Find the volume of this container.



16 The cylindrical glass is full of water, which is poured into the rectangular pan. Will the pan overflow?



18 A cistern is to be built of cement. The walls and bottom will be 1 ft thick. The outer height will be 20 ft. The inner diameter will be 10 ft. To the nearest cubic foot, how much cement will be needed for the job?

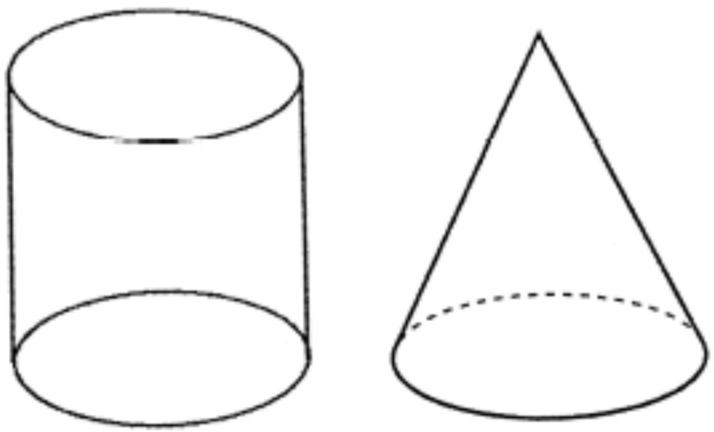


Objective

Students will be able to find the volumes of pyramids and cones.

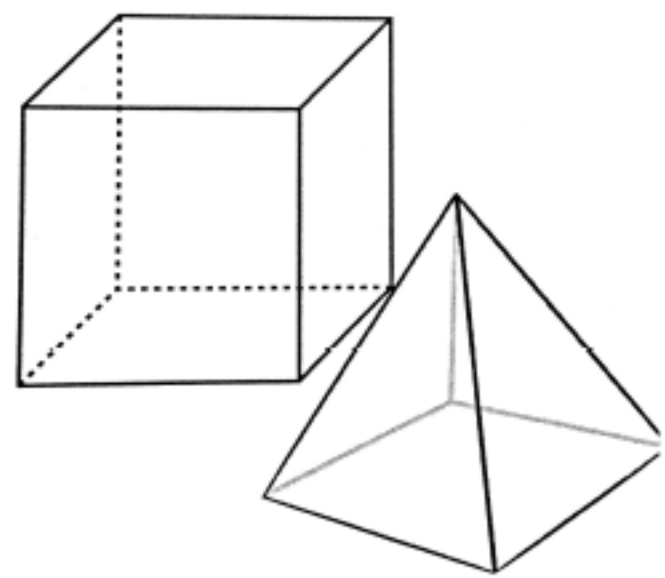
Surface Area and Volume Test on Tuesday!

Volume Comparison



Volume of a cone VS Volume of a cylinder

Volume of a pyramid VS Volume of a cube



Volume of a Pyramid

$$V_{pyramid} = \frac{1}{3}Bh$$

where *B* is the area of the base and *h* is the height (length of the altitude)

Volume of a Cone

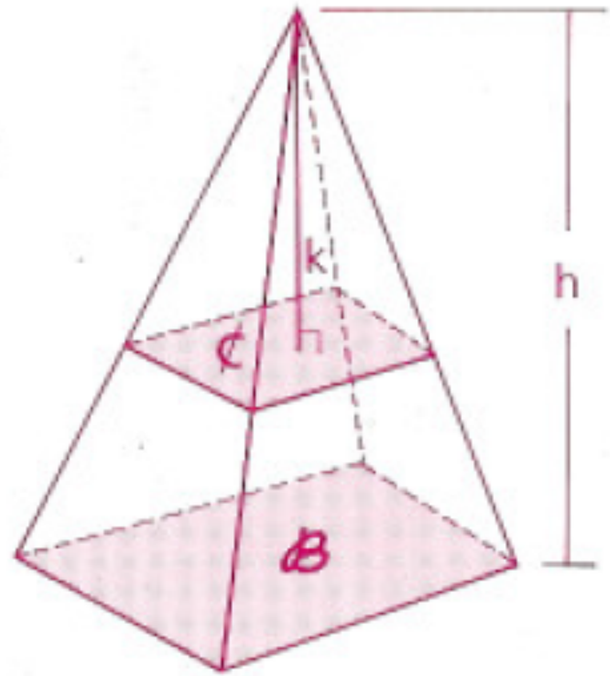
$$V_{cone} = \frac{1}{3}Bh$$

where *B* is the area of the base and *h* is the height (length of the altitude)

Cross Section of a Pyramid or a Cone

Unlike a cross section of a prism or cylinder, a cross section of a pyramid or a cone is not congruent to the figure's base, it is *similar*.

In a pyramid or a cone, the ratio of the area of a cross section to the area of the base equals the square of the ratio of the figure's respective distances from the vertex.



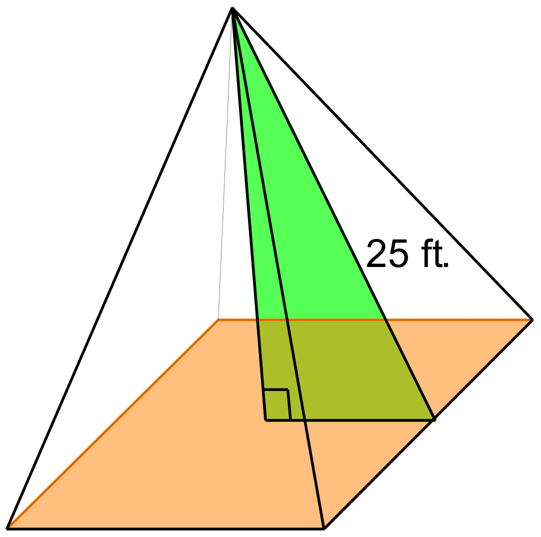
$$\frac{C}{B}=\left(\frac{k}{h}\right)^2$$

where *C* is the area of the cross section, *B* is the area of the base, *k* is the distance from the vertex to the cross section, and *h* is the height of the pyramid or cone.

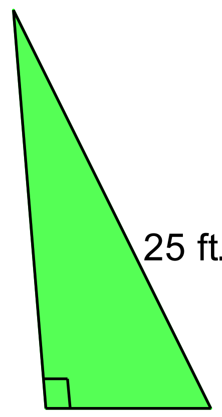
Find the volume of the following:

1) Regular Pyramid

In order to find the volume, we need to know the height of the pyramid. We can use the slant height and Pythagorean Theorem to find this.



$25^2 = 20^2 + h^2 \quad 225 = h^2 \quad 15 = h$

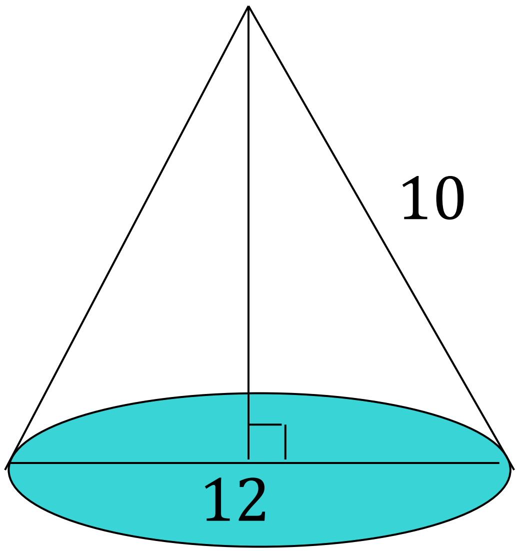


$$V_{pyramid} = \frac{1}{3}Bh \quad = \frac{1}{3}(Bh)h \quad = \frac{1}{3}(40 \cdot 40)15$$

$$= 8000\, ft^3$$

Find the volume of the following:

2) Cone with a diameter of 12 in and a slant height of 10 in.



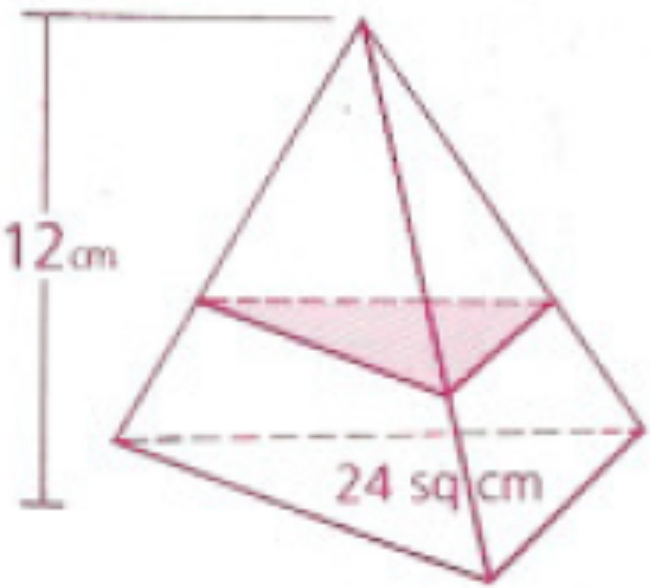
In order to find the volume, we need to know the height of the pyramid. We can use the slant height and radius Pythagorean Theorem to find this.

$10^2 = 6^2 + h^2$ $\quad$ $64 = h^2$ $\quad$ $8 = h$

$$V_{cone} = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi 6^2)8 = 96\pi in^3$$

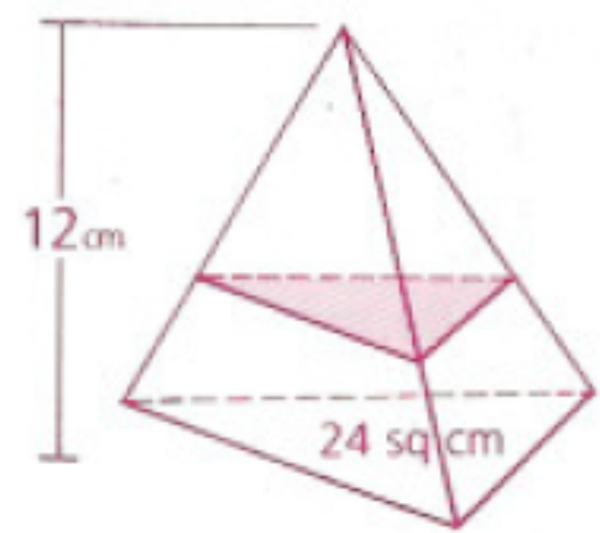
A pyramid has a base area of 24 cm^2 and a height of 12 cm. A cross section is cut 3 cm from the base.

a) Find the volume of the upper pyramid (the solid above the cross section)



b) Find the volume of the <u>frustum</u> (the solid below the cross section)

a) Find the volume of the upper pyramid

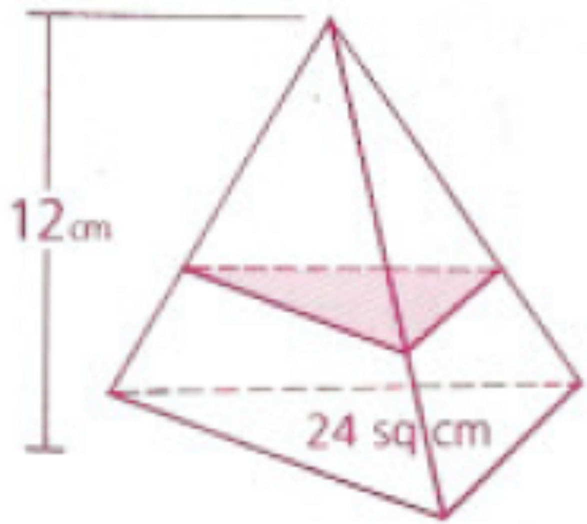


Since the cross section is 3 cm from the base, its distance, *k*, from the peak is 9 cm

$$\frac{C}{B}=\left(\frac{k}{h}\right)^2 \quad \frac{C}{24}=\left(\frac{9}{12}\right)^2 \quad \frac{C}{24}=\frac{81}{144} \quad C=\frac{27}{2}$$

$$V_{upp.pyramid}=\frac{1}{3}Ck=\frac{1}{3}\left(\frac{27}{2}\right)9=40.5cm^3$$

b) Find the volume of the frustum



To find the volume of the frustum, we subtract the volume of the upper pyramid from the volume of the whole pyramid

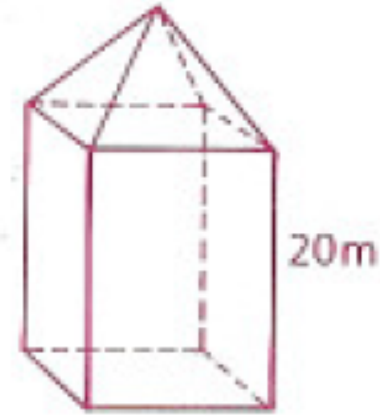
$$V_{frustum} = V_{pyramid} - V_{upp.pyramid}$$

$$= \frac{1}{3}(24)(12) - 40.5$$

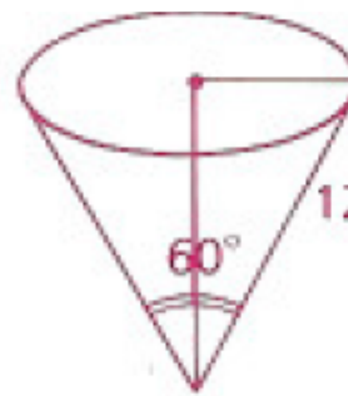
$$= 96 - 40.5 \quad = 55.5cm^3$$

Homework p. 586: 6, 8, 9, 12-14

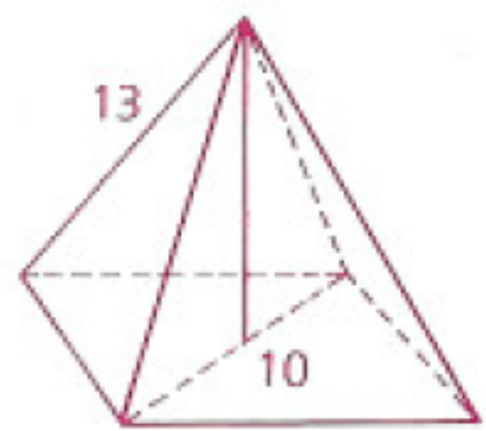
6 A tower has a total height of 24 m. The height of the wall is 20 m. The base is a rectangle with an area of 25 sq m. Find the total volume of the tower to the nearest cubic meter.



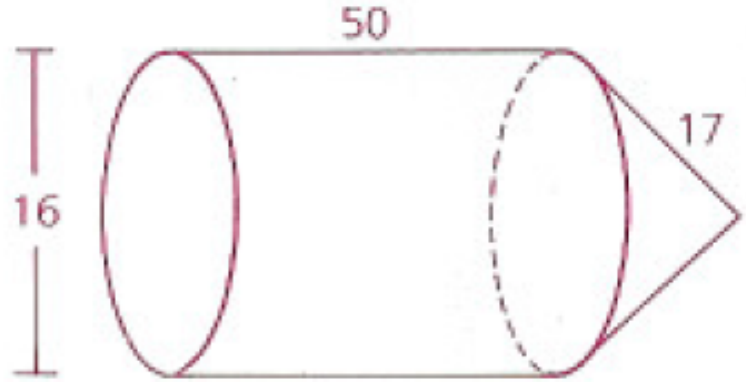
8 Find, to the nearest tenth, the volume of a cone with a 60° vertex angle and a slant height of 12.



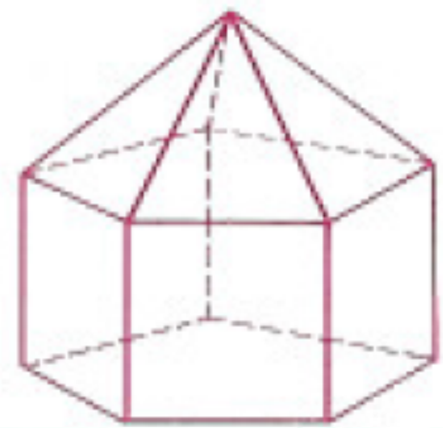
9 A pyramid has a square base with a diagonal of 10. Each lateral edge measures 13. Find the volume of the pyramid.



12 A rocket has the dimensions shown. If 60% of the space in the rocket is needed for fuel, what is the volume, to the nearest whole unit, of the portion of the rocket that is available for nonfuel items?



13 A gazebo (garden house) has a pentagonal base with an area of 60 sq m. The total height to the peak is 16 m. The height of the pyramidal roof is 6 m. Find the gazebo's total volume.



14 Use the diagram at the right to find

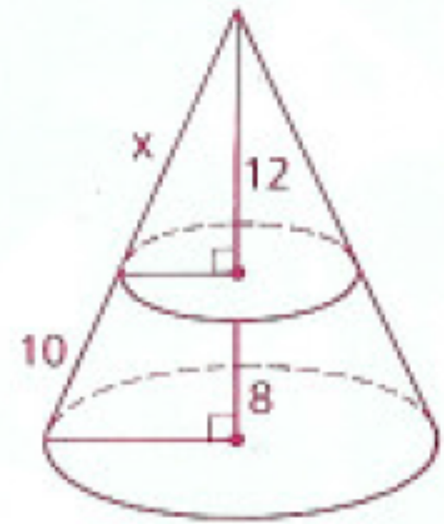
a x

b The radii of the circles

c The volume of the smaller cone

d The volume of the larger cone

e The volume of the frustum

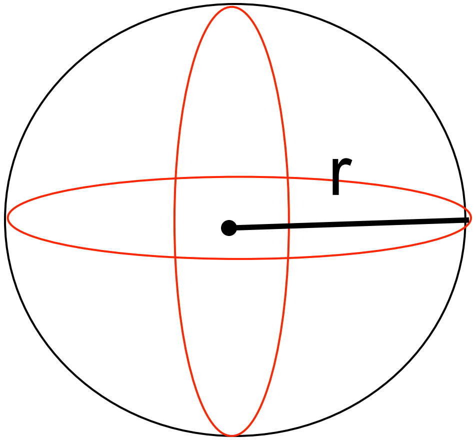


Objective

Students will be able to find the volumes of spheres.

Surface Area and Volume Test on Tuesday!

Volume of a Sphere



$$V_{sphere} = \frac{4}{3}\pi r^3$$

where *r* is the radius of the sphere

Find the volume of a hemisphere with a radius of 6 cm.

Since a hemisphere is half of a sphere, we can do the following:

$$V_{sphere} = \frac{4}{3}\pi r^3$$

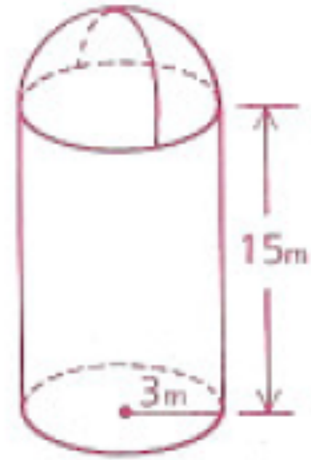
$$V_{hemisphere} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) \quad = \frac{1}{2}\left(\frac{4}{3}\pi (6)^3\right) \quad = \frac{1}{2}\left(\frac{4}{3}\pi 216\right)$$

$$= \frac{1}{2}(288\pi) \qquad = 144\pi cm^3$$

Homework

p. 590: 3, 4, 7, 9, 11

3 Find the volume of the grain silo to the nearest cubic meter.

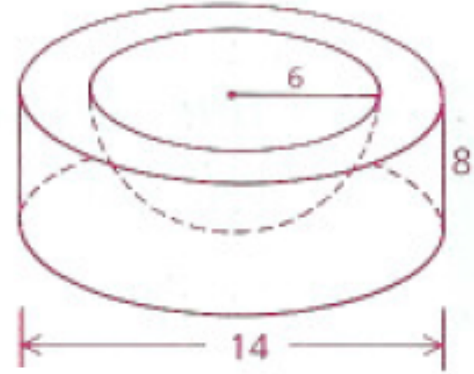


4 A plastic bowl is in the shape of a cylinder with a hemisphere cut out. The dimensions are shown.

a What is the volume of the cylinder?

b What is the volume of the hemisphere?

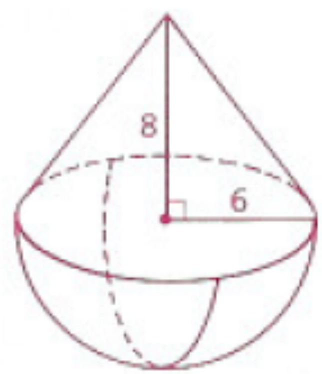
c What is the volume of plastic used to make the bowl?



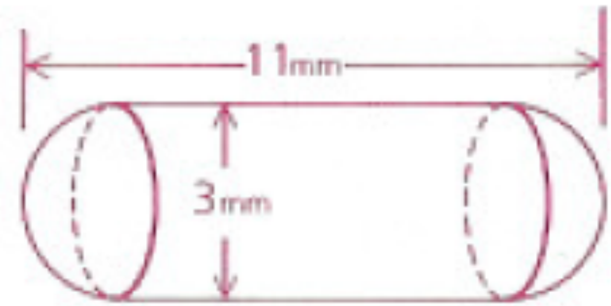
7 Given: A cone and a hemisphere as marked

Find: **a** The total volume of the solid

b The total surface area of the solid



9 A cold capsule is 11 mm long and 3 mm in diameter. Find, to the nearest cubic millimeter, the volume of medicine it contains.



11 A minisubmarine has the dimensions shown.

a What is the sub's total volume?

b Knowing the sub's surface area is important in determining how much pressure it will withstand. What is the sub's total surface area?