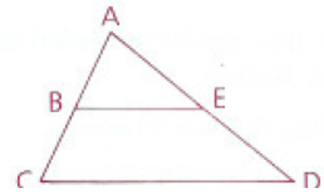
Three Theorems Involving Proportions (8.5) Name: ______________________________ Period: ________

Honors Geometry, Glawe

Side-Splitter Theorem (Theorem 65): If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally.

Given: $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$

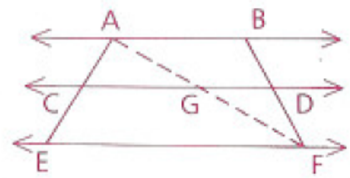
Prove: $\frac{AB}{BC} = \frac{AE}{ED}$



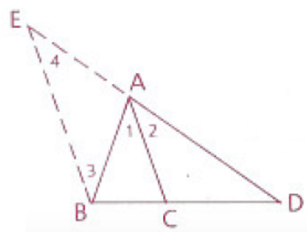
Theorem 66: If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

Conclusion: $\frac{AC}{CE} = \frac{BD}{DF}$



Angle Bisector Theorem (Theorem 67): If a ray bisects an angle of a triangle, it divides the opposite side into segments that are proportional to the adjacent sides.



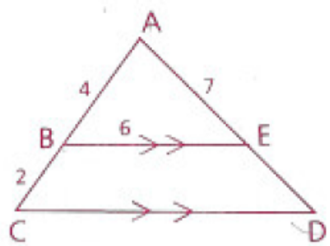
Given: $\triangle ABD$;
$\overrightarrow{AC}$ bisects $\angle BAD$.

Prove: $\frac{BC}{CD} = \frac{AB}{AD}$

Examples:

1) *Given:* $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$, *lengths as shown*

Find: **a** ED
b CD



Solution:

Be alert. In problems involving this type of figure, you may need to use both the Side-Splitter Theorem and the properties of similar triangles.

a By the Side-Splitter Theorem,

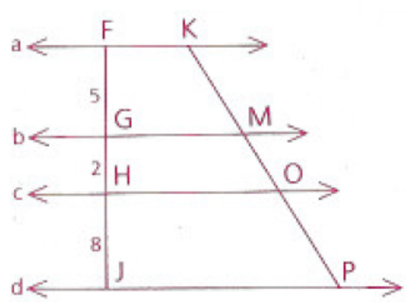
$$\frac{AB}{BC} = \frac{AE}{ED}$$
$$\frac{4}{2} = \frac{7}{ED}$$
$$\frac{2}{1} = \frac{7}{ED}$$
$$2(ED) = 7$$
$$ED = 3\tfrac{1}{2}$$

b Since the parallel segments are involved, use the fact that $\triangle ABE \sim \triangle ACD$ to write a proportion.

$$\frac{AB}{AC} = \frac{BE}{CD}$$
$$\frac{4}{4+2} = \frac{6}{CD}$$
$$\frac{2}{3} = \frac{6}{CD}$$
$$2(CD) = 18$$
$$CD = 9$$

2) Given: $a \parallel b \parallel c \parallel d$,
lengths as shown,
$KP = 24$

Find: KM



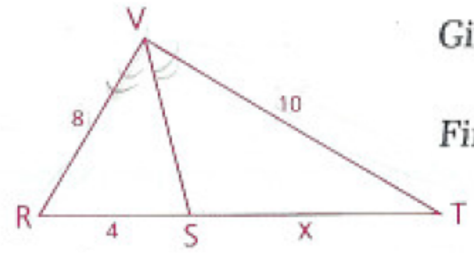
Solution:

According to Theorem 66, the ratio KM:MO:OP is equal to 5:2:8. Therefore, we let $KM = 5x$, $MO = 2x$, and $OP = 8x$. Since $KP = 24$,

$$5x + 2x + 8x = 24$$
$$15x = 24$$
$$x = \frac{24}{15} = \frac{8}{5}$$

$$\text{Thus, } KM = 5\left(\frac{8}{5}\right) = 8$$

3)



Given: $\angle RVS \cong \angle SVT$, *lengths as shown*

Find: ST

Solution:

By Theorem 67,

$$\frac{VR}{VT} = \frac{RS}{ST}$$

$$\frac{8}{10} = \frac{4}{ST}$$

$$\frac{4}{5} = \frac{4}{ST}$$

$$4(ST) = 20$$

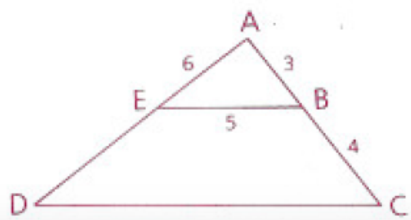
$$ST = 5$$

<u>HOMEWORK: p. 354: 1, 4, 6, 10, 20, 22</u>

For problems 1–3, see sample problem 1.

1 Given: $\overline{BE} \parallel \overline{CD}$, lengths as shown

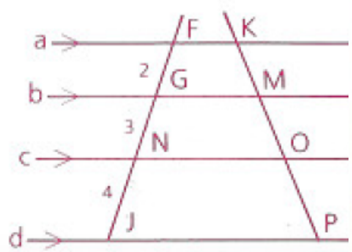
Find: **a** ED
b CD



For problems 4 and 5, see sample problem 2.

4 Given: a ∥ b ∥ c ∥ d, lengths as shown, KP = 15

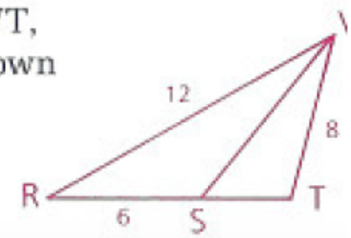
Find: KM, MO, and OP



For problems 6 and 7, see sample problem 3.

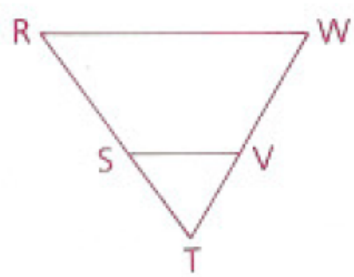
6 Given: $\angle RVS \cong \angle SVT$, lengths as shown

Find: ST



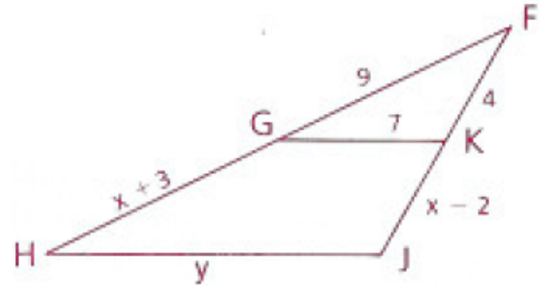
10 Given: $\overleftrightarrow{SV} \parallel \overleftrightarrow{RW}$, RW = 15, RS = 10, ST = 3, WV = 8

Find: SV and VT



20 Given: $\overleftrightarrow{GK} \parallel \overleftrightarrow{HJ}$, lengths as shown

Find: The perimeter of $\triangle HJF$



22 Given: $\overleftrightarrow{VS} \parallel \overleftrightarrow{MR}$, TV = 12, VM = 8, TS = 15, SR = TW = TX

Find: XP